

Mathematica 11.3 Integration Test Results

Test results for the 241 problems in "4.5.2.1 $(a+b \sec)^m (c+d \sec)^n \cdot m^n$ "

Problem 5: Result more than twice size of optimal antiderivative.

$$\int (a + a \operatorname{Sec}[e + f x])^2 (c - c \operatorname{Sec}[e + f x]) dx$$

Optimal (type 3, 55 leaves, 4 steps):

$$\frac{a^2 c x}{2 f} + \frac{a^2 c \operatorname{ArcTanh}[\operatorname{Sin}[e + f x]]}{2 f} - \frac{c (2 a^2 + a^2 \operatorname{Sec}[e + f x]) \operatorname{Tan}[e + f x]}{2 f}$$

Result (type 3, 141 leaves):

$$-\frac{1}{16 f} a^2 c (-1 + \operatorname{Cos}[e + f x]) (1 + \operatorname{Cos}[e + f x])^2 \operatorname{Csc}\left[\frac{1}{2} (e + f x)\right]^2 \operatorname{Sec}\left[\frac{1}{2} (e + f x)\right]^4 \\ \operatorname{Sec}[e + f x] \left(\operatorname{Cos}[e + f x] \left(2 e + 2 f x - \operatorname{Log}[\operatorname{Cos}\left[\frac{1}{2} (e + f x)\right] - \operatorname{Sin}\left[\frac{1}{2} (e + f x)\right]] + \operatorname{Log}[\operatorname{Cos}\left[\frac{1}{2} (e + f x)\right] + \operatorname{Sin}\left[\frac{1}{2} (e + f x)\right]] \right) - (1 + 2 \operatorname{Cos}[e + f x]) \operatorname{Tan}[e + f x] \right)$$

Problem 6: Result more than twice size of optimal antiderivative.

$$\int \frac{(a + a \operatorname{Sec}[e + f x])^2}{c - c \operatorname{Sec}[e + f x]} dx$$

Optimal (type 3, 56 leaves, 8 steps):

$$\frac{a^2 x}{c} - \frac{a^2 \operatorname{ArcTanh}[\operatorname{Sin}[e + f x]]}{c f} - \frac{4 a^2 \operatorname{Tan}[e + f x]}{c f (1 - \operatorname{Sec}[e + f x])}$$

Result (type 3, 169 leaves):

$$\left(a^2 \operatorname{Csc}\left[\frac{e}{2}\right] \left(-\operatorname{Cos}\left[\frac{f x}{2}\right] \left(f x + \operatorname{Log}[\operatorname{Cos}\left[\frac{1}{2} (e + f x)\right] - \operatorname{Sin}\left[\frac{1}{2} (e + f x)\right]] - \operatorname{Log}[\operatorname{Cos}\left[\frac{1}{2} (e + f x)\right] + \operatorname{Sin}\left[\frac{1}{2} (e + f x)\right]] \right) + \operatorname{Cos}\left[e + \frac{f x}{2}\right] \right. \\ \left. \left(f x + \operatorname{Log}[\operatorname{Cos}\left[\frac{1}{2} (e + f x)\right] - \operatorname{Sin}\left[\frac{1}{2} (e + f x)\right]] - \operatorname{Log}[\operatorname{Cos}\left[\frac{1}{2} (e + f x)\right] + \operatorname{Sin}\left[\frac{1}{2} (e + f x)\right]] \right) + 8 \operatorname{Sin}\left[\frac{f x}{2}\right] \operatorname{Sin}\left[\frac{1}{2} (e + f x)\right] \right) / (c f (-1 + \operatorname{Cos}[e + f x]))$$

Problem 16: Result more than twice size of optimal antiderivative.

$$\int \frac{(a + a \sec(e + f x))^3}{c - c \sec(e + f x)} dx$$

Optimal (type 3, 78 leaves, 15 steps):

$$\frac{a^3 x}{c} - \frac{4 a^3 \operatorname{ArcTanh}[\sin(e + f x)]}{c f} + \frac{8 a^3 \cot(e + f x)}{c f} + \frac{8 a^3 \csc(e + f x)}{c f} - \frac{a^3 \tan(e + f x)}{c f}$$

Result (type 3, 240 leaves):

$$\begin{aligned} & \frac{1}{4 f (c - c \sec(e + f x))} a^3 \cos(e + f x)^2 \sec\left(\frac{1}{2} (e + f x)\right)^4 \\ & (1 + \sec(e + f x))^3 \tan\left(\frac{1}{2} (e + f x)\right) \left(8 \csc\left(\frac{e}{2}\right) \sec\left(\frac{1}{2} (e + f x)\right) \sin\left(\frac{f x}{2}\right) + \left(-f x - \right. \right. \\ & \left. \left. 4 \log[\cos\left(\frac{1}{2} (e + f x)\right) - \sin\left(\frac{1}{2} (e + f x)\right)] + 4 \log[\cos\left(\frac{1}{2} (e + f x)\right) + \sin\left(\frac{1}{2} (e + f x)\right)] + \right. \right. \\ & \left. \left. \sin(f x) \right/ \left(\left(\cos\left(\frac{e}{2}\right) - \sin\left(\frac{e}{2}\right) \right) \left(\cos\left(\frac{e}{2}\right) + \sin\left(\frac{e}{2}\right) \right) \left(\cos\left(\frac{1}{2} (e + f x)\right) - \sin\left(\frac{1}{2} (e + f x)\right) \right) \right. \right. \\ & \left. \left. \left(\cos\left(\frac{1}{2} (e + f x)\right) + \sin\left(\frac{1}{2} (e + f x)\right) \right) \right) \tan\left(\frac{1}{2} (e + f x)\right) \right) \end{aligned}$$

Problem 17: Result more than twice size of optimal antiderivative.

$$\int \frac{(a + a \sec(e + f x))^3}{(c - c \sec(e + f x))^2} dx$$

Optimal (type 3, 88 leaves, 13 steps):

$$\frac{a^3 x}{c^2} + \frac{a^3 \operatorname{ArcTanh}[\sin(e + f x)]}{c^2 f} - \frac{8 a^3 \tan(e + f x)}{3 c^2 f (1 - \sec(e + f x))^2} + \frac{4 a^3 \tan(e + f x)}{3 c^2 f (1 - \sec(e + f x))}$$

Result (type 3, 177 leaves):

$$\begin{aligned} & \left(a^3 (1 + \cos(e + f x))^3 \sec\left(\frac{1}{2} (e + f x)\right)^2 \tan\left(\frac{1}{2} (e + f x)\right) \right. \\ & \left(4 \csc\left(\frac{e}{2}\right) \sec\left(\frac{1}{2} (e + f x)\right) \sin\left(\frac{f x}{2}\right) - 4 \cot\left(\frac{e}{2}\right) \sec\left(\frac{1}{2} (e + f x)\right)^2 \tan\left(\frac{1}{2} (e + f x)\right) + \right. \\ & \left. 3 \left(f x - \log[\cos\left(\frac{1}{2} (e + f x)\right) - \sin\left(\frac{1}{2} (e + f x)\right)] + \log[\cos\left(\frac{1}{2} (e + f x)\right) + \sin\left(\frac{1}{2} (e + f x)\right)] \right) \right. \\ & \left. \left. \tan\left(\frac{1}{2} (e + f x)\right)^3 \right) \right/ \left(6 c^2 f (-1 + \cos(e + f x))^2 \right) \end{aligned}$$

Problem 21: Result more than twice size of optimal antiderivative.

$$\int \frac{(c - c \sec(e + f x))^5}{(a + a \sec(e + f x))^2} dx$$

Optimal (type 3, 136 leaves, 26 steps):

$$\begin{aligned} & \frac{c^5 x}{a^2} - \frac{47 c^5 \operatorname{ArcTanh}[\sin(e + f x)]}{2 a^2 f} + \frac{13 c^5 \tan(e + f x)}{2 a^2 f} + \\ & \frac{112 c^5 \tan(e + f x)}{3 a^2 f (1 + \sec(e + f x))} - \frac{32 c^5 \tan(e + f x)}{3 f (a + a \sec(e + f x))^2} + \frac{(c^5 - c^5 \sec(e + f x)) \tan(e + f x)}{2 a^2 f} \end{aligned}$$

Result (type 3, 384 leaves):

$$\begin{aligned} & \frac{1}{96 a^2 (1 + \sec(e + f x))^2} \cos(e + f x)^3 \cot\left(\frac{1}{2}(e + f x)\right) \csc\left(\frac{1}{2}(e + f x)\right)^6 \\ & (c - c \sec(e + f x))^5 \left(-\frac{320 \cot\left(\frac{1}{2}(e + f x)\right)^2 \csc\left(\frac{1}{2}(e + f x)\right) \sec\left(\frac{e}{2}\right) \sin\left(\frac{f x}{2}\right)}{f} - \right. \\ & \frac{64 \csc\left(\frac{1}{2}(e + f x)\right)^3 \sec\left(\frac{e}{2}\right) \sin\left(\frac{f x}{2}\right)}{f} + \\ & 3 \cot\left(\frac{1}{2}(e + f x)\right)^3 \left(-4x - \frac{94 \log[\cos\left(\frac{1}{2}(e + f x)\right) - \sin\left(\frac{1}{2}(e + f x)\right)]}{f} + \right. \\ & \frac{94 \log[\cos\left(\frac{1}{2}(e + f x)\right) + \sin\left(\frac{1}{2}(e + f x)\right)]}{f} + \frac{1}{f (\cos\left(\frac{1}{2}(e + f x)\right) - \sin\left(\frac{1}{2}(e + f x)\right))^2} - \\ & \frac{1}{f (\cos\left(\frac{1}{2}(e + f x)\right) + \sin\left(\frac{1}{2}(e + f x)\right))^2} - (28 \sin(f x)) / \\ & \left. \left(f \left(\cos\left(\frac{e}{2}\right) - \sin\left(\frac{e}{2}\right) \right) \left(\cos\left(\frac{e}{2}\right) + \sin\left(\frac{e}{2}\right) \right) \left(\cos\left(\frac{1}{2}(e + f x)\right) - \sin\left(\frac{1}{2}(e + f x)\right) \right) \right. \\ & \left. \left(\cos\left(\frac{1}{2}(e + f x)\right) + \sin\left(\frac{1}{2}(e + f x)\right) \right) \right) - \frac{64 \cot\left(\frac{1}{2}(e + f x)\right) \csc\left(\frac{1}{2}(e + f x)\right)^2 \tan\left(\frac{e}{2}\right)}{f} \end{aligned}$$

Problem 22: Result more than twice size of optimal antiderivative.

$$\int \frac{(c - c \sec(e + f x))^4}{(a + a \sec(e + f x))^2} dx$$

Optimal (type 3, 102 leaves, 21 steps):

$$\begin{aligned} & \frac{c^4 x}{a^2} - \frac{6 c^4 \operatorname{ArcTanh}[\sin(e + f x)]}{a^2 f} - \frac{16 c^4 \cot(e + f x)}{a^2 f} - \\ & \frac{32 c^4 \cot(e + f x)^3}{3 a^2 f} + \frac{32 c^4 \csc(e + f x)^3}{3 a^2 f} + \frac{c^4 \tan(e + f x)}{a^2 f} \end{aligned}$$

Result (type 3, 753 leaves):

$$\begin{aligned}
& \frac{x \cos[e+f x]^2 \cot[\frac{e}{2}+\frac{f x}{2}]^4 \csc[\frac{e}{2}+\frac{f x}{2}]^4 (c-c \sec[e+f x])^4}{4 (a+a \sec[e+f x])^2} + \\
& \left(3 \cos[e+f x]^2 \cot[\frac{e}{2}+\frac{f x}{2}]^4 \csc[\frac{e}{2}+\frac{f x}{2}]^4 \right. \\
& \quad \left. \log[\cos[\frac{e}{2}+\frac{f x}{2}]-\sin[\frac{e}{2}+\frac{f x}{2}]] (c-c \sec[e+f x])^4 \right) / \left(2 f (a+a \sec[e+f x])^2 \right) - \\
& \left(3 \cos[e+f x]^2 \cot[\frac{e}{2}+\frac{f x}{2}]^4 \csc[\frac{e}{2}+\frac{f x}{2}]^4 \log[\cos[\frac{e}{2}+\frac{f x}{2}]+\sin[\frac{e}{2}+\frac{f x}{2}]] \right. \\
& \quad \left. (c-c \sec[e+f x])^4 \right) / \left(2 f (a+a \sec[e+f x])^2 \right) + \\
& \left(4 \cos[e+f x]^2 \cot[\frac{e}{2}+\frac{f x}{2}]^3 \csc[\frac{e}{2}+\frac{f x}{2}]^5 \sec[\frac{e}{2}] (c-c \sec[e+f x])^4 \sin[\frac{f x}{2}] \right) / \\
& \quad \left(3 f (a+a \sec[e+f x])^2 \right) + \\
& \left(2 \cos[e+f x]^2 \cot[\frac{e}{2}+\frac{f x}{2}] \csc[\frac{e}{2}+\frac{f x}{2}]^7 \sec[\frac{e}{2}] (c-c \sec[e+f x])^4 \sin[\frac{f x}{2}] \right) / \\
& \quad \left(3 f (a+a \sec[e+f x])^2 \right) + \\
& \left(\cos[e+f x]^2 \cot[\frac{e}{2}+\frac{f x}{2}]^4 \csc[\frac{e}{2}+\frac{f x}{2}]^4 (c-c \sec[e+f x])^4 \sin[\frac{f x}{2}] \right) / \\
& \quad \left(4 f (a+a \sec[e+f x])^2 \left(\cos[\frac{e}{2}]-\sin[\frac{e}{2}] \right) \left(\cos[\frac{e}{2}+\frac{f x}{2}]-\sin[\frac{e}{2}+\frac{f x}{2}] \right) \right) + \\
& \left(\cos[e+f x]^2 \cot[\frac{e}{2}+\frac{f x}{2}]^4 \csc[\frac{e}{2}+\frac{f x}{2}]^4 (c-c \sec[e+f x])^4 \sin[\frac{f x}{2}] \right) / \\
& \quad \left(4 f (a+a \sec[e+f x])^2 \left(\cos[\frac{e}{2}]+\sin[\frac{e}{2}] \right) \left(\cos[\frac{e}{2}+\frac{f x}{2}]+\sin[\frac{e}{2}+\frac{f x}{2}] \right) \right) + \\
& \left(2 \cos[e+f x]^2 \cot[\frac{e}{2}+\frac{f x}{2}]^2 \csc[\frac{e}{2}+\frac{f x}{2}]^6 (c-c \sec[e+f x])^4 \tan[\frac{e}{2}] \right) / \\
& \quad \left(3 f (a+a \sec[e+f x])^2 \right)
\end{aligned}$$

Problem 23: Result more than twice size of optimal antiderivative.

$$\int \frac{(c-c \sec[e+f x])^3}{(a+a \sec[e+f x])^2} dx$$

Optimal (type 3, 85 leaves, 13 steps):

$$\frac{c^3 x}{a^2} - \frac{c^3 \operatorname{ArcTanh}[\sin[e+f x]]}{a^2 f} - \frac{8 c^3 \tan[e+f x]}{3 a^2 f (1+\sec[e+f x])^2} + \frac{4 c^3 \tan[e+f x]}{3 a^2 f (1+\sec[e+f x])}$$

Result (type 3, 216 leaves):

$$\begin{aligned}
& -\frac{1}{6 a^2 f (1 + \cos[e + f x])^2} \\
& c^3 (-1 + \cos[e + f x])^3 \cot[\frac{1}{2} (e + f x)] \csc[\frac{1}{2} (e + f x)]^2 \left(3 \cot[\frac{1}{2} (e + f x)]^3\right. \\
& \left(f x + \log[\cos[\frac{1}{2} (e + f x)] - \sin[\frac{1}{2} (e + f x)]] - \log[\cos[\frac{1}{2} (e + f x)] + \sin[\frac{1}{2} (e + f x)]]\right) - \\
& 4 \cot[\frac{1}{2} (e + f x)]^2 \csc[\frac{1}{2} (e + f x)] \sec[\frac{e}{2}] \sin[\frac{f x}{2}] + \\
& \left. 4 \csc[\frac{1}{2} (e + f x)]^3 \sec[\frac{e}{2}] \sin[\frac{f x}{2}] + 4 \cot[\frac{1}{2} (e + f x)] \csc[\frac{1}{2} (e + f x)]^2 \tan[\frac{e}{2}]\right)
\end{aligned}$$

Problem 28: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{(a + a \sec[e + f x])^2 (c - c \sec[e + f x])^3} dx$$

Optimal (type 3, 98 leaves, 5 steps):

$$\begin{aligned}
& \frac{x}{a^2 c^3} + \frac{\cot[e + f x]^5 (1 + \sec[e + f x])}{5 a^2 c^3 f} - \\
& \frac{\cot[e + f x]^3 (5 + 4 \sec[e + f x])}{15 a^2 c^3 f} + \frac{\cot[e + f x] (15 + 8 \sec[e + f x])}{15 a^2 c^3 f}
\end{aligned}$$

Result (type 3, 257 leaves):

$$\begin{aligned}
& \frac{1}{30720 a^2 c^3 f} \csc[\frac{e}{2}] \csc[\frac{1}{2} (e + f x)]^5 \sec[\frac{e}{2}] \sec[\frac{1}{2} (e + f x)]^3 \\
& (360 f x \cos[f x] - 360 f x \cos[2 e + f x] - 120 f x \cos[e + 2 f x] + 120 f x \cos[3 e + 2 f x] - \\
& 120 f x \cos[2 e + 3 f x] + 120 f x \cos[4 e + 3 f x] + 60 f x \cos[3 e + 4 f x] - \\
& 60 f x \cos[5 e + 4 f x] + 200 \sin[e] - 584 \sin[f x] - 534 \sin[e + f x] + 178 \sin[2 (e + f x)] + \\
& 178 \sin[3 (e + f x)] - 89 \sin[4 (e + f x)] - 520 \sin[2 e + f x] + 248 \sin[e + 2 f x] + \\
& 120 \sin[3 e + 2 f x] + 248 \sin[2 e + 3 f x] + 120 \sin[4 e + 3 f x] - 184 \sin[3 e + 4 f x])
\end{aligned}$$

Problem 31: Result more than twice size of optimal antiderivative.

$$\int \frac{(c - c \sec[e + f x])^5}{(a + a \sec[e + f x])^3} dx$$

Optimal (type 3, 162 leaves, 29 steps):

$$\begin{aligned}
& \frac{c^5 x}{a^3} + \frac{8 c^5 \operatorname{ArcTanh}[\sin[e + f x]]}{a^3 f} + \frac{32 c^5 \cot[e + f x]}{a^3 f} + \frac{128 c^5 \cot[e + f x]^3}{3 a^3 f} + \frac{128 c^5 \cot[e + f x]^5}{5 a^3 f} - \\
& \frac{16 c^5 \csc[e + f x]}{a^3 f} + \frac{64 c^5 \csc[e + f x]^3}{3 a^3 f} - \frac{128 c^5 \csc[e + f x]^5}{5 a^3 f} - \frac{c^5 \tan[e + f x]}{a^3 f}
\end{aligned}$$

Result (type 3, 908 leaves):

$$\begin{aligned}
& - \frac{x \cos[e+f x]^2 \cot[\frac{e}{2} + \frac{f x}{2}]^6 \csc[\frac{e}{2} + \frac{f x}{2}]^4 (c - c \sec[e+f x])^5}{4 (a + a \sec[e+f x])^3} + \\
& \left(2 \cos[e+f x]^2 \cot[\frac{e}{2} + \frac{f x}{2}]^6 \csc[\frac{e}{2} + \frac{f x}{2}]^4 \right. \\
& \quad \left. \log[\cos[\frac{e}{2} + \frac{f x}{2}] - \sin[\frac{e}{2} + \frac{f x}{2}]] (c - c \sec[e+f x])^5 \right) / \left(f (a + a \sec[e+f x])^3 \right) - \\
& \left(2 \cos[e+f x]^2 \cot[\frac{e}{2} + \frac{f x}{2}]^6 \csc[\frac{e}{2} + \frac{f x}{2}]^4 \log[\cos[\frac{e}{2} + \frac{f x}{2}] + \sin[\frac{e}{2} + \frac{f x}{2}]] \right. \\
& \quad \left. (c - c \sec[e+f x])^5 \right) / \left(f (a + a \sec[e+f x])^3 \right) + \\
& \left(56 \cos[e+f x]^2 \cot[\frac{e}{2} + \frac{f x}{2}]^5 \csc[\frac{e}{2} + \frac{f x}{2}]^5 \sec[\frac{e}{2}] (c - c \sec[e+f x])^5 \sin[\frac{f x}{2}] \right) / \\
& \quad \left(15 f (a + a \sec[e+f x])^3 \right) - \\
& \left(2 \cos[e+f x]^2 \cot[\frac{e}{2} + \frac{f x}{2}]^3 \csc[\frac{e}{2} + \frac{f x}{2}]^7 \sec[\frac{e}{2}] (c - c \sec[e+f x])^5 \sin[\frac{f x}{2}] \right) / \\
& \quad \left(15 f (a + a \sec[e+f x])^3 \right) + \\
& \left(2 \cos[e+f x]^2 \cot[\frac{e}{2} + \frac{f x}{2}] \csc[\frac{e}{2} + \frac{f x}{2}]^9 \sec[\frac{e}{2}] (c - c \sec[e+f x])^5 \sin[\frac{f x}{2}] \right) / \\
& \quad \left(5 f (a + a \sec[e+f x])^3 \right) + \\
& \left(\cos[e+f x]^2 \cot[\frac{e}{2} + \frac{f x}{2}]^6 \csc[\frac{e}{2} + \frac{f x}{2}]^4 (c - c \sec[e+f x])^5 \sin[\frac{f x}{2}] \right) / \\
& \quad \left(4 f (a + a \sec[e+f x])^3 \left(\cos[\frac{e}{2}] - \sin[\frac{e}{2}] \right) \left(\cos[\frac{e}{2} + \frac{f x}{2}] - \sin[\frac{e}{2} + \frac{f x}{2}] \right) \right) + \\
& \left(\cos[e+f x]^2 \cot[\frac{e}{2} + \frac{f x}{2}]^6 \csc[\frac{e}{2} + \frac{f x}{2}]^4 (c - c \sec[e+f x])^5 \sin[\frac{f x}{2}] \right) / \\
& \quad \left(4 f (a + a \sec[e+f x])^3 \left(\cos[\frac{e}{2}] + \sin[\frac{e}{2}] \right) \left(\cos[\frac{e}{2} + \frac{f x}{2}] + \sin[\frac{e}{2} + \frac{f x}{2}] \right) \right) - \\
& \left(2 \cos[e+f x]^2 \cot[\frac{e}{2} + \frac{f x}{2}]^4 \csc[\frac{e}{2} + \frac{f x}{2}]^6 (c - c \sec[e+f x])^5 \tan[\frac{e}{2}] \right) / \\
& \quad \left(15 f (a + a \sec[e+f x])^3 \right) + \\
& \left(2 \cos[e+f x]^2 \cot[\frac{e}{2} + \frac{f x}{2}]^2 \csc[\frac{e}{2} + \frac{f x}{2}]^8 (c - c \sec[e+f x])^5 \tan[\frac{e}{2}] \right) / \\
& \quad \left(5 f (a + a \sec[e+f x])^3 \right)
\end{aligned}$$

Problem 37: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{(a + a \sec[e+f x])^3 (c - c \sec[e+f x])^2} dx$$

Optimal (type 3, 100 leaves, 5 steps):

$$\begin{aligned}
& \frac{x}{a^3 c^2} + \frac{\cot[e+f x] (15 - 8 \sec[e+f x])}{15 a^3 c^2 f} - \\
& \frac{\cot[e+f x]^3 (5 - 4 \sec[e+f x])}{15 a^3 c^2 f} + \frac{\cot[e+f x]^5 (1 - \sec[e+f x])}{5 a^3 c^2 f}
\end{aligned}$$

Result (type 3, 257 leaves):

$$\frac{1}{30720 a^3 c^2 f} \csc\left[\frac{e}{2}\right] \csc\left[\frac{1}{2}(e+f x)\right]^3 \sec\left[\frac{e}{2}\right] \sec\left[\frac{1}{2}(e+f x)\right]^5 \\ (360 f x \cos[f x] - 360 f x \cos[2 e + f x] + 120 f x \cos[e + 2 f x] - 120 f x \cos[3 e + 2 f x] - \\ 120 f x \cos[2 e + 3 f x] + 120 f x \cos[4 e + 3 f x] - 60 f x \cos[3 e + 4 f x] + \\ 60 f x \cos[5 e + 4 f x] - 200 \sin[e] - 584 \sin[f x] + 534 \sin[e + f x] + 178 \sin[2(e + f x)] - \\ 178 \sin[3(e + f x)] - 89 \sin[4(e + f x)] - 520 \sin[2 e + f x] - 248 \sin[e + 2 f x] - \\ 120 \sin[3 e + 2 f x] + 248 \sin[2 e + 3 f x] + 120 \sin[4 e + 3 f x] + 184 \sin[3 e + 4 f x])$$

Problem 39: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{(a + a \sec[e + f x])^3 (c - c \sec[e + f x])^4} dx$$

Optimal (type 3, 129 leaves, 6 steps):

$$\frac{x}{a^3 c^4} - \frac{\cot[e + f x]^7 (1 + \sec[e + f x])}{7 a^3 c^4 f} + \frac{\cot[e + f x]^5 (7 + 6 \sec[e + f x])}{35 a^3 c^4 f} + \\ \frac{\cot[e + f x] (35 + 16 \sec[e + f x])}{35 a^3 c^4 f} - \frac{\cot[e + f x]^3 (35 + 24 \sec[e + f x])}{105 a^3 c^4 f}$$

Result (type 3, 362 leaves):

$$\frac{1}{6881280 a^3 c^4 f} \csc\left[\frac{e}{2}\right] \csc\left[\frac{1}{2}(e+f x)\right]^7 \sec\left[\frac{e}{2}\right] \sec\left[\frac{1}{2}(e+f x)\right]^5 \\ (16800 f x \cos[f x] - 16800 f x \cos[2 e + f x] - 4200 f x \cos[e + 2 f x] + \\ 4200 f x \cos[3 e + 2 f x] - 8400 f x \cos[2 e + 3 f x] + 8400 f x \cos[4 e + 3 f x] + \\ 3360 f x \cos[3 e + 4 f x] - 3360 f x \cos[5 e + 4 f x] + 1680 f x \cos[4 e + 5 f x] - \\ 1680 f x \cos[6 e + 5 f x] - 840 f x \cos[5 e + 6 f x] + 840 f x \cos[7 e + 6 f x] + 3136 \sin[e] - \\ 30112 \sin[f x] - 22860 \sin[e + f x] + 5715 \sin[2(e + f x)] + 11430 \sin[3(e + f x)] - \\ 4572 \sin[4(e + f x)] - 2286 \sin[5(e + f x)] + 1143 \sin[6(e + f x)] - 26208 \sin[2 e + f x] + \\ 14080 \sin[e + 2 f x] + 16400 \sin[2 e + 3 f x] + 11760 \sin[4 e + 3 f x] - 7904 \sin[3 e + 4 f x] - \\ 3360 \sin[5 e + 4 f x] - 3952 \sin[4 e + 5 f x] - 1680 \sin[6 e + 5 f x] + 2816 \sin[5 e + 6 f x])$$

Problem 40: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{(a + a \sec[e + f x])^3 (c - c \sec[e + f x])^5} dx$$

Optimal (type 3, 210 leaves, 14 steps):

$$\frac{x}{a^3 c^5} + \frac{\cot[e + f x]}{a^3 c^5 f} - \frac{\cot[e + f x]^3}{3 a^3 c^5 f} + \frac{\cot[e + f x]^5}{5 a^3 c^5 f} - \frac{\cot[e + f x]^7}{7 a^3 c^5 f} + \frac{2 \cot[e + f x]^9}{9 a^3 c^5 f} + \\ \frac{2 \csc[e + f x]}{a^3 c^5 f} - \frac{8 \csc[e + f x]^3}{3 a^3 c^5 f} + \frac{12 \csc[e + f x]^5}{5 a^3 c^5 f} - \frac{8 \csc[e + f x]^7}{7 a^3 c^5 f} + \frac{2 \csc[e + f x]^9}{9 a^3 c^5 f}$$

Result (type 3, 441 leaves):

$$\frac{1}{2580480 a^3 c^5 f \left(-1 + \sec[e + f x]\right)^5 \left(1 + \sec[e + f x]\right)^3} \\ \csc\left[\frac{e}{2}\right] \sec\left[\frac{e}{2}\right] \sec[e + f x]^7 (453600 f x \cos[f x] - 453600 f x \cos[2 e + f x] - 201600 f x \cos[e + 2 f x] + 201600 f x \cos[3 e + 2 f x] - 191520 f x \cos[2 e + 3 f x] + 191520 f x \cos[4 e + 3 f x] + 161280 f x \cos[3 e + 4 f x] - 161280 f x \cos[5 e + 4 f x] + 10080 f x \cos[4 e + 5 f x] - 10080 f x \cos[6 e + 5 f x] - 40320 f x \cos[5 e + 6 f x] + 40320 f x \cos[7 e + 6 f x] + 10080 f x \cos[6 e + 7 f x] - 10080 f x \cos[8 e + 7 f x] + 259584 \sin[e] - 897024 \sin[f x] - 1152405 \sin[e + f x] + 512180 \sin[2(e + f x)] + 486571 \sin[3(e + f x)] - 409744 \sin[4(e + f x)] - 25609 \sin[5(e + f x)] + 102436 \sin[6(e + f x)] - 25609 \sin[7(e + f x)] - 825216 \sin[2e + f x] + 622976 \sin[e + 2f x] + 142464 \sin[3e + 2f x] + 297088 \sin[2e + 3f x] + 430080 \sin[4e + 3f x] - 424192 \sin[3e + 4f x] - 188160 \sin[5e + 4f x] + 2048 \sin[4e + 5f x] - 40320 \sin[6e + 5f x] + 112768 \sin[5e + 6f x] + 40320 \sin[7e + 6f x] - 38272 \sin[6e + 7f x]) \tan[e + f x]$$

Problem 47: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{a + a \sec[e + f x]}}{(c - c \sec[e + f x])^2} dx$$

Optimal (type 3, 104 leaves, 5 steps) :

$$\frac{2 \sqrt{a} \operatorname{ArcTan}\left[\frac{\sqrt{a} \tan[e+f x]}{\sqrt{a+a \sec[e+f x]}}\right]}{c^2 f} + \frac{2 \cot[e+f x] \sqrt{a+a \sec[e+f x]}}{c^2 f} - \frac{2 \cot[e+f x]^3 (a+a \sec[e+f x])^{3/2}}{3 a c^2 f}$$

Result (type 4, 471 leaves) :

$$\begin{aligned}
& \left(\sec \left[\frac{1}{2} (e + f x) \right] \sec [e + f x]^2 \sqrt{a (1 + \sec [e + f x])} \sin \left[\frac{e}{2} + \frac{f x}{2} \right]^4 \right. \\
& \left. \left(\frac{20}{3} \csc \left[\frac{1}{2} (e + f x) \right] - \frac{2}{3} \csc \left[\frac{1}{2} (e + f x) \right]^3 - \frac{32}{3} \sin \left[\frac{1}{2} (e + f x) \right] \right) \right) / \left(f (c - c \sec [e + f x])^2 \right) - \\
& \frac{1}{f (c - c \sec [e + f x])^2} 32 (-3 - 2 \sqrt{2}) \cos \left[\frac{1}{4} (e + f x) \right]^4 \\
& \sqrt{\frac{7 - 5 \sqrt{2} + (10 - 7 \sqrt{2}) \cos \left[\frac{1}{2} (e + f x) \right]}{1 + \cos \left[\frac{1}{2} (e + f x) \right]}} \sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \cos \left[\frac{1}{2} (e + f x) \right]}{1 + \cos \left[\frac{1}{2} (e + f x) \right]}} \\
& \left(1 - \sqrt{2} + (-2 + \sqrt{2}) \cos \left[\frac{1}{2} (e + f x) \right] \right) \left(\text{EllipticF} \left[\text{ArcSin} \left[\frac{\tan \left[\frac{1}{4} (e + f x) \right]}{\sqrt{3 - 2 \sqrt{2}}} \right], 17 - 12 \sqrt{2} \right] + \right. \\
& \left. 2 \text{EllipticPi} \left[-3 + 2 \sqrt{2}, -\text{ArcSin} \left[\frac{\tan \left[\frac{1}{4} (e + f x) \right]}{\sqrt{3 - 2 \sqrt{2}}} \right], 17 - 12 \sqrt{2} \right] \right) \\
& \sqrt{\left(-1 - \sqrt{2} + (2 + \sqrt{2}) \cos \left[\frac{1}{2} (e + f x) \right] \right) \sec \left[\frac{1}{4} (e + f x) \right]^2 \sec \left[\frac{1}{2} (e + f x) \right]} \\
& \sec [e + f x]^3 \sqrt{a (1 + \sec [e + f x])} \sin \left[\frac{e}{2} + \frac{f x}{2} \right]^4 \sqrt{3 - 2 \sqrt{2} - \tan \left[\frac{1}{4} (e + f x) \right]^2}
\end{aligned}$$

Problem 48: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{a + a \sec [e + f x]}}{(c - c \sec [e + f x])^3} dx$$

Optimal (type 3, 139 leaves, 6 steps):

$$\begin{aligned}
& \frac{2 \sqrt{a} \text{ArcTan} \left[\frac{\sqrt{a} \tan [e + f x]}{\sqrt{a + a \sec [e + f x]}} \right]}{c^3 f} + \frac{2 \cot [e + f x] \sqrt{a + a \sec [e + f x]}}{c^3 f} - \\
& \frac{2 \cot [e + f x]^3 (a + a \sec [e + f x])^{3/2}}{3 a c^3 f} + \frac{2 \cot [e + f x]^5 (a + a \sec [e + f x])^{5/2}}{5 a^2 c^3 f}
\end{aligned}$$

Result (type 4, 487 leaves):

$$\begin{aligned}
& \left(\sec \left[\frac{1}{2} (e + f x) \right] \sec [e + f x]^3 \sqrt{a (1 + \sec [e + f x])} \sin \left[\frac{e}{2} + \frac{f x}{2} \right]^6 \left(-\frac{272}{15} \csc \left[\frac{1}{2} (e + f x) \right] + \right. \right. \\
& \quad \left. \left. \frac{56}{15} \csc \left[\frac{1}{2} (e + f x) \right]^3 - \frac{2}{5} \csc \left[\frac{1}{2} (e + f x) \right]^5 + \frac{368}{15} \sin \left[\frac{1}{2} (e + f x) \right] \right) \right) / \\
& \quad \left(f (c - c \sec [e + f x])^3 \right) + \frac{1}{f (c - c \sec [e + f x])^3} 64 (-3 - 2 \sqrt{2}) \cos \left[\frac{1}{4} (e + f x) \right]^4 \\
& \quad \sqrt{\frac{7 - 5 \sqrt{2} + (10 - 7 \sqrt{2}) \cos \left[\frac{1}{2} (e + f x) \right]}{1 + \cos \left[\frac{1}{2} (e + f x) \right]}} \sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \cos \left[\frac{1}{2} (e + f x) \right]}{1 + \cos \left[\frac{1}{2} (e + f x) \right]}} \\
& \quad \left(1 - \sqrt{2} + (-2 + \sqrt{2}) \cos \left[\frac{1}{2} (e + f x) \right] \right) \left(\text{EllipticF} \left[\text{ArcSin} \left[\frac{\tan \left[\frac{1}{4} (e + f x) \right]}{\sqrt{3 - 2 \sqrt{2}}} \right], 17 - 12 \sqrt{2} \right] + \right. \\
& \quad \left. 2 \text{EllipticPi} \left[-3 + 2 \sqrt{2}, -\text{ArcSin} \left[\frac{\tan \left[\frac{1}{4} (e + f x) \right]}{\sqrt{3 - 2 \sqrt{2}}} \right], 17 - 12 \sqrt{2} \right] \right) \\
& \quad \sqrt{\left(-1 - \sqrt{2} + (2 + \sqrt{2}) \cos \left[\frac{1}{2} (e + f x) \right] \right) \sec \left[\frac{1}{4} (e + f x) \right]^2 \sec \left[\frac{1}{2} (e + f x) \right]} \\
& \quad \sec [e + f x]^4 \sqrt{a (1 + \sec [e + f x])} \sin \left[\frac{e}{2} + \frac{f x}{2} \right]^6 \sqrt{3 - 2 \sqrt{2} - \tan \left[\frac{1}{4} (e + f x) \right]^2}
\end{aligned}$$

Problem 49: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{a + a \sec [e + f x]}}{(c - c \sec [e + f x])^4} dx$$

Optimal (type 3, 174 leaves, 7 steps):

$$\begin{aligned}
& \frac{2 \sqrt{a} \text{ArcTan} \left[\frac{\sqrt{a} \tan [e + f x]}{\sqrt{a + a \sec [e + f x]}} \right]}{c^4 f} + \\
& \frac{2 \cot [e + f x] \sqrt{a + a \sec [e + f x]}}{c^4 f} - \frac{2 \cot [e + f x]^3 (a + a \sec [e + f x])^{3/2}}{3 a c^4 f} + \\
& \frac{2 \cot [e + f x]^5 (a + a \sec [e + f x])^{5/2}}{5 a^2 c^4 f} - \frac{2 \cot [e + f x]^7 (a + a \sec [e + f x])^{7/2}}{7 a^3 c^4 f}
\end{aligned}$$

Result (type 4, 503 leaves):

$$\begin{aligned}
& \left(\sec \left[\frac{1}{2} (e + f x) \right] \sec [e + f x]^4 \sqrt{a (1 + \sec [e + f x])} \right. \\
& \quad \left. \sin \left[\frac{e}{2} + \frac{f x}{2} \right]^8 \left(\frac{4768}{105} \csc \left[\frac{1}{2} (e + f x) \right] - \frac{1504}{105} \csc \left[\frac{1}{2} (e + f x) \right]^3 + \right. \right. \\
& \quad \left. \left. \frac{108}{35} \csc \left[\frac{1}{2} (e + f x) \right]^5 - \frac{2}{7} \csc \left[\frac{1}{2} (e + f x) \right]^7 - \frac{5632}{105} \sin \left[\frac{1}{2} (e + f x) \right] \right) \right) / \\
& \left(f (c - c \sec [e + f x])^4 \right) - \frac{1}{f (c - c \sec [e + f x])^4} \frac{128 (-3 - 2 \sqrt{2}) \cos \left[\frac{1}{4} (e + f x) \right]^4}{1 + \cos \left[\frac{1}{2} (e + f x) \right]} \\
& \sqrt{\frac{7 - 5 \sqrt{2} + (10 - 7 \sqrt{2}) \cos \left[\frac{1}{2} (e + f x) \right]}{1 + \cos \left[\frac{1}{2} (e + f x) \right]}} \sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \cos \left[\frac{1}{2} (e + f x) \right]}{1 + \cos \left[\frac{1}{2} (e + f x) \right]}} \\
& \left(1 - \sqrt{2} + (-2 + \sqrt{2}) \cos \left[\frac{1}{2} (e + f x) \right] \right) \left(\text{EllipticF}[\text{ArcSin}\left[\frac{\tan \left[\frac{1}{4} (e + f x) \right]}{\sqrt{3 - 2 \sqrt{2}}} \right], 17 - 12 \sqrt{2}] + \right. \\
& \quad \left. 2 \text{EllipticPi}[-3 + 2 \sqrt{2}, -\text{ArcSin}\left[\frac{\tan \left[\frac{1}{4} (e + f x) \right]}{\sqrt{3 - 2 \sqrt{2}}} \right], 17 - 12 \sqrt{2}] \right) \\
& \sqrt{\left(-1 - \sqrt{2} + (2 + \sqrt{2}) \cos \left[\frac{1}{2} (e + f x) \right] \right) \sec \left[\frac{1}{4} (e + f x) \right]^2 \sec \left[\frac{1}{2} (e + f x) \right]} \\
& \sec [e + f x]^5 \sqrt{a (1 + \sec [e + f x])} \sin \left[\frac{e}{2} + \frac{f x}{2} \right]^8 \sqrt{3 - 2 \sqrt{2} - \tan \left[\frac{1}{4} (e + f x) \right]^2}
\end{aligned}$$

Problem 54: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{(a + a \sec [e + f x])^{3/2}}{(c - c \sec [e + f x])^2} dx$$

Optimal (type 3, 102 leaves, 5 steps):

$$\begin{aligned}
& \frac{2 a^{3/2} \text{ArcTan} \left[\frac{\sqrt{a} \tan [e + f x]}{\sqrt{a + a \sec [e + f x]}} \right]}{c^2 f} + \\
& \frac{2 a \cot [e + f x] \sqrt{a + a \sec [e + f x]}}{c^2 f} - \frac{4 \cot [e + f x]^3 (a + a \sec [e + f x])^{3/2}}{3 c^2 f}
\end{aligned}$$

Result (type 4, 473 leaves):

$$\begin{aligned}
& \left(\sec \left[\frac{1}{2} (e + f x) \right]^3 \sec [e + f x] \left(a (1 + \sec [e + f x]) \right)^{3/2} \sin \left[\frac{e}{2} + \frac{f x}{2} \right]^4 \right. \\
& \left. \left(\frac{14}{3} \csc \left[\frac{1}{2} (e + f x) \right] - \frac{2}{3} \csc \left[\frac{1}{2} (e + f x) \right]^3 - \frac{20}{3} \sin \left[\frac{1}{2} (e + f x) \right] \right) \right) / \left(f (c - c \sec [e + f x])^2 \right) - \\
& \frac{1}{f (c - c \sec [e + f x])^2} 16 \left(-3 - 2 \sqrt{2} \right) \cos \left[\frac{1}{4} (e + f x) \right]^4 \\
& \sqrt{\frac{7 - 5 \sqrt{2} + (10 - 7 \sqrt{2}) \cos \left[\frac{1}{2} (e + f x) \right]}{1 + \cos \left[\frac{1}{2} (e + f x) \right]}} \sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \cos \left[\frac{1}{2} (e + f x) \right]}{1 + \cos \left[\frac{1}{2} (e + f x) \right]}} \\
& \left(1 - \sqrt{2} + (-2 + \sqrt{2}) \cos \left[\frac{1}{2} (e + f x) \right] \right) \left(\text{EllipticF} \left[\text{ArcSin} \left[\frac{\tan \left[\frac{1}{4} (e + f x) \right]}{\sqrt{3 - 2 \sqrt{2}}} \right], 17 - 12 \sqrt{2} \right] + \right. \\
& \left. 2 \text{EllipticPi} \left[-3 + 2 \sqrt{2}, -\text{ArcSin} \left[\frac{\tan \left[\frac{1}{4} (e + f x) \right]}{\sqrt{3 - 2 \sqrt{2}}} \right], 17 - 12 \sqrt{2} \right] \right) \\
& \sqrt{\left(-1 - \sqrt{2} + (2 + \sqrt{2}) \cos \left[\frac{1}{2} (e + f x) \right] \right) \sec \left[\frac{1}{4} (e + f x) \right]^2 \sec \left[\frac{1}{2} (e + f x) \right]^3} \\
& \sec [e + f x]^2 \left(a (1 + \sec [e + f x]) \right)^{3/2} \sin \left[\frac{e}{2} + \frac{f x}{2} \right]^4 \sqrt{3 - 2 \sqrt{2} - \tan \left[\frac{1}{4} (e + f x) \right]^2}
\end{aligned}$$

Problem 55: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{(a + a \sec [e + f x])^{3/2}}{(c - c \sec [e + f x])^3} dx$$

Optimal (type 3, 137 leaves, 6 steps):

$$\begin{aligned}
& \frac{2 a^{3/2} \text{ArcTan} \left[\frac{\sqrt{a} \tan [e + f x]}{\sqrt{a + a \sec [e + f x]}} \right]}{c^3 f} + \frac{2 a \cot [e + f x] \sqrt{a + a \sec [e + f x]}}{c^3 f} - \\
& \frac{2 \cot [e + f x]^3 (a + a \sec [e + f x])^{3/2}}{3 c^3 f} + \frac{4 \cot [e + f x]^5 (a + a \sec [e + f x])^{5/2}}{5 a c^3 f}
\end{aligned}$$

Result (type 4, 491 leaves):

$$\begin{aligned}
& \left(\sec \left[\frac{1}{2} (e + f x) \right]^3 \sec [e + f x]^2 (a (1 + \sec [e + f x]))^{3/2} \sin \left[\frac{e}{2} + \frac{f x}{2} \right]^6 \left(-\frac{172}{15} \csc \left[\frac{1}{2} (e + f x) \right] + \right. \right. \\
& \quad \left. \left. \frac{46}{15} \csc \left[\frac{1}{2} (e + f x) \right]^3 - \frac{2}{5} \csc \left[\frac{1}{2} (e + f x) \right]^5 + \frac{208}{15} \sin \left[\frac{1}{2} (e + f x) \right] \right) \right) / \\
& \quad \left(f (c - c \sec [e + f x])^3 \right) + \frac{1}{f (c - c \sec [e + f x])^3} 32 (-3 - 2 \sqrt{2}) \cos \left[\frac{1}{4} (e + f x) \right]^4 \\
& \quad \sqrt{\frac{7 - 5 \sqrt{2} + (10 - 7 \sqrt{2}) \cos \left[\frac{1}{2} (e + f x) \right]}{1 + \cos \left[\frac{1}{2} (e + f x) \right]}} \sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \cos \left[\frac{1}{2} (e + f x) \right]}{1 + \cos \left[\frac{1}{2} (e + f x) \right]}} \\
& \quad \left. \left(1 - \sqrt{2} + (-2 + \sqrt{2}) \cos \left[\frac{1}{2} (e + f x) \right] \right) \left(\text{EllipticF} \left[\text{ArcSin} \left[\frac{\tan \left[\frac{1}{4} (e + f x) \right]}{\sqrt{3 - 2 \sqrt{2}}} \right], 17 - 12 \sqrt{2} \right] + \right. \right. \\
& \quad \left. \left. 2 \text{EllipticPi} \left[-3 + 2 \sqrt{2}, -\text{ArcSin} \left[\frac{\tan \left[\frac{1}{4} (e + f x) \right]}{\sqrt{3 - 2 \sqrt{2}}} \right], 17 - 12 \sqrt{2} \right] \right) \right. \\
& \quad \sqrt{\left(-1 - \sqrt{2} + (2 + \sqrt{2}) \cos \left[\frac{1}{2} (e + f x) \right] \right) \sec \left[\frac{1}{4} (e + f x) \right]^2 \sec \left[\frac{1}{2} (e + f x) \right]^3} \\
& \quad \sec [e + f x]^3 (a (1 + \sec [e + f x]))^{3/2} \sin \left[\frac{e}{2} + \frac{f x}{2} \right]^6 \sqrt{3 - 2 \sqrt{2} - \tan \left[\frac{1}{4} (e + f x) \right]^2}
\end{aligned}$$

Problem 56: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{(a + a \sec [e + f x])^{3/2}}{(c - c \sec [e + f x])^4} dx$$

Optimal (type 3, 172 leaves, 7 steps):

$$\begin{aligned}
& \frac{2 a^{3/2} \text{ArcTan} \left[\frac{\sqrt{a} \tan [e + f x]}{\sqrt{a + a \sec [e + f x]}} \right]}{c^4 f} + \\
& \frac{2 a \cot [e + f x] \sqrt{a + a \sec [e + f x]}}{c^4 f} - \frac{2 \cot [e + f x]^3 (a + a \sec [e + f x])^{3/2}}{3 c^4 f} + \\
& \frac{2 \cot [e + f x]^5 (a + a \sec [e + f x])^{5/2}}{5 a c^4 f} - \frac{4 \cot [e + f x]^7 (a + a \sec [e + f x])^{7/2}}{7 a^2 c^4 f}
\end{aligned}$$

Result (type 4, 507 leaves):

$$\begin{aligned}
& \left(\sec \left[\frac{1}{2} (e + f x) \right]^3 \sec [e + f x]^3 (a (1 + \sec [e + f x]))^{3/2} \right. \\
& \quad \left. \sin \left[\frac{e}{2} + \frac{f x}{2} \right]^8 \left(\frac{2864}{105} \csc \left[\frac{1}{2} (e + f x) \right] - \frac{1112}{105} \csc \left[\frac{1}{2} (e + f x) \right]^3 + \right. \right. \\
& \quad \left. \left. \frac{94}{35} \csc \left[\frac{1}{2} (e + f x) \right]^5 - \frac{2}{7} \csc \left[\frac{1}{2} (e + f x) \right]^7 - \frac{3056}{105} \sin \left[\frac{1}{2} (e + f x) \right] \right) \right) / \\
& \quad \left(f (c - c \sec [e + f x])^4 \right) - \frac{1}{f (c - c \sec [e + f x])^4} 64 (-3 - 2 \sqrt{2}) \cos \left[\frac{1}{4} (e + f x) \right]^4 \\
& \quad \sqrt{\frac{7 - 5 \sqrt{2} + (10 - 7 \sqrt{2}) \cos \left[\frac{1}{2} (e + f x) \right]}{1 + \cos \left[\frac{1}{2} (e + f x) \right]}} \sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \cos \left[\frac{1}{2} (e + f x) \right]}{1 + \cos \left[\frac{1}{2} (e + f x) \right]}} \\
& \quad \left(1 - \sqrt{2} + (-2 + \sqrt{2}) \cos \left[\frac{1}{2} (e + f x) \right] \right) \left(\text{EllipticF} \left[\text{ArcSin} \left[\frac{\tan \left[\frac{1}{4} (e + f x) \right]}{\sqrt{3 - 2 \sqrt{2}}} \right], 17 - 12 \sqrt{2} \right] + \right. \\
& \quad \left. 2 \text{EllipticPi} \left[-3 + 2 \sqrt{2}, -\text{ArcSin} \left[\frac{\tan \left[\frac{1}{4} (e + f x) \right]}{\sqrt{3 - 2 \sqrt{2}}} \right], 17 - 12 \sqrt{2} \right] \right) \\
& \quad \sqrt{\left(-1 - \sqrt{2} + (2 + \sqrt{2}) \cos \left[\frac{1}{2} (e + f x) \right] \right) \sec \left[\frac{1}{4} (e + f x) \right]^2 \sec \left[\frac{1}{2} (e + f x) \right]^3} \\
& \quad \sec [e + f x]^4 (a (1 + \sec [e + f x]))^{3/2} \sin \left[\frac{e}{2} + \frac{f x}{2} \right]^8 \sqrt{3 - 2 \sqrt{2} - \tan \left[\frac{1}{4} (e + f x) \right]^2}
\end{aligned}$$

Problem 61: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{(a + a \sec [e + f x])^{5/2}}{(c - c \sec [e + f x])^2} dx$$

Optimal (type 3, 74 leaves, 5 steps):

$$\frac{2 a^{5/2} \text{ArcTan} \left[\frac{\sqrt{a} \tan [e + f x]}{\sqrt{a + a \sec [e + f x]}} \right]}{c^2 f} - \frac{8 a \cot [e + f x]^3 (a + a \sec [e + f x])^{3/2}}{3 c^2 f}$$

Result (type 4, 465 leaves):

$$\begin{aligned}
& \left(\sec \left[\frac{1}{2} (e + f x) \right]^5 (a (1 + \sec [e + f x]))^{5/2} \sin \left[\frac{e}{2} + \frac{f x}{2} \right]^4 \right. \\
& \left. \left(\frac{8}{3} \csc \left[\frac{1}{2} (e + f x) \right] - \frac{2}{3} \csc \left[\frac{1}{2} (e + f x) \right]^3 - \frac{8}{3} \sin \left[\frac{1}{2} (e + f x) \right] \right) \right) / \left(f (c - c \sec [e + f x])^2 \right) - \\
& \frac{1}{f (c - c \sec [e + f x])^2} 8 (-3 - 2 \sqrt{2}) \cos \left[\frac{1}{4} (e + f x) \right]^4 \sqrt{\frac{7 - 5 \sqrt{2} + (10 - 7 \sqrt{2}) \cos \left[\frac{1}{2} (e + f x) \right]}{1 + \cos \left[\frac{1}{2} (e + f x) \right]}} \\
& \sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \cos \left[\frac{1}{2} (e + f x) \right]}{1 + \cos \left[\frac{1}{2} (e + f x) \right]}} \left(1 - \sqrt{2} + (-2 + \sqrt{2}) \cos \left[\frac{1}{2} (e + f x) \right] \right) \\
& \left(\text{EllipticF} \left[\text{ArcSin} \left[\frac{\tan \left[\frac{1}{4} (e + f x) \right]}{\sqrt{3 - 2 \sqrt{2}}} \right], 17 - 12 \sqrt{2} \right] + \right. \\
& \left. 2 \text{EllipticPi} \left[-3 + 2 \sqrt{2}, -\text{ArcSin} \left[\frac{\tan \left[\frac{1}{4} (e + f x) \right]}{\sqrt{3 - 2 \sqrt{2}}} \right], 17 - 12 \sqrt{2} \right] \right) \\
& \sqrt{\left(-1 - \sqrt{2} + (2 + \sqrt{2}) \cos \left[\frac{1}{2} (e + f x) \right] \right) \sec \left[\frac{1}{4} (e + f x) \right]^2 \sec \left[\frac{1}{2} (e + f x) \right]^5} \\
& \sec [e + f x] (a (1 + \sec [e + f x]))^{5/2} \sin \left[\frac{e}{2} + \frac{f x}{2} \right]^4 \sqrt{3 - 2 \sqrt{2} - \tan \left[\frac{1}{4} (e + f x) \right]^2}
\end{aligned}$$

Problem 62: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{(a + a \sec [e + f x])^{5/2}}{(c - c \sec [e + f x])^3} dx$$

Optimal (type 3, 104 leaves, 5 steps):

$$\begin{aligned}
& \frac{2 a^{5/2} \text{ArcTan} \left[\frac{\sqrt{a} \tan [e + f x]}{\sqrt{a + a \sec [e + f x]}} \right]}{c^3 f} + \\
& \frac{2 a^2 \cot [e + f x] \sqrt{a + a \sec [e + f x]}}{c^3 f} + \frac{8 \cot [e + f x]^5 (a + a \sec [e + f x])^{5/2}}{5 c^3 f}
\end{aligned}$$

Result (type 4, 489 leaves):

$$\begin{aligned}
& \left(\sec \left[\frac{1}{2} (e + f x) \right]^5 \sec [e + f x] (a (1 + \sec [e + f x]))^{5/2} \sin \left[\frac{e}{2} + \frac{f x}{2} \right]^6 \right. \\
& \quad \left. \left(-\frac{34}{5} \csc \left[\frac{1}{2} (e + f x) \right] + \frac{12}{5} \csc \left[\frac{1}{2} (e + f x) \right]^3 - \frac{2}{5} \csc \left[\frac{1}{2} (e + f x) \right]^5 + \frac{36}{5} \sin \left[\frac{1}{2} (e + f x) \right] \right) \right) / \\
& \quad \left(f (c - c \sec [e + f x])^3 \right) + \frac{1}{f (c - c \sec [e + f x])^3} 16 (-3 - 2 \sqrt{2}) \cos \left[\frac{1}{4} (e + f x) \right]^4 \\
& \quad \sqrt{\frac{7 - 5 \sqrt{2} + (10 - 7 \sqrt{2}) \cos \left[\frac{1}{2} (e + f x) \right]}{1 + \cos \left[\frac{1}{2} (e + f x) \right]}} \sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \cos \left[\frac{1}{2} (e + f x) \right]}{1 + \cos \left[\frac{1}{2} (e + f x) \right]}} \\
& \quad \left(1 - \sqrt{2} + (-2 + \sqrt{2}) \cos \left[\frac{1}{2} (e + f x) \right] \right) \left(\text{EllipticF} \left[\text{ArcSin} \left[\frac{\tan \left[\frac{1}{4} (e + f x) \right]}{\sqrt{3 - 2 \sqrt{2}}} \right], 17 - 12 \sqrt{2} \right] + \right. \\
& \quad \left. 2 \text{EllipticPi} \left[-3 + 2 \sqrt{2}, -\text{ArcSin} \left[\frac{\tan \left[\frac{1}{4} (e + f x) \right]}{\sqrt{3 - 2 \sqrt{2}}} \right], 17 - 12 \sqrt{2} \right] \right) \\
& \quad \sqrt{\left(-1 - \sqrt{2} + (2 + \sqrt{2}) \cos \left[\frac{1}{2} (e + f x) \right] \right) \sec \left[\frac{1}{4} (e + f x) \right]^2 \sec \left[\frac{1}{2} (e + f x) \right]^5} \\
& \quad \sec [e + f x]^2 (a (1 + \sec [e + f x]))^{5/2} \sin \left[\frac{e}{2} + \frac{f x}{2} \right]^6 \sqrt{3 - 2 \sqrt{2} - \tan \left[\frac{1}{4} (e + f x) \right]^2}
\end{aligned}$$

Problem 63: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{(a + a \sec [e + f x])^{5/2}}{(c - c \sec [e + f x])^4} dx$$

Optimal (type 3, 140 leaves, 5 steps):

$$\begin{aligned}
& \frac{2 a^{5/2} \text{ArcTan} \left[\frac{\sqrt{a} \tan [e + f x]}{\sqrt{a + a \sec [e + f x]}} \right]}{c^4 f} + \frac{2 a^2 \cot [e + f x] \sqrt{a + a \sec [e + f x]}}{c^4 f} - \\
& \frac{2 a \cot [e + f x]^3 (a + a \sec [e + f x])^{3/2}}{3 c^4 f} - \frac{8 \cot [e + f x]^7 (a + a \sec [e + f x])^{7/2}}{7 a c^4 f}
\end{aligned}$$

Result (type 4, 507 leaves):

$$\begin{aligned}
& \left(\sec \left[\frac{1}{2} (e + f x) \right]^5 \sec [e + f x]^2 (a (1 + \sec [e + f x]))^{5/2} \right. \\
& \quad \left. \sin \left[\frac{e}{2} + \frac{f x}{2} \right]^8 \left(\frac{332}{21} \csc \left[\frac{1}{2} (e + f x) \right] - \frac{158}{21} \csc \left[\frac{1}{2} (e + f x) \right]^3 + \right. \right. \\
& \quad \left. \left. \frac{16}{7} \csc \left[\frac{1}{2} (e + f x) \right]^5 - \frac{2}{7} \csc \left[\frac{1}{2} (e + f x) \right]^7 - \frac{320}{21} \sin \left[\frac{1}{2} (e + f x) \right] \right) \right) / \\
& \left(f (c - c \sec [e + f x])^4 \right) - \frac{1}{f (c - c \sec [e + f x])^4} 32 (-3 - 2 \sqrt{2}) \cos \left[\frac{1}{4} (e + f x) \right]^4 \\
& \sqrt{\frac{7 - 5 \sqrt{2} + (10 - 7 \sqrt{2}) \cos \left[\frac{1}{2} (e + f x) \right]}{1 + \cos \left[\frac{1}{2} (e + f x) \right]}} \sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \cos \left[\frac{1}{2} (e + f x) \right]}{1 + \cos \left[\frac{1}{2} (e + f x) \right]}} \\
& \left(1 - \sqrt{2} + (-2 + \sqrt{2}) \cos \left[\frac{1}{2} (e + f x) \right] \right) \left(\text{EllipticF} \left[\text{ArcSin} \left[\frac{\tan \left[\frac{1}{4} (e + f x) \right]}{\sqrt{3 - 2 \sqrt{2}}} \right], 17 - 12 \sqrt{2} \right] + \right. \\
& \quad \left. 2 \text{EllipticPi} \left[-3 + 2 \sqrt{2}, -\text{ArcSin} \left[\frac{\tan \left[\frac{1}{4} (e + f x) \right]}{\sqrt{3 - 2 \sqrt{2}}} \right], 17 - 12 \sqrt{2} \right] \right) \\
& \sqrt{\left(-1 - \sqrt{2} + (2 + \sqrt{2}) \cos \left[\frac{1}{2} (e + f x) \right] \right) \sec \left[\frac{1}{4} (e + f x) \right]^2 \sec \left[\frac{1}{2} (e + f x) \right]^5} \\
& \sec [e + f x]^3 (a (1 + \sec [e + f x]))^{5/2} \sin \left[\frac{e}{2} + \frac{f x}{2} \right]^8 \sqrt{3 - 2 \sqrt{2} - \tan \left[\frac{1}{4} (e + f x) \right]^2}
\end{aligned}$$

Problem 64: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{(a + a \sec [e + f x])^{5/2}}{(c - c \sec [e + f x])^5} dx$$

Optimal (type 3, 172 leaves, 5 steps):

$$\begin{aligned}
& \frac{2 a^{5/2} \text{ArcTan} \left[\frac{\sqrt{a} \tan [e + f x]}{\sqrt{a + a \sec [e + f x]}} \right]}{c^5 f} + \\
& \frac{2 a^2 \cot [e + f x] \sqrt{a + a \sec [e + f x]}}{c^5 f} - \frac{2 a \cot [e + f x]^3 (a + a \sec [e + f x])^{3/2}}{3 c^5 f} + \\
& \frac{2 \cot [e + f x]^5 (a + a \sec [e + f x])^{5/2}}{5 c^5 f} + \frac{8 \cot [e + f x]^9 (a + a \sec [e + f x])^{9/2}}{9 a^2 c^5 f}
\end{aligned}$$

Result (type 4, 523 leaves):

$$\begin{aligned}
& \left(\sec \left[\frac{1}{2} (e + f x) \right]^5 \sec [e + f x]^3 (a (1 + \sec [e + f x]))^{5/2} \sin \left[\frac{e}{2} + \frac{f x}{2} \right]^{10} \right. \\
& \quad \left(-\frac{1616}{45} \csc \left[\frac{1}{2} (e + f x) \right] + \frac{968}{45} \csc \left[\frac{1}{2} (e + f x) \right]^3 - \frac{418}{45} \csc \left[\frac{1}{2} (e + f x) \right]^5 + \right. \\
& \quad \left. \frac{20}{9} \csc \left[\frac{1}{2} (e + f x) \right]^7 - \frac{2}{9} \csc \left[\frac{1}{2} (e + f x) \right]^9 + \frac{1424}{45} \sin \left[\frac{1}{2} (e + f x) \right] \right) / \\
& \quad \left(f (c - c \sec [e + f x])^5 \right) + \frac{1}{f (c - c \sec [e + f x])^5} 64 (-3 - 2 \sqrt{2}) \cos \left[\frac{1}{4} (e + f x) \right]^4 \\
& \quad \sqrt{\frac{7 - 5 \sqrt{2} + (10 - 7 \sqrt{2}) \cos \left[\frac{1}{2} (e + f x) \right]}{1 + \cos \left[\frac{1}{2} (e + f x) \right]}} \sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \cos \left[\frac{1}{2} (e + f x) \right]}{1 + \cos \left[\frac{1}{2} (e + f x) \right]}} \\
& \quad \left(1 - \sqrt{2} + (-2 + \sqrt{2}) \cos \left[\frac{1}{2} (e + f x) \right] \right) \left(\text{EllipticF} \left[\text{ArcSin} \left[\frac{\tan \left[\frac{1}{4} (e + f x) \right]}{\sqrt{3 - 2 \sqrt{2}}} \right], 17 - 12 \sqrt{2} \right] + \right. \\
& \quad \left. 2 \text{EllipticPi} \left[-3 + 2 \sqrt{2}, -\text{ArcSin} \left[\frac{\tan \left[\frac{1}{4} (e + f x) \right]}{\sqrt{3 - 2 \sqrt{2}}} \right], 17 - 12 \sqrt{2} \right] \right) \\
& \quad \sqrt{\left(-1 - \sqrt{2} + (2 + \sqrt{2}) \cos \left[\frac{1}{2} (e + f x) \right] \right) \sec \left[\frac{1}{4} (e + f x) \right]^2 \sec \left[\frac{1}{2} (e + f x) \right]^5} \\
& \quad \sec [e + f x]^4 (a (1 + \sec [e + f x]))^{5/2} \sin \left[\frac{e}{2} + \frac{f x}{2} \right]^{10} \sqrt{3 - 2 \sqrt{2} - \tan \left[\frac{1}{4} (e + f x) \right]^2}
\end{aligned}$$

Problem 73: Result more than twice size of optimal antiderivative.

$$\int \frac{(c - c \sec [e + f x])^3}{(a + a \sec [e + f x])^{3/2}} dx$$

Optimal (type 3, 169 leaves, 7 steps):

$$\begin{aligned}
& \frac{2 c^3 \text{ArcTan} \left[\frac{\sqrt{a} \tan [e + f x]}{\sqrt{a + a \sec [e + f x]}} \right]}{a^{3/2} f} + \frac{2 \sqrt{2} c^3 \text{ArcTan} \left[\frac{\sqrt{a} \tan [e + f x]}{\sqrt{2} \sqrt{a + a \sec [e + f x]}} \right]}{a^{3/2} f} - \\
& \frac{4 c^3 \tan [e + f x]}{a f \sqrt{a + a \sec [e + f x]}} + \frac{c^3 \sec \left[\frac{1}{2} (e + f x) \right]^2 \sin [e + f x] \tan [e + f x]^2}{f (a + a \sec [e + f x])^{3/2}}
\end{aligned}$$

Result (type 3, 564 leaves):

$$\begin{aligned}
& \left(\cos [e + fx]^3 \csc \left[\frac{e}{2} + \frac{fx}{2} \right]^6 (1 + \sec [e + fx])^{3/2} \right. \\
& (c - c \sec [e + fx])^3 \left(\left(3 \sqrt{2} \operatorname{ArcTan} \left[\frac{\sqrt{2}}{\sqrt{-1 + \sec [e + fx]}} \right] \cos [e + fx]^2 \right. \right. \\
& \left. \left. \sqrt{-1 + \sec [e + fx]} (1 + \sec [e + fx])^{3/2} \sin [e + fx] \right) \right) / \left(f (1 + \cos [e + fx]) \right. \\
& \left. \left. \sqrt{1 - \cos [e + fx]^2} \sqrt{\cos [e + fx]^2 (-1 + \sec [e + fx]) (1 + \sec [e + fx])} \right) - \right. \\
& \left(\left(\sqrt{2} \operatorname{ArcTan} \left[\frac{\sqrt{2}}{\sqrt{-1 + \sec [e + fx]}} \right] + \operatorname{ArcTan} \left[\frac{-2 + \sqrt{1 + \sec [e + fx]}}{\sqrt{-1 + \sec [e + fx]}} \right] - \right. \right. \\
& \left. \left. \operatorname{ArcTan} \left[\frac{2 + \sqrt{1 + \sec [e + fx]}}{\sqrt{-1 + \sec [e + fx]}} \right] \right) \cos [e + fx]^2 \sqrt{-1 + \sec [e + fx]} \right. \\
& \left. \left. (1 + \sec [e + fx])^{3/2} \sin [e + fx] \right) \right) / \left(f (1 + \cos [e + fx]) \sqrt{1 - \cos [e + fx]^2} \right. \\
& \left. \left. \sqrt{\cos [e + fx]^2 (-1 + \sec [e + fx]) (1 + \sec [e + fx])} \right) \right) \right) / \\
& \left(8 (a (1 + \sec [e + fx]))^{3/2} \right) + \left(\cos [e + fx]^3 \csc \left[\frac{e}{2} + \frac{fx}{2} \right]^6 \right. \\
& \left. \sqrt{(1 + \cos [e + fx]) \sec [e + fx]} \right. \\
& \left. (1 + \sec [e + fx])^{3/2} \right. \\
& \left. (c - c \sec [e + fx])^3 \right. \\
& \left. \left(\frac{3 \sec \left[\frac{e}{2} \right] \sec \left[\frac{e}{2} + \frac{fx}{2} \right] \sin \left[\frac{fx}{2} \right]}{4f} - \frac{\sec \left[\frac{e}{2} \right] \sec \left[\frac{e}{2} + \frac{fx}{2} \right]^3 \sin \left[\frac{fx}{2} \right]}{4f} + \right. \right. \\
& \left. \left. \frac{3 \tan \left[\frac{e}{2} \right]}{4f} - \frac{\sec \left[\frac{e}{2} + \frac{fx}{2} \right]^2 \tan \left[\frac{e}{2} \right]}{4f} \right) \right) / (a (1 + \sec [e + fx]))^{3/2}
\end{aligned}$$

Problem 79: Result more than twice size of optimal antiderivative.

$$\int \frac{(c - c \operatorname{Sec}[e + f x])^5}{(a + a \operatorname{Sec}[e + f x])^{5/2}} dx$$

Optimal (type 3, 260 leaves, 9 steps):

$$\begin{aligned}
& \frac{2 c^5 \operatorname{ArcTan}\left[\frac{\sqrt{a} \tan [e+f x]}{\sqrt{a+a \sec [e+f x]}}\right]}{a^{5/2} f}-\frac{23 \sqrt{2} c^5 \operatorname{ArcTan}\left[\frac{\sqrt{a} \tan [e+f x]}{\sqrt{2} \sqrt{a+a \sec [e+f x]}}\right]}{a^{5/2} f}+ \\
& \frac{21 c^5 \tan [e+f x]}{a^2 f \sqrt{a+a \sec [e+f x]}}-\frac{19 c^5 \tan [e+f x]^3}{6 a f (a+a \sec [e+f x])^{3/2}}+ \\
& \frac{3 c^5 \sec \left[\frac{1}{2} (e+f x)\right]^2 \sin [e+f x] \tan [e+f x]^4}{4 f (a+a \sec [e+f x])^{5/2}}+\frac{a c^5 \sec \left[\frac{1}{2} (e+f x)\right]^4 \sin [e+f x]^2 \tan [e+f x]^5}{4 f (a+a \sec [e+f x])^{7/2}}
\end{aligned}$$

Result (type 3, 667 leaves):

$$\begin{aligned}
& \left(\cos[e + fx]^5 \csc\left[\frac{e}{2} + \frac{fx}{2}\right]^{10} (1 + \sec[e + fx])^{5/2} \right. \\
& \quad \left. (c - c \sec[e + fx])^5 \left(- \left(\left(22 \sqrt{2} \operatorname{ArcTan}\left[\frac{\sqrt{2}}{\sqrt{-1 + \sec[e + fx]}}\right] \cos[e + fx]^2 \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. \sqrt{-1 + \sec[e + fx]} (1 + \sec[e + fx])^{3/2} \sin[e + fx] \right) \right) \right) \Big/ \left(f (1 + \cos[e + fx]) \right. \\
& \quad \left. \left. \left. \left. \sqrt{1 - \cos[e + fx]^2} \sqrt{\cos[e + fx]^2 (-1 + \sec[e + fx]) (1 + \sec[e + fx])} \right) \right) \right) - \\
& \quad \left(\left(\sqrt{2} \operatorname{ArcTan}\left[\frac{\sqrt{2}}{\sqrt{-1 + \sec[e + fx]}}\right] + \operatorname{ArcTan}\left[\frac{-2 + \sqrt{1 + \sec[e + fx]}}{\sqrt{-1 + \sec[e + fx]}}\right] - \right. \right. \\
& \quad \left. \left. \operatorname{ArcTan}\left[\frac{2 + \sqrt{1 + \sec[e + fx]}}{\sqrt{-1 + \sec[e + fx]}}\right] \right) \cos[e + fx]^2 \sqrt{-1 + \sec[e + fx]} \right. \\
& \quad \left. \left. \left. (1 + \sec[e + fx])^{3/2} \sin[e + fx] \right) \right) \Big/ \left(f (1 + \cos[e + fx]) \sqrt{1 - \cos[e + fx]^2} \right. \\
& \quad \left. \left. \left. \sqrt{\cos[e + fx]^2 (-1 + \sec[e + fx]) (1 + \sec[e + fx])} \right) \right) \right) \Big/ \\
& \quad \left(32 (a (1 + \sec[e + fx]))^{5/2} \right) + \left(\cos[e + fx]^5 \csc\left[\frac{e}{2} + \frac{fx}{2}\right]^{10} \right. \\
& \quad \left. \sqrt{(1 + \cos[e + fx]) \sec[e + fx]} \right. \\
& \quad \left. (1 + \sec[e + fx])^{5/2} \right. \\
& \quad \left. (c - c \sec[e + fx])^5 \right. \\
& \quad \left. \left(- \frac{(-1 + 37 \cos[e]) \sin[\frac{e}{2}]}{24 f (\cos[\frac{e}{2}] + \cos[\frac{3e}{2}])} - \frac{19 \sec[\frac{e}{2}] \sec[\frac{e}{2} + \frac{fx}{2}] \sin[\frac{fx}{2}]}{24 f} + \right. \right. \\
& \quad \left. \left. \frac{\sec[\frac{e}{2}] \sec[\frac{e}{2} + \frac{fx}{2}]^3 \sin[\frac{fx}{2}]}{32 f} + \frac{\sec[\frac{e}{2}] \sec[\frac{e}{2} + \frac{fx}{2}]^5 \sin[\frac{fx}{2}]}{16 f} + \right. \right. \\
& \quad \left. \left. \frac{\sec[e] \sec[e + fx] \sin[fx]}{48 f} + \frac{\sec[\frac{e}{2} + \frac{fx}{2}]^2 \tan[\frac{e}{2}]}{32 f} + \right. \right. \\
& \quad \left. \left. \frac{\sec[\frac{e}{2} + \frac{fx}{2}]^4 \tan[\frac{e}{2}]}{16 f} \right) \right) \Big/ (a (1 + \sec[e + fx]))^{5/2}
\end{aligned}$$

Problem 80: Result more than twice size of optimal antiderivative.

$$\int \frac{(c - c \sec[e + fx])^4}{(a + a \sec[e + fx])^{5/2}} dx$$

Optimal (type 3, 229 leaves, 8 steps):

$$\frac{2 c^4 \operatorname{ArcTan}\left[\frac{\sqrt{a} \tan [e+f x]}{\sqrt{a+a \sec [e+f x]}}\right]}{a^{5/2} f}-\frac{11 c^4 \operatorname{ArcTan}\left[\frac{\sqrt{a} \tan [e+f x]}{\sqrt{2} \sqrt{a+a \sec [e+f x]}}\right]}{\sqrt{2} a^{5/2} f}+\frac{7 c^4 \tan [e+f x]}{2 a^2 f \sqrt{a+a \sec [e+f x]}}-$$

$$\frac{c^4 \sec ^{\left[\frac{1}{2} (e+f x)\right]^2} \sin [e+f x] \tan [e+f x]^2}{4 a f \left(a+a \sec [e+f x]\right)^{3/2}}-\frac{c^4 \sec ^{\left[\frac{1}{2} (e+f x)\right]^4} \sin [e+f x]^2 \tan [e+f x]^3}{4 f \left(a+a \sec [e+f x]\right)^{5/2}}$$

Result (type 3, 627 leaves) :

$$\begin{aligned} & \left(\cos [e+f x]^4 \csc \left[\frac{e}{2}+\frac{f x}{2}\right]^8 (1+\sec [e+f x])^{5/2} \right. \\ & (c-c \sec [e+f x])^4 \left(\left(9 \sqrt{2} \operatorname{ArcTan}\left[\frac{\sqrt{2}}{\sqrt{-1+\sec [e+f x]}}\right] \cos [e+f x]^2 \right. \right. \\ & \left. \left. \sqrt{-1+\sec [e+f x]} (1+\sec [e+f x])^{3/2} \sin [e+f x] \right) \middle/ (f (1+\cos [e+f x]) \right. \\ & \left. \sqrt{1-\cos [e+f x]^2} \sqrt{\cos [e+f x]^2 (-1+\sec [e+f x]) (1+\sec [e+f x])} \right) + \\ & \left(2 \left(\sqrt{2} \operatorname{ArcTan}\left[\frac{\sqrt{2}}{\sqrt{-1+\sec [e+f x]}}\right] + \operatorname{ArcTan}\left[\frac{-2+\sqrt{1+\sec [e+f x]}}{\sqrt{-1+\sec [e+f x]}}\right] - \right. \right. \\ & \left. \left. \operatorname{ArcTan}\left[\frac{2+\sqrt{1+\sec [e+f x]}}{\sqrt{-1+\sec [e+f x]}}\right] \right) \cos [e+f x]^2 \sqrt{-1+\sec [e+f x]} \right. \\ & \left. (1+\sec [e+f x])^{3/2} \sin [e+f x] \right) \middle/ \left(f (1+\cos [e+f x]) \sqrt{1-\cos [e+f x]^2} \right. \\ & \left. \left. \sqrt{\cos [e+f x]^2 (-1+\sec [e+f x]) (1+\sec [e+f x])} \right) \right) \middle/ \\ & \left(32 (a (1+\sec [e+f x]))^{5/2} \right) + \left(\cos [e+f x]^4 \csc \left[\frac{e}{2}+\frac{f x}{2}\right]^8 \right. \\ & \left. \sqrt{(1+\cos [e+f x]) \sec [e+f x]} \right. \\ & \left. (1+\sec [e+f x])^{5/2} \right. \\ & \left. (c-c \sec [e+f x])^4 \right. \\ & \left(\frac{3 \sec \left[\frac{e}{2}\right] \sec \left[\frac{e}{2}+\frac{f x}{2}\right] \sin \left[\frac{f x}{2}\right]}{16 f} + \frac{3 \sec \left[\frac{e}{2}\right] \sec \left[\frac{e}{2}+\frac{f x}{2}\right]^3 \sin \left[\frac{f x}{2}\right]}{32 f} - \right. \\ & \left. \frac{\sec \left[\frac{e}{2}\right] \sec \left[\frac{e}{2}+\frac{f x}{2}\right]^5 \sin \left[\frac{f x}{2}\right]}{16 f} + \frac{3 \tan \left[\frac{e}{2}\right]}{16 f} + \frac{3 \sec \left[\frac{e}{2}+\frac{f x}{2}\right]^2 \tan \left[\frac{e}{2}\right]}{32 f} - \right. \\ & \left. \left. \frac{\sec \left[\frac{e}{2}+\frac{f x}{2}\right]^4 \tan \left[\frac{e}{2}\right]}{16 f} \right) \right) \middle/ (a (1+\sec [e+f x]))^{5/2} \end{aligned}$$

Problem 86: Result unnecessarily involves imaginary or complex numbers.

$$\int \sqrt{a + a \sec[e + f x]} (c - c \sec[e + f x])^{7/2} dx$$

Optimal (type 3, 185 leaves, 5 steps) :

$$\begin{aligned} & \frac{a c^4 \log[\cos[e + f x]] \tan[e + f x]}{f \sqrt{a + a \sec[e + f x]} \sqrt{c - c \sec[e + f x]}} - \frac{a c^3 \sqrt{c - c \sec[e + f x]} \tan[e + f x]}{f \sqrt{a + a \sec[e + f x]}} - \\ & \frac{a c^2 (c - c \sec[e + f x])^{3/2} \tan[e + f x]}{2 f \sqrt{a + a \sec[e + f x]}} - \frac{a c (c - c \sec[e + f x])^{5/2} \tan[e + f x]}{3 f \sqrt{a + a \sec[e + f x]}} \end{aligned}$$

Result (type 3, 149 leaves) :

$$\begin{aligned} & \frac{1}{24 f} c^3 \csc\left[\frac{1}{2} (e + f x)\right] (-22 - 18 \cos[2 (e + f x)] + 3 i f x \cos[3 (e + f x)]) + \\ & 9 \cos[e + f x] (2 + i f x - \log[1 + e^{2 i (e + f x)}]) - 3 \cos[3 (e + f x)] \log[1 + e^{2 i (e + f x)}] \\ & \sec\left[\frac{1}{2} (e + f x)\right] \sec[e + f x]^2 \sqrt{a (1 + \sec[e + f x])} \sqrt{c - c \sec[e + f x]} \end{aligned}$$

Problem 87: Result unnecessarily involves imaginary or complex numbers.

$$\int \sqrt{a + a \sec[e + f x]} (c - c \sec[e + f x])^{5/2} dx$$

Optimal (type 3, 139 leaves, 4 steps) :

$$\begin{aligned} & \frac{a c^3 \log[\cos[e + f x]] \tan[e + f x]}{f \sqrt{a + a \sec[e + f x]} \sqrt{c - c \sec[e + f x]}} - \\ & \frac{a c^2 \sqrt{c - c \sec[e + f x]} \tan[e + f x]}{f \sqrt{a + a \sec[e + f x]}} - \frac{a c (c - c \sec[e + f x])^{3/2} \tan[e + f x]}{2 f \sqrt{a + a \sec[e + f x]}} \end{aligned}$$

Result (type 3, 162 leaves) :

$$\begin{aligned} & - \left(\left(c^2 e^{-3 i (e + f x)} (1 + e^{2 i (e + f x)})^3 \left(i + \cot\left[\frac{1}{2} (e + f x)\right] \right) \right. \right. \\ & \left. \left. (-1 - i f x + 4 \cos[e + f x] + \log[1 + e^{2 i (e + f x)}] + \cos[2 (e + f x)] (-i f x + \log[1 + e^{2 i (e + f x)}])) \right) \right. \\ & \left. \sec[e + f x]^4 \sqrt{a (1 + \sec[e + f x])} \sqrt{c - c \sec[e + f x]} \right) / (16 (1 + e^{i (e + f x)}) f) \end{aligned}$$

Problem 88: Result unnecessarily involves imaginary or complex numbers.

$$\int \sqrt{a + a \sec[e + f x]} (c - c \sec[e + f x])^{3/2} dx$$

Optimal (type 3, 93 leaves, 3 steps) :

$$\begin{aligned} & \frac{a c^2 \log[\cos[e + f x]] \tan[e + f x]}{f \sqrt{a + a \sec[e + f x]} \sqrt{c - c \sec[e + f x]}} - \frac{a c \sqrt{c - c \sec[e + f x]} \tan[e + f x]}{f \sqrt{a + a \sec[e + f x]}} \end{aligned}$$

Result (type 3, 99 leaves):

$$\frac{1}{(1 + e^{i(e+f x)})^{\frac{1}{2}} c} \left(\frac{1}{2} \operatorname{Cot} \left[\frac{1}{2} (e + f x) \right] \right) \\ (\frac{1}{2} + \operatorname{Cos}[e + f x] (f x + \frac{i}{2} \operatorname{Log}[1 + e^{2 i (e+f x)}])) \sqrt{a (1 + \operatorname{Sec}[e + f x])} \sqrt{c - c \operatorname{Sec}[e + f x]}$$

Problem 89: Result unnecessarily involves imaginary or complex numbers.

$$\int \sqrt{a + a \operatorname{Sec}[e + f x]} \sqrt{c - c \operatorname{Sec}[e + f x]} dx$$

Optimal (type 3, 48 leaves, 2 steps):

$$\frac{a c \operatorname{Log}[\operatorname{Cos}[e + f x]] \operatorname{Tan}[e + f x]}{f \sqrt{a + a \operatorname{Sec}[e + f x]} \sqrt{c - c \operatorname{Sec}[e + f x]}}$$

Result (type 3, 84 leaves):

$$-\left(\left((1 + e^{2 i (e+f x)}) (f x + \frac{i}{2} \operatorname{Log}[1 + e^{2 i (e+f x)}]) \sqrt{a (1 + \operatorname{Sec}[e + f x])} \sqrt{c - c \operatorname{Sec}[e + f x]} \right) / \right. \\ \left. \left((-1 + e^{2 i (e+f x)}) f \right) \right)$$

Problem 90: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{a + a \operatorname{Sec}[e + f x]}}{\sqrt{c - c \operatorname{Sec}[e + f x]}} dx$$

Optimal (type 3, 51 leaves, 2 steps):

$$\frac{a \operatorname{Log}[1 - \operatorname{Cos}[e + f x]] \operatorname{Tan}[e + f x]}{f \sqrt{a + a \operatorname{Sec}[e + f x]} \sqrt{c - c \operatorname{Sec}[e + f x]}}$$

Result (type 3, 86 leaves):

$$-\frac{(-1 + e^{i (e+f x)}) (f x + 2 \frac{i}{2} \operatorname{Log}[1 - e^{i (e+f x)}]) \sqrt{a (1 + \operatorname{Sec}[e + f x])}}{(1 + e^{i (e+f x)}) f \sqrt{c - c \operatorname{Sec}[e + f x]}}$$

Problem 91: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{a + a \operatorname{Sec}[e + f x]}}{(c - c \operatorname{Sec}[e + f x])^{3/2}} dx$$

Optimal (type 3, 96 leaves, 3 steps):

$$-\frac{a \operatorname{Tan}[e + f x]}{f \sqrt{a + a \operatorname{Sec}[e + f x]} (c - c \operatorname{Sec}[e + f x])^{3/2}} + \frac{a \operatorname{Log}[1 - \operatorname{Cos}[e + f x]] \operatorname{Tan}[e + f x]}{c f \sqrt{a + a \operatorname{Sec}[e + f x]} \sqrt{c - c \operatorname{Sec}[e + f x]}}$$

Result (type 3, 107 leaves):

$$\left(\left(-1 + i f x - 2 \log[1 - e^{i(e+f x)}] + \cos[e + f x] (-i f x + 2 \log[1 - e^{i(e+f x)}]) \right) \right. \\ \left. \sec[e + f x] \sqrt{a (1 + \sec[e + f x])} \tan\left[\frac{1}{2} (e + f x)\right] \right) / \left(f (c - c \sec[e + f x])^{3/2} \right)$$

Problem 92: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{a + a \sec[e + f x]}}{(c - c \sec[e + f x])^{5/2}} dx$$

Optimal (type 3, 142 leaves, 4 steps):

$$- \frac{a \tan[e + f x]}{2 f \sqrt{a + a \sec[e + f x]} (c - c \sec[e + f x])^{5/2}} - \\ \frac{a \tan[e + f x]}{c f \sqrt{a + a \sec[e + f x]} (c - c \sec[e + f x])^{3/2}} + \frac{a \log[1 - \cos[e + f x]] \tan[e + f x]}{c^2 f \sqrt{a + a \sec[e + f x]} \sqrt{c - c \sec[e + f x]}}$$

Result (type 3, 152 leaves):

$$\left((3 - 3 i f x + \cos[e + f x] (-4 + 4 i f x - 8 \log[1 - e^{i(e+f x)}])) + 6 \log[1 - e^{i(e+f x)}] + \right. \\ \left. \cos[2 (e + f x)] (-i f x + 2 \log[1 - e^{i(e+f x)}]) \right) \sqrt{a (1 + \sec[e + f x])} \tan\left[\frac{1}{2} (e + f x)\right] / \\ (2 c^2 f (-1 + \cos[e + f x])^2 \sqrt{c - c \sec[e + f x]})$$

Problem 93: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{a + a \sec[e + f x]}}{(c - c \sec[e + f x])^{7/2}} dx$$

Optimal (type 3, 188 leaves, 5 steps):

$$- \frac{a \tan[e + f x]}{3 f \sqrt{a + a \sec[e + f x]} (c - c \sec[e + f x])^{7/2}} - \frac{a \tan[e + f x]}{2 c f \sqrt{a + a \sec[e + f x]} (c - c \sec[e + f x])^{5/2}} - \\ \frac{a \tan[e + f x]}{c^2 f \sqrt{a + a \sec[e + f x]} (c - c \sec[e + f x])^{3/2}} + \frac{a \log[1 - \cos[e + f x]] \tan[e + f x]}{c^3 f \sqrt{a + a \sec[e + f x]} \sqrt{c - c \sec[e + f x]}}$$

Result (type 3, 198 leaves):

$$\left((-40 + 30 i f x - 3 i f x \cos[3 (e + f x)] + 18 i \cos[2 (e + f x)] (i + f x + 2 i \log[1 - e^{i(e+f x)}]) - \right. \\ \left. 60 \log[1 - e^{i(e+f x)}] + 6 \cos[3 (e + f x)] \log[1 - e^{i(e+f x)}] + \right. \\ \left. 9 \cos[e + f x] (6 - 5 i f x + 10 \log[1 - e^{i(e+f x)}]) \right) \sqrt{a (1 + \sec[e + f x])} \tan\left[\frac{1}{2} (e + f x)\right] / \\ (12 c^3 f (-1 + \cos[e + f x])^3 \sqrt{c - c \sec[e + f x]})$$

Problem 94: Result unnecessarily involves imaginary or complex numbers.

$$\int (a + a \sec(e + f x))^{3/2} (c - c \sec(e + f x))^{5/2} dx$$

Optimal (type 3, 190 leaves, 5 steps):

$$\begin{aligned} & \frac{a^2 c^3 \log[\cos(e + f x)] \tan[e + f x]}{f \sqrt{a + a \sec[e + f x]} \sqrt{c - c \sec[e + f x]}} - \frac{a^2 c^2 \sqrt{c - c \sec[e + f x]} \tan[e + f x]}{f \sqrt{a + a \sec[e + f x]}} - \\ & \frac{a^2 c (c - c \sec[e + f x])^{3/2} \tan[e + f x]}{2 f \sqrt{a + a \sec[e + f x]}} + \frac{a^2 (c - c \sec[e + f x])^{5/2} \tan[e + f x]}{3 f \sqrt{a + a \sec[e + f x]}} \end{aligned}$$

Result (type 3, 157 leaves):

$$\begin{aligned} & \frac{1}{24 f} i a c^2 \csc\left(\frac{1}{2} (e + f x)\right) (2 i + 6 i \cos[2 (e + f x)] + 3 f x \cos[3 (e + f x)] + \\ & \cos[e + f x] (6 i + 9 f x + 9 i \log[1 + e^{2 i (e + f x)}]) + 3 i \cos[3 (e + f x)] \log[1 + e^{2 i (e + f x)}]) \\ & \sec\left(\frac{1}{2} (e + f x)\right) \sec[e + f x]^2 \sqrt{a (1 + \sec[e + f x])} \sqrt{c - c \sec[e + f x]} \end{aligned}$$

Problem 95: Result unnecessarily involves imaginary or complex numbers.

$$\int (a + a \sec(e + f x))^{3/2} (c - c \sec(e + f x))^{3/2} dx$$

Optimal (type 3, 103 leaves, 3 steps):

$$\begin{aligned} & \frac{a^2 c^2 \log[\cos(e + f x)] \tan[e + f x]}{f \sqrt{a + a \sec[e + f x]} \sqrt{c - c \sec[e + f x]}} + \frac{a^2 c^2 \tan[e + f x]^3}{2 f \sqrt{a + a \sec[e + f x]} \sqrt{c - c \sec[e + f x]}} \end{aligned}$$

Result (type 3, 159 leaves):

$$\begin{aligned} & \frac{1}{8 (1 + e^{i (e + f x)}) f} i a c e^{-2 i (e + f x)} (1 + e^{2 i (e + f x)})^2 \left(i + \cot\left(\frac{1}{2} (e + f x)\right) \right) \\ & (\dot{x} + f x + \cos[2 (e + f x)] (f x + i \log[1 + e^{2 i (e + f x)}]) + i \log[1 + e^{2 i (e + f x)}]) \\ & \sec[e + f x]^3 \sqrt{a (1 + \sec[e + f x])} \sqrt{c - c \sec[e + f x]} \end{aligned}$$

Problem 96: Result unnecessarily involves imaginary or complex numbers.

$$\int (a + a \sec(e + f x))^{3/2} \sqrt{c - c \sec[e + f x]} dx$$

Optimal (type 3, 93 leaves, 3 steps):

$$\begin{aligned} & \frac{a^2 c \log[\cos(e + f x)] \tan[e + f x]}{f \sqrt{a + a \sec[e + f x]} \sqrt{c - c \sec[e + f x]}} - \frac{a c \sqrt{a + a \sec[e + f x]} \tan[e + f x]}{f \sqrt{c - c \sec[e + f x]}} \end{aligned}$$

Result (type 3, 128 leaves):

$$\frac{1}{2 (1 + e^{i (e+f x)})^f} a e^{-i (e+f x)} (1 + e^{2 i (e+f x)}) \left(\frac{i}{2} + \cot \left[\frac{1}{2} (e + f x) \right] \right) \\ (1 + \cos [e + f x] (i f x - \log [1 + e^{2 i (e+f x)}])) \sec [e + f x] \sqrt{a (1 + \sec [e + f x])} \sqrt{c - c \sec [e + f x]}$$

Problem 97: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(a + a \sec [e + f x])^{3/2}}{\sqrt{c - c \sec [e + f x]}} dx$$

Optimal (type 3, 104 leaves, 3 steps):

$$\frac{a^2 \log [\cos [e + f x]] \tan [e + f x]}{f \sqrt{a + a \sec [e + f x]} \sqrt{c - c \sec [e + f x]}} + \frac{2 a^2 \log [1 - \sec [e + f x]] \tan [e + f x]}{f \sqrt{a + a \sec [e + f x]} \sqrt{c - c \sec [e + f x]}}$$

Result (type 3, 105 leaves):

$$- \left(\left(a (-1 + e^{i (e+f x)}) (f x + 4 i \log [1 - e^{i (e+f x)}] - i \log [1 + e^{2 i (e+f x)}]) \sqrt{a (1 + \sec [e + f x])} \right) / \right. \\ \left. \left((1 + e^{i (e+f x)}) f \sqrt{c - c \sec [e + f x]} \right) \right)$$

Problem 98: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(a + a \sec [e + f x])^{3/2}}{(c - c \sec [e + f x])^{3/2}} dx$$

Optimal (type 3, 100 leaves, 3 steps):

$$- \frac{2 a^2 \tan [e + f x]}{f \sqrt{a + a \sec [e + f x]} (c - c \sec [e + f x])^{3/2}} + \frac{a^2 \log [1 - \cos [e + f x]] \tan [e + f x]}{c f \sqrt{a + a \sec [e + f x]} \sqrt{c - c \sec [e + f x]}}$$

Result (type 3, 115 leaves):

$$\left(a (-2 + i f x - 2 \log [1 - e^{i (e+f x)}] + \cos [e + f x] (-i f x + 2 \log [1 - e^{i (e+f x)}])) \right. \\ \left. \sqrt{a (1 + \sec [e + f x])} \tan \left[\frac{1}{2} (e + f x) \right] \right) / \left(c f (-1 + \cos [e + f x]) \sqrt{c - c \sec [e + f x]} \right)$$

Problem 99: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(a + a \sec [e + f x])^{3/2}}{(c - c \sec [e + f x])^{5/2}} dx$$

Optimal (type 3, 146 leaves, 4 steps):

$$- \frac{a^2 \tan [e + f x]}{f \sqrt{a + a \sec [e + f x]} (c - c \sec [e + f x])^{5/2}} - \\ \frac{a^2 \tan [e + f x]}{c f \sqrt{a + a \sec [e + f x]} (c - c \sec [e + f x])^{3/2}} + \frac{a^2 \log [1 - \cos [e + f x]] \tan [e + f x]}{c^2 f \sqrt{a + a \sec [e + f x]} \sqrt{c - c \sec [e + f x]}}$$

Result (type 3, 153 leaves) :

$$\begin{aligned} & \left(a \left(4 - 3 i f x + \cos[e + f x] \left(-6 + 4 i f x - 8 \log[1 - e^{i(e+f x)}] \right) + 6 \log[1 - e^{i(e+f x)}] \right. \right. \\ & \quad \left. \left. + \cos[2(e + f x)] (-i f x + 2 \log[1 - e^{i(e+f x)}]) \right) \sqrt{a (1 + \sec[e + f x])} \tan\left[\frac{1}{2}(e + f x)\right] \right) / \\ & \quad \left(2 c^2 f (-1 + \cos[e + f x])^2 \sqrt{c - c \sec[e + f x]} \right) \end{aligned}$$

Problem 100: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{(a + a \sec[e + f x])^{3/2}}{(c - c \sec[e + f x])^{7/2}} dx$$

Optimal (type 3, 196 leaves, 5 steps) :

$$\begin{aligned} & - \frac{2 a^2 \tan[e + f x]}{3 f \sqrt{a + a \sec[e + f x]} (c - c \sec[e + f x])^{7/2}} - \frac{a^2 \tan[e + f x]}{2 c f \sqrt{a + a \sec[e + f x]} (c - c \sec[e + f x])^{5/2}} - \\ & \frac{a^2 \tan[e + f x]}{c^2 f \sqrt{a + a \sec[e + f x]} (c - c \sec[e + f x])^{3/2}} + \frac{a^2 \log[1 - \cos[e + f x]] \tan[e + f x]}{c^3 f \sqrt{a + a \sec[e + f x]} \sqrt{c - c \sec[e + f x]}} \end{aligned}$$

Result (type 3, 489 leaves) :

$$\begin{aligned}
& \left(8 \sqrt[2]{e^{\frac{1}{2} i (e+f x)}} \sqrt{\frac{(1 + e^{i (e+f x)})^2}{1 + e^{2 i (e+f x)}}} \right. \\
& \left. \left(f x + 2 i \operatorname{Log}[1 - e^{i (e+f x)}] \right) \operatorname{Sec}[e + f x]^{7/2} (a (1 + \operatorname{Sec}[e + f x]))^{3/2} \sin\left[\frac{e}{2} + \frac{f x}{2}\right]^7 \right) / \\
& \left((1 + e^{i (e+f x)}) \sqrt{\frac{e^{i (e+f x)}}{1 + e^{2 i (e+f x)}}} f (1 + \operatorname{Sec}[e + f x])^{3/2} (c - c \operatorname{Sec}[e + f x])^{7/2} \right) + \\
& \left(\operatorname{Sec}[e + f x]^4 \sqrt{(1 + \cos[e + f x]) \operatorname{Sec}[e + f x]} (a (1 + \operatorname{Sec}[e + f x]))^{3/2} \right. \\
& \left(-\frac{61 \operatorname{Cot}\left[\frac{e}{2}\right] \operatorname{Csc}\left[\frac{e}{2} + \frac{f x}{2}\right]}{3 f} + \frac{17 \operatorname{Cot}\left[\frac{e}{2}\right] \operatorname{Csc}\left[\frac{e}{2} + \frac{f x}{2}\right]^3}{3 f} - \right. \\
& \left. \left. \frac{2 \operatorname{Cot}\left[\frac{e}{2}\right] \operatorname{Csc}\left[\frac{e}{2} + \frac{f x}{2}\right]^5}{3 f} + \frac{35 \operatorname{Sec}\left[\frac{e}{2} + \frac{f x}{2}\right]}{3 f} + \frac{61 \operatorname{Csc}\left[\frac{e}{2}\right] \operatorname{Csc}\left[\frac{e}{2} + \frac{f x}{2}\right]^2 \sin\left[\frac{f x}{2}\right]}{3 f} - \right. \right. \\
& \left. \left. \frac{17 \operatorname{Csc}\left[\frac{e}{2}\right] \operatorname{Csc}\left[\frac{e}{2} + \frac{f x}{2}\right]^4 \sin\left[\frac{f x}{2}\right]}{3 f} + \frac{2 \operatorname{Csc}\left[\frac{e}{2}\right] \operatorname{Csc}\left[\frac{e}{2} + \frac{f x}{2}\right]^6 \sin\left[\frac{f x}{2}\right]}{3 f} \right) \right. \\
& \left. \sin\left[\frac{e}{2} + \frac{f x}{2}\right]^7 \right) / ((1 + \operatorname{Sec}[e + f x])^{3/2} (c - c \operatorname{Sec}[e + f x])^{7/2})
\end{aligned}$$

Problem 101: Result unnecessarily involves imaginary or complex numbers.

$$\int (a + a \operatorname{Sec}[e + f x])^{5/2} (c - c \operatorname{Sec}[e + f x])^{5/2} dx$$

Optimal (type 3, 153 leaves, 4 steps) :

$$\begin{aligned}
& \frac{a^3 c^3 \operatorname{Log}[\cos[e + f x]] \tan[e + f x]}{f \sqrt{a + a \operatorname{Sec}[e + f x]} \sqrt{c - c \operatorname{Sec}[e + f x]}} + \\
& \frac{a^3 c^3 \tan[e + f x]^3}{2 f \sqrt{a + a \operatorname{Sec}[e + f x]} \sqrt{c - c \operatorname{Sec}[e + f x]}} - \frac{a^3 c^3 \tan[e + f x]^5}{4 f \sqrt{a + a \operatorname{Sec}[e + f x]} \sqrt{c - c \operatorname{Sec}[e + f x]}}
\end{aligned}$$

Result (type 3, 164 leaves) :

$$\begin{aligned}
& \frac{1}{16 f} i a^2 c^2 \operatorname{Csc}\left[\frac{1}{2} (e + f x)\right] (2 i + 3 f x + \cos[4 (e + f x)] (f x + i \operatorname{Log}[1 + e^{2 i (e+f x)}])) + \\
& 4 \cos[2 (e + f x)] (i + f x + i \operatorname{Log}[1 + e^{2 i (e+f x)}]) + 3 i \operatorname{Log}[1 + e^{2 i (e+f x)}] \\
& \operatorname{Sec}\left[\frac{1}{2} (e + f x)\right] \operatorname{Sec}[e + f x]^3 \sqrt{a (1 + \operatorname{Sec}[e + f x])} \sqrt{c - c \operatorname{Sec}[e + f x]}
\end{aligned}$$

Problem 102: Result unnecessarily involves imaginary or complex numbers.

$$\int (a + a \operatorname{Sec}[e + f x])^{5/2} (c - c \operatorname{Sec}[e + f x])^{3/2} dx$$

Optimal (type 3, 190 leaves, 5 steps) :

$$\begin{aligned} & \frac{a^3 c^2 \operatorname{Log}[\operatorname{Cos}[e + f x]] \operatorname{Tan}[e + f x]}{f \sqrt{a + a \operatorname{Sec}[e + f x]} \sqrt{c - c \operatorname{Sec}[e + f x]}} - \frac{a^2 c^2 \sqrt{a + a \operatorname{Sec}[e + f x]} \operatorname{Tan}[e + f x]}{f \sqrt{c - c \operatorname{Sec}[e + f x]}} - \\ & \frac{a c^2 (a + a \operatorname{Sec}[e + f x])^{3/2} \operatorname{Tan}[e + f x]}{2 f \sqrt{c - c \operatorname{Sec}[e + f x]}} + \frac{c^2 (a + a \operatorname{Sec}[e + f x])^{5/2} \operatorname{Tan}[e + f x]}{3 f \sqrt{c - c \operatorname{Sec}[e + f x]}} \end{aligned}$$

Result (type 3, 149 leaves) :

$$\begin{aligned} & \frac{1}{24 f} a^2 c \operatorname{Csc}\left[\frac{1}{2} (e + f x)\right] (2 + 6 \operatorname{Cos}[2 (e + f x)] + 3 i f x \operatorname{Cos}[3 (e + f x)]) + \\ & \operatorname{Cos}[e + f x] (-6 + 9 i f x - 9 \operatorname{Log}[1 + e^{2 i (e + f x)}]) - 3 \operatorname{Cos}[3 (e + f x)] \operatorname{Log}[1 + e^{2 i (e + f x)}] \\ & \operatorname{Sec}\left[\frac{1}{2} (e + f x)\right] \operatorname{Sec}[e + f x]^2 \sqrt{a (1 + \operatorname{Sec}[e + f x])} \sqrt{c - c \operatorname{Sec}[e + f x]} \end{aligned}$$

Problem 103: Result unnecessarily involves imaginary or complex numbers.

$$\int (a + a \operatorname{Sec}[e + f x])^{5/2} \sqrt{c - c \operatorname{Sec}[e + f x]} dx$$

Optimal (type 3, 139 leaves, 4 steps) :

$$\begin{aligned} & \frac{a^3 c \operatorname{Log}[\operatorname{Cos}[e + f x]] \operatorname{Tan}[e + f x]}{f \sqrt{a + a \operatorname{Sec}[e + f x]} \sqrt{c - c \operatorname{Sec}[e + f x]}} - \\ & \frac{a^2 c \sqrt{a + a \operatorname{Sec}[e + f x]} \operatorname{Tan}[e + f x]}{f \sqrt{c - c \operatorname{Sec}[e + f x]}} - \frac{a c (a + a \operatorname{Sec}[e + f x])^{3/2} \operatorname{Tan}[e + f x]}{2 f \sqrt{c - c \operatorname{Sec}[e + f x]}} \end{aligned}$$

Result (type 3, 164 leaves) :

$$\begin{aligned} & \frac{1}{4 (1 + e^{i (e + f x)}) f} a^2 e^{-i (e + f x)} (1 + e^{2 i (e + f x)}) \left(i + \operatorname{Cot}\left[\frac{1}{2} (e + f x)\right] \right) \\ & (1 + i f x + 4 \operatorname{Cos}[e + f x] + \operatorname{Cos}[2 (e + f x)] (i f x - \operatorname{Log}[1 + e^{2 i (e + f x)}]) - \operatorname{Log}[1 + e^{2 i (e + f x)}]) \\ & \operatorname{Sec}[e + f x]^2 \sqrt{a (1 + \operatorname{Sec}[e + f x])} \sqrt{c - c \operatorname{Sec}[e + f x]} \end{aligned}$$

Problem 104: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(a + a \operatorname{Sec}[e + f x])^{5/2}}{\sqrt{c - c \operatorname{Sec}[e + f x]}} dx$$

Optimal (type 3, 152 leaves, 3 steps) :

$$\frac{a^3 \log[\cos[e+f x]] \tan[e+f x]}{f \sqrt{a+a \sec[e+f x]} \sqrt{c-c \sec[e+f x]}} + \frac{4 a^3 \log[1-\sec[e+f x]] \tan[e+f x]}{f \sqrt{a+a \sec[e+f x]} \sqrt{c-c \sec[e+f x]}} + \frac{a^3 \sec[e+f x] \tan[e+f x]}{f \sqrt{a+a \sec[e+f x]} \sqrt{c-c \sec[e+f x]}}$$

Result (type 3, 198 leaves):

$$\left(\left(1 + \cos[e+f x] \left(-\frac{i}{2} f x + 8 \log[1 - e^{i(e+f x)}] - 3 \log[1 + e^{2i(e+f x)}] \right) \right) \sqrt{\sec\left[\frac{1}{2}(e+f x)\right]^2} \right. \\ \left. \sec[e+f x] (a (1 + \sec[e+f x]))^{5/2} \left(\cos\left[\frac{1}{2}(e+f x)\right] + i \sin\left[\frac{1}{2}(e+f x)\right] \right) \sin\left[\frac{1}{2}(e+f x)\right] \right) / \\ \left(\left(1 + e^{i(e+f x)} \right) \sqrt{\frac{e^{i(e+f x)}}{1 + e^{2i(e+f x)}}} f (1 + \sec[e+f x])^{3/2} \sqrt{c - c \sec[e+f x]} \right)$$

Problem 105: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(a+a \sec[e+f x])^{5/2}}{(c-c \sec[e+f x])^{3/2}} dx$$

Optimal (type 3, 96 leaves, 3 steps):

$$- \frac{4 a^3 \tan[e+f x]}{f \sqrt{a+a \sec[e+f x]} (c-c \sec[e+f x])^{3/2}} + \frac{a^3 \log[\cos[e+f x]] \tan[e+f x]}{c f \sqrt{a+a \sec[e+f x]} \sqrt{c-c \sec[e+f x]}}$$

Result (type 3, 111 leaves):

$$\left(a^2 (-4 + i f x - \log[1 + e^{2i(e+f x)}] + \cos[e+f x] (-i f x + \log[1 + e^{2i(e+f x)}])) \right. \\ \left. \sqrt{a (1 + \sec[e+f x])} \tan\left[\frac{1}{2}(e+f x)\right] \right) / \left(c f (-1 + \cos[e+f x]) \sqrt{c - c \sec[e+f x]} \right)$$

Problem 106: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(a+a \sec[e+f x])^{5/2}}{(c-c \sec[e+f x])^{5/2}} dx$$

Optimal (type 3, 100 leaves, 3 steps):

$$- \frac{2 a^3 \tan[e+f x]}{f \sqrt{a+a \sec[e+f x]} (c-c \sec[e+f x])^{5/2}} + \frac{a^3 \log[1 - \cos[e+f x]] \tan[e+f x]}{c^2 f \sqrt{a+a \sec[e+f x]} \sqrt{c-c \sec[e+f x]}}$$

Result (type 3, 155 leaves):

$$\left(a^2 \left(4 - 3 \operatorname{f} x + \cos[e + f x] \left(-8 + 4 \operatorname{f} x - 8 \log[1 - e^{i (e+f x)}] \right) + 6 \log[1 - e^{i (e+f x)}] + \cos[2 (e + f x)] (-\operatorname{f} x + 2 \log[1 - e^{i (e+f x)}]) \right) \sqrt{a (1 + \sec[e + f x])} \tan\left[\frac{1}{2} (e + f x)\right] \right) / \\ \left(2 c^2 f (-1 + \cos[e + f x])^2 \sqrt{c - c \sec[e + f x]} \right)$$

Problem 107: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{(a + a \sec[e + f x])^{5/2}}{(c - c \sec[e + f x])^{7/2}} dx$$

Optimal (type 3, 148 leaves, 4 steps):

$$- \frac{4 a^3 \tan[e + f x]}{3 f \sqrt{a + a \sec[e + f x]} (c - c \sec[e + f x])^{7/2}} - \\ \frac{a^3 \tan[e + f x]}{c^2 f \sqrt{a + a \sec[e + f x]} (c - c \sec[e + f x])^{3/2}} + \frac{a^3 \log[1 - \cos[e + f x]] \tan[e + f x]}{c^3 f \sqrt{a + a \sec[e + f x]} \sqrt{c - c \sec[e + f x]}}$$

Result (type 3, 489 leaves):

$$\begin{aligned}
& \left(8 \pm \sqrt{2} e^{\frac{1}{2} i (e+fx)} \sqrt{\frac{(1 + e^{i(e+fx)})^2}{1 + e^{2i(e+fx)}}} \right. \\
& \quad \left. \left(fx + 2 \pm \log[1 - e^{i(e+fx)}] \right) \sec[e+fx]^{7/2} (a(1 + \sec[e+fx]))^{5/2} \sin\left[\frac{e}{2} + \frac{fx}{2}\right]^7 \right) / \\
& \quad \left((1 + e^{i(e+fx)}) \sqrt{\frac{e^{i(e+fx)}}{1 + e^{2i(e+fx)}}} f (1 + \sec[e+fx])^{5/2} (c - c \sec[e+fx])^{7/2} \right) + \\
& \quad \left(\sec[e+fx]^4 \sqrt{(1 + \cos[e+fx]) \sec[e+fx]} (a(1 + \sec[e+fx]))^{5/2} \right. \\
& \quad \left. - \frac{80 \cot\left[\frac{e}{2}\right] \csc\left[\frac{e}{2} + \frac{fx}{2}\right]}{3f} + \frac{28 \cot\left[\frac{e}{2}\right] \csc\left[\frac{e}{2} + \frac{fx}{2}\right]^3}{3f} - \right. \\
& \quad \left. \frac{4 \cot\left[\frac{e}{2}\right] \csc\left[\frac{e}{2} + \frac{fx}{2}\right]^5}{3f} + \frac{40 \sec\left[\frac{e}{2} + \frac{fx}{2}\right]}{3f} + \frac{80 \csc\left[\frac{e}{2}\right] \csc\left[\frac{e}{2} + \frac{fx}{2}\right]^2 \sin\left[\frac{fx}{2}\right]}{3f} - \right. \\
& \quad \left. \frac{28 \csc\left[\frac{e}{2}\right] \csc\left[\frac{e}{2} + \frac{fx}{2}\right]^4 \sin\left[\frac{fx}{2}\right]}{3f} + \frac{4 \csc\left[\frac{e}{2}\right] \csc\left[\frac{e}{2} + \frac{fx}{2}\right]^6 \sin\left[\frac{fx}{2}\right]}{3f} \right) \\
& \quad \left. \sin\left[\frac{e}{2} + \frac{fx}{2}\right]^7 \right) / ((1 + \sec[e+fx])^{5/2} (c - c \sec[e+fx])^{7/2})
\end{aligned}$$

Problem 108: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(a + a \sec[e+fx])^{5/2}}{(c - c \sec[e+fx])^{9/2}} dx$$

Optimal (type 3, 194 leaves, 5 steps):

$$\begin{aligned}
& - \frac{a^3 \tan[e+fx]}{f \sqrt{a + a \sec[e+fx]} (c - c \sec[e+fx])^{9/2}} - \frac{a^3 \tan[e+fx]}{2c^2 f \sqrt{a + a \sec[e+fx]} (c - c \sec[e+fx])^{5/2}} - \\
& \quad \frac{a^3 \tan[e+fx]}{c^3 f \sqrt{a + a \sec[e+fx]} (c - c \sec[e+fx])^{3/2}} + \frac{a^3 \log[1 - \cos[e+fx]] \tan[e+fx]}{c^4 f \sqrt{a + a \sec[e+fx]} \sqrt{c - c \sec[e+fx]}}
\end{aligned}$$

Result (type 3, 285 leaves):

$$\begin{aligned}
& \left(\frac{\operatorname{Sec}[e+f x]^{9/2} (a (1 + \operatorname{Sec}[e+f x]))^{5/2}}{16 \sqrt{2} e^{\frac{1}{2} i (e+f x)} \sqrt{\frac{(1+e^i (e+f x))^2}{1+e^{2 i} (e+f x)}} (-i f x + 2 \operatorname{Log}[1 - e^{i (e+f x)}])} + \frac{1}{8 f} \right. \\
& \left. \left(1 + e^{i (e+f x)} \right) \sqrt{\frac{e^i (e+f x)}{1+e^{2 i} (e+f x)}} f \right. \\
& \left. (-54 + 89 \cos[e+f x] - 60 \cos[2 (e+f x)] + 23 \cos[3 (e+f x)] - 6 \cos[4 (e+f x)]) \right. \\
& \left. \csc[\frac{1}{2} (e+f x)]^8 \operatorname{Sec}[\frac{1}{2} (e+f x)] \sqrt{\operatorname{Sec}[e+f x]} \sqrt{1 + \operatorname{Sec}[e+f x]} \right. \\
& \left. \sin[\frac{1}{2} (e+f x)]^9 \right) / \left((1 + \operatorname{Sec}[e+f x])^{5/2} (c - c \operatorname{Sec}[e+f x])^{9/2} \right)
\end{aligned}$$

Problem 109: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{(a + a \operatorname{Sec}[e+f x])^{5/2}}{(c - c \operatorname{Sec}[e+f x])^{11/2}} \mathrm{d}x$$

Optimal (type 3, 244 leaves, 6 steps) :

$$\begin{aligned}
& - \frac{4 a^3 \tan[e+f x]}{5 f \sqrt{a+a \operatorname{Sec}[e+f x]} (c - c \operatorname{Sec}[e+f x])^{11/2}} - \\
& - \frac{a^3 \tan[e+f x]}{3 c^2 f \sqrt{a+a \operatorname{Sec}[e+f x]} (c - c \operatorname{Sec}[e+f x])^{7/2}} - \frac{a^3 \tan[e+f x]}{2 c^3 f \sqrt{a+a \operatorname{Sec}[e+f x]} (c - c \operatorname{Sec}[e+f x])^{5/2}} - \\
& + \frac{a^3 \tan[e+f x]}{c^4 f \sqrt{a+a \operatorname{Sec}[e+f x]} (c - c \operatorname{Sec}[e+f x])^{3/2}} + \frac{a^3 \operatorname{Log}[1 - \cos[e+f x]] \tan[e+f x]}{c^5 f \sqrt{a+a \operatorname{Sec}[e+f x]} \sqrt{c - c \operatorname{Sec}[e+f x]}}
\end{aligned}$$

Result (type 3, 615 leaves) :

$$\begin{aligned}
& \left(32 \pm \sqrt{2} e^{\frac{1}{2} i (e+f x)} \sqrt{\frac{(1 + e^{i (e+f x)})^2}{1 + e^{2 i (e+f x)}}} (f x + 2 \pm \text{Log}[1 - e^{i (e+f x)}]) \right. \\
& \quad \left. \text{Sec}[e + f x]^{11/2} (a (1 + \text{Sec}[e + f x]))^{5/2} \sin\left[\frac{e}{2} + \frac{f x}{2}\right]^{11} \right) / \\
& \left((1 + e^{i (e+f x)}) \sqrt{\frac{e^{i (e+f x)}}{1 + e^{2 i (e+f x)}}} f (1 + \text{Sec}[e + f x])^{5/2} (c - c \text{Sec}[e + f x])^{11/2} \right) + \\
& \frac{1}{(1 + \text{Sec}[e + f x])^{5/2} (c - c \text{Sec}[e + f x])^{11/2}} \\
& \text{Sec}[e + f x]^6 \sqrt{(1 + \cos[e + f x]) \text{Sec}[e + f x]} (a (1 + \text{Sec}[e + f x]))^{5/2} \\
& \left(-\frac{2428 \cot\left[\frac{e}{2}\right] \csc\left[\frac{e}{2} + \frac{f x}{2}\right]}{15 f} + \frac{1532 \cot\left[\frac{e}{2}\right] \csc\left[\frac{e}{2} + \frac{f x}{2}\right]^3}{15 f} - \frac{608 \cot\left[\frac{e}{2}\right] \csc\left[\frac{e}{2} + \frac{f x}{2}\right]^5}{15 f} + \right. \\
& \quad \frac{44 \cot\left[\frac{e}{2}\right] \csc\left[\frac{e}{2} + \frac{f x}{2}\right]^7}{5 f} - \frac{4 \cot\left[\frac{e}{2}\right] \csc\left[\frac{e}{2} + \frac{f x}{2}\right]^9}{5 f} + \frac{932 \text{Sec}\left[\frac{e}{2} + \frac{f x}{2}\right]}{15 f} + \\
& \quad \frac{2428 \csc\left[\frac{e}{2}\right] \csc\left[\frac{e}{2} + \frac{f x}{2}\right]^2 \sin\left[\frac{f x}{2}\right]}{15 f} - \frac{1532 \csc\left[\frac{e}{2}\right] \csc\left[\frac{e}{2} + \frac{f x}{2}\right]^4 \sin\left[\frac{f x}{2}\right]}{15 f} + \\
& \quad \frac{608 \csc\left[\frac{e}{2}\right] \csc\left[\frac{e}{2} + \frac{f x}{2}\right]^6 \sin\left[\frac{f x}{2}\right]}{15 f} - \frac{44 \csc\left[\frac{e}{2}\right] \csc\left[\frac{e}{2} + \frac{f x}{2}\right]^8 \sin\left[\frac{f x}{2}\right]}{15 f} + \\
& \quad \left. \frac{4 \csc\left[\frac{e}{2}\right] \csc\left[\frac{e}{2} + \frac{f x}{2}\right]^{10} \sin\left[\frac{f x}{2}\right]}{5 f} \right) \sin\left[\frac{e}{2} + \frac{f x}{2}\right]^{11}
\end{aligned}$$

Problem 110: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(c - c \text{Sec}[e + f x])^{7/2}}{\sqrt{a + a \text{Sec}[e + f x]}} dx$$

Optimal (type 3, 204 leaves, 3 steps):

$$\begin{aligned}
& \frac{c^4 \text{Log}[\cos[e + f x]] \tan[e + f x]}{f \sqrt{a + a \text{Sec}[e + f x]} \sqrt{c - c \text{Sec}[e + f x]}} + \frac{8 c^4 \text{Log}[1 + \text{Sec}[e + f x]] \tan[e + f x]}{f \sqrt{a + a \text{Sec}[e + f x]} \sqrt{c - c \text{Sec}[e + f x]}} - \\
& \frac{4 c^4 \text{Sec}[e + f x] \tan[e + f x]}{f \sqrt{a + a \text{Sec}[e + f x]} \sqrt{c - c \text{Sec}[e + f x]}} + \frac{c^4 \text{Sec}[e + f x]^2 \tan[e + f x]}{2 f \sqrt{a + a \text{Sec}[e + f x]} \sqrt{c - c \text{Sec}[e + f x]}}
\end{aligned}$$

Result (type 3, 153 leaves):

$$\left(c^3 \operatorname{Cot} \left[\frac{1}{2} (e + f x) \right] (-1 + i f x + 8 \cos[e + f x] - 16 \log[1 + e^{i(e+f x)}] + 7 \log[1 + e^{2i(e+f x)}] + \cos[2(e + f x)] (i f x - 16 \log[1 + e^{i(e+f x)}] + 7 \log[1 + e^{2i(e+f x)}])) \right. \\ \left. \sec[e + f x]^2 \sqrt{c - c \sec[e + f x]} \right) / \left(2 f \sqrt{a (1 + \sec[e + f x])} \right)$$

Problem 111: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{(c - c \sec[e + f x])^{5/2}}{\sqrt{a + a \sec[e + f x]}} dx$$

Optimal (type 3, 151 leaves, 3 steps):

$$\frac{c^3 \log[\cos[e + f x]] \tan[e + f x]}{f \sqrt{a + a \sec[e + f x]} \sqrt{c - c \sec[e + f x]}} + \frac{4 c^3 \log[1 + \sec[e + f x]] \tan[e + f x]}{f \sqrt{a + a \sec[e + f x]} \sqrt{c - c \sec[e + f x]}} - \frac{c^3 \sec[e + f x] \tan[e + f x]}{f \sqrt{a + a \sec[e + f x]} \sqrt{c - c \sec[e + f x]}}$$

Result (type 3, 315 leaves):

$$\left(e^{\frac{1}{2} i (e+f x)} \sqrt{\frac{(1 + e^{i(e+f x)})^2}{1 + e^{2i(e+f x)}}} \csc\left[\frac{e}{2} + \frac{f x}{2}\right]^5 \right. \\ \left. \left(\frac{i f x - 8 \log[1 + e^{i(e+f x)}] + 3 \log[1 + e^{2i(e+f x)}]}{\sqrt{1 + \sec[e + f x]} (c - c \sec[e + f x])^{5/2}} \right) \right) / \\ \left(4 \sqrt{2} (1 + e^{i(e+f x)}) \sqrt{\frac{e^{i(e+f x)}}{1 + e^{2i(e+f x)}}} f \sec[e + f x]^{5/2} \sqrt{a (1 + \sec[e + f x])} \right. \\ \left. \left(\cos[e + f x]^2 \csc\left[\frac{e}{2} + \frac{f x}{2}\right]^5 \sec\left[\frac{e}{2} + \frac{f x}{2}\right] \sqrt{(1 + \cos[e + f x]) \sec[e + f x]} \right. \right. \\ \left. \left. \sqrt{1 + \sec[e + f x]} (c - c \sec[e + f x])^{5/2} \right) \right) / \left(8 f \sqrt{a (1 + \sec[e + f x])} \right)$$

Problem 112: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(c - c \sec[e + f x])^{3/2}}{\sqrt{a + a \sec[e + f x]}} dx$$

Optimal (type 3, 102 leaves, 3 steps):

$$\frac{c^2 \log[\cos[e + f x]] \tan[e + f x]}{f \sqrt{a + a \sec[e + f x]} \sqrt{c - c \sec[e + f x]}} + \frac{2 c^2 \log[1 + \sec[e + f x]] \tan[e + f x]}{f \sqrt{a + a \sec[e + f x]} \sqrt{c - c \sec[e + f x]}}$$

Result (type 3, 103 leaves):

$$-\left(\left(c \left(1 + e^{i(e+f x)} \right) \left(f x + 4 i \log \left[1 + e^{i(e+f x)} \right] - i \log \left[1 + e^{2i(e+f x)} \right] \right) \sqrt{c - c \sec[e+f x]} \right) \right. \\ \left. \left((-1 + e^{i(e+f x)}) f \sqrt{a (1 + \sec[e+f x])} \right) \right)$$

Problem 113: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{c - c \sec[e+f x]}}{\sqrt{a + a \sec[e+f x]}} dx$$

Optimal (type 3, 49 leaves, 2 steps):

$$\frac{c \log[1 + \cos[e+f x]] \tan[e+f x]}{f \sqrt{a + a \sec[e+f x]} \sqrt{c - c \sec[e+f x]}}$$

Result (type 3, 105 leaves):

$$-\frac{\left(1 + e^{i(e+f x)} \right) \sqrt{\frac{c (-1 + e^{i(e+f x)})^2}{1 + e^{2i(e+f x)}}} \left(f x + 2 i \log \left[1 + e^{i(e+f x)} \right] \right)}{(-1 + e^{i(e+f x)}) f \sqrt{a (1 + \sec[e+f x])}}$$

Problem 114: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{a + a \sec[e+f x]} \sqrt{c - c \sec[e+f x]}} dx$$

Optimal (type 3, 46 leaves, 2 steps):

$$\frac{\log[\sin[e+f x]] \tan[e+f x]}{f \sqrt{a + a \sec[e+f x]} \sqrt{c - c \sec[e+f x]}}$$

Result (type 3, 122 leaves):

$$-\left(\left(2 (-1 + e^{i(e+f x)}) \cos \left[\frac{1}{2} (e+f x) \right]^2 (f x + i \log[1 - e^{i(e+f x)}] + i \log[1 + e^{i(e+f x)}]) \right. \right. \\ \left. \left. \sec[e+f x] \right) \right) \left/ \left((1 + e^{i(e+f x)}) f \sqrt{a (1 + \sec[e+f x])} \sqrt{c - c \sec[e+f x]} \right) \right)$$

Problem 115: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{a + a \sec[e+f x]} (c - c \sec[e+f x])^{3/2}} dx$$

Optimal (type 3, 168 leaves, 3 steps):

$$\frac{\tan[e+f x]}{2 c f (1 - \cos[e+f x]) \sqrt{a + a \sec[e+f x]} \sqrt{c - c \sec[e+f x]}} + \frac{3 \log[1 - \cos[e+f x]] \tan[e+f x]}{4 c f \sqrt{a + a \sec[e+f x]} \sqrt{c - c \sec[e+f x]}} + \frac{\log[1 + \cos[e+f x]] \tan[e+f x]}{4 c f \sqrt{a + a \sec[e+f x]} \sqrt{c - c \sec[e+f x]}}$$

Result (type 3, 143 leaves):

$$\left(\left(-1 + 2 i f x - 3 \log[1 - e^{i (e+f x)}] - \log[1 + e^{i (e+f x)}] + \cos[e+f x] (-2 i f x + 3 \log[1 - e^{i (e+f x)}] + \log[1 + e^{i (e+f x)}]) \right) \tan[e+f x] \right) / \left(2 c f (-1 + \cos[e+f x]) \sqrt{a (1 + \sec[e+f x])} \sqrt{c - c \sec[e+f x]} \right)$$

Problem 116: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{a + a \sec[e+f x]} (c - c \sec[e+f x])^{5/2}} dx$$

Optimal (type 3, 274 leaves, 3 steps):

$$\begin{aligned} & \frac{\log[\cos[e+f x]] \tan[e+f x]}{c^2 f \sqrt{a + a \sec[e+f x]} \sqrt{c - c \sec[e+f x]}} + \\ & \frac{7 \log[1 - \sec[e+f x]] \tan[e+f x]}{8 c^2 f \sqrt{a + a \sec[e+f x]} \sqrt{c - c \sec[e+f x]}} + \frac{\log[1 + \sec[e+f x]] \tan[e+f x]}{8 c^2 f \sqrt{a + a \sec[e+f x]} \sqrt{c - c \sec[e+f x]}} - \\ & \frac{\tan[e+f x]}{4 c^2 f (1 - \sec[e+f x])^2 \sqrt{a + a \sec[e+f x]} \sqrt{c - c \sec[e+f x]}} - \\ & \frac{3 \tan[e+f x]}{4 c^2 f (1 - \sec[e+f x]) \sqrt{a + a \sec[e+f x]} \sqrt{c - c \sec[e+f x]}} \end{aligned}$$

Result (type 3, 194 leaves):

$$\left(\left(8 - 12 i f x + 21 \log[1 - e^{i (e+f x)}] + \cos[e+f x] (-10 + 16 i f x - 28 \log[1 - e^{i (e+f x)}] - 4 \log[1 + e^{i (e+f x)}]) + 3 \log[1 + e^{i (e+f x)}] + \cos[2 (e+f x)] (-4 i f x + 7 \log[1 - e^{i (e+f x)}] + \log[1 + e^{i (e+f x)}]) \right) \tan[e+f x] \right) / \left(8 c^2 f (-1 + \cos[e+f x])^2 \sqrt{a (1 + \sec[e+f x])} \sqrt{c - c \sec[e+f x]} \right)$$

Problem 117: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(c - c \sec[e+f x])^{7/2}}{(a + a \sec[e+f x])^{3/2}} dx$$

Optimal (type 3, 215 leaves, 3 steps):

$$\begin{aligned} & \frac{c^4 \log[\cos[e+f x]] \tan[e+f x]}{a f \sqrt{a+a \sec[e+f x]} \sqrt{c-c \sec[e+f x]}} - \\ & \frac{4 c^4 \log[1+\sec[e+f x]] \tan[e+f x]}{a f \sqrt{a+a \sec[e+f x]} \sqrt{c-c \sec[e+f x]}} + \frac{c^4 \sec[e+f x] \tan[e+f x]}{a f \sqrt{a+a \sec[e+f x]} \sqrt{c-c \sec[e+f x]}} - \\ & \frac{8 c^4 \tan[e+f x]}{a f (1+\sec[e+f x]) \sqrt{a+a \sec[e+f x]} \sqrt{c-c \sec[e+f x]}} \end{aligned}$$

Result (type 3, 204 leaves):

$$\begin{aligned} & \left(c^3 \cot\left(\frac{1}{2}(e+f x)\right) (-2 + i f x + 8 \log[1 + e^{i(e+f x)}]) + \right. \\ & 2 \cos[e+f x] (-9 + i f x + 8 \log[1 + e^{i(e+f x)}] - 5 \log[1 + e^{2i(e+f x)}]) + \\ & \cos[2(e+f x)] (i f x + 8 \log[1 + e^{i(e+f x)}] - 5 \log[1 + e^{2i(e+f x)}] - 5 \log[1 + e^{4i(e+f x)}]) \\ & \left. \sec[e+f x] \sqrt{c-c \sec[e+f x]}\right) / \left(2 a f (1 + \cos[e+f x]) \sqrt{a (1 + \sec[e+f x])}\right) \end{aligned}$$

Problem 118: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(c-c \sec[e+f x])^{5/2}}{(a+a \sec[e+f x])^{3/2}} dx$$

Optimal (type 3, 96 leaves, 3 steps):

$$-\frac{4 c^3 \tan[e+f x]}{f (a+a \sec[e+f x])^{3/2} \sqrt{c-c \sec[e+f x]}} + \frac{c^3 \log[\cos[e+f x]] \tan[e+f x]}{a f \sqrt{a+a \sec[e+f x]} \sqrt{c-c \sec[e+f x]}}$$

Result (type 3, 116 leaves):

$$\begin{aligned} & \left(\frac{i}{2} c^2 \cot\left(\frac{1}{2}(e+f x)\right) (4 \frac{i}{2} + f x + \cos[e+f x] (f x + i \log[1 + e^{2i(e+f x)}])) + i \log[1 + e^{2i(e+f x)}] \right. \\ & \left. \sqrt{c-c \sec[e+f x]}\right) / \left(a f (1 + \cos[e+f x]) \sqrt{a (1 + \sec[e+f x])}\right) \end{aligned}$$

Problem 119: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(c-c \sec[e+f x])^{3/2}}{(a+a \sec[e+f x])^{3/2}} dx$$

Optimal (type 3, 98 leaves, 3 steps):

$$-\frac{2 c^2 \tan[e+f x]}{f (a+a \sec[e+f x])^{3/2} \sqrt{c-c \sec[e+f x]}} + \frac{c^2 \log[1+\cos[e+f x]] \tan[e+f x]}{a f \sqrt{a+a \sec[e+f x]} \sqrt{c-c \sec[e+f x]}}$$

Result (type 3, 114 leaves):

$$\begin{aligned} & \left(\frac{i}{2} c \cot\left(\frac{1}{2}(e+f x)\right) (2 \frac{i}{2} + f x + \cos[e+f x] (f x + 2 i \log[1 + e^{i(e+f x)}])) + 2 i \log[1 + e^{i(e+f x)}] \right. \\ & \left. \sqrt{c-c \sec[e+f x]}\right) / \left(a f (1 + \cos[e+f x]) \sqrt{a (1 + \sec[e+f x])}\right) \end{aligned}$$

Problem 120: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{c - c \sec[e + f x]}}{(a + a \sec[e + f x])^{3/2}} dx$$

Optimal (type 3, 94 leaves, 3 steps):

$$-\frac{c \tan[e + f x]}{f (a + a \sec[e + f x])^{3/2} \sqrt{c - c \sec[e + f x]}} + \frac{c \log[1 + \cos[e + f x]] \tan[e + f x]}{a f \sqrt{a + a \sec[e + f x]} \sqrt{c - c \sec[e + f x]}}$$

Result (type 3, 106 leaves):

$$\left(\frac{1}{2} \cot\left(\frac{1}{2}(e + f x)\right) \left(\frac{1}{2} + f x + \cos[e + f x] (f x + 2 \frac{1}{2} \log[1 + e^{\frac{1}{2}(e+f x)}]) + 2 \frac{1}{2} \log[1 + e^{\frac{1}{2}(e+f x)}] \right) \right. \\ \left. \sec[e + f x] \sqrt{c - c \sec[e + f x]} \right) \Big/ \left(f (a (1 + \sec[e + f x]))^{3/2} \right)$$

Problem 121: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(a + a \sec[e + f x])^{3/2} \sqrt{c - c \sec[e + f x]}} dx$$

Optimal (type 3, 215 leaves, 3 steps):

$$\frac{\log[\cos[e + f x]] \tan[e + f x]}{a f \sqrt{a + a \sec[e + f x]} \sqrt{c - c \sec[e + f x]}} + \\ \frac{\log[1 - \sec[e + f x]] \tan[e + f x]}{4 a f \sqrt{a + a \sec[e + f x]} \sqrt{c - c \sec[e + f x]}} + \frac{3 \log[1 + \sec[e + f x]] \tan[e + f x]}{4 a f \sqrt{a + a \sec[e + f x]} \sqrt{c - c \sec[e + f x]}} - \\ \frac{\tan[e + f x]}{2 a f (1 + \sec[e + f x]) \sqrt{a + a \sec[e + f x]} \sqrt{c - c \sec[e + f x]}}$$

Result (type 3, 141 leaves):

$$\left((1 - 2 \frac{1}{2} f x + \log[1 - e^{\frac{1}{2}(e+f x)}] + 3 \log[1 + e^{\frac{1}{2}(e+f x)}] + \right. \\ \left. \cos[e + f x] (-2 \frac{1}{2} f x + \log[1 - e^{\frac{1}{2}(e+f x)}] + 3 \log[1 + e^{\frac{1}{2}(e+f x)}]) \right) \tan[e + f x] \Big/ \\ \left(2 a f (1 + \cos[e + f x]) \sqrt{a (1 + \sec[e + f x])} \sqrt{c - c \sec[e + f x]} \right)$$

Problem 122: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(a + a \sec[e + f x])^{3/2} (c - c \sec[e + f x])^{3/2}} dx$$

Optimal (type 3, 101 leaves, 3 steps):

$$\frac{\cot[e + f x]}{2 a c f \sqrt{a + a \sec[e + f x]} \sqrt{c - c \sec[e + f x]}} + \frac{\log[\sin[e + f x]] \tan[e + f x]}{a c f \sqrt{a + a \sec[e + f x]} \sqrt{c - c \sec[e + f x]}}$$

Result (type 3, 151 leaves):

$$\left(\left(1 - \frac{1}{2} f x + \log[1 - e^{i(e+f x)}] + \cos[2(e+f x)] \right) \left(\frac{1}{2} f x - \log[1 - e^{i(e+f x)}] - \log[1 + e^{i(e+f x)}] \right) + \log[1 + e^{i(e+f x)}] \right) \sec[e+f x]^2 \tan[e+f x] \Big/ \\ \left(2 c f (-1 + \sec[e+f x]) (a (1 + \sec[e+f x]))^{3/2} \sqrt{c - c \sec[e+f x]} \right)$$

Problem 123: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(a + a \sec[e+f x])^{3/2} (c - c \sec[e+f x])^{5/2}} dx$$

Optimal (type 3, 347 leaves, 3 steps):

$$\frac{\log[\cos[e+f x]] \tan[e+f x]}{a c^2 f \sqrt{a + a \sec[e+f x]} \sqrt{c - c \sec[e+f x]}} + \frac{11 \log[1 - \sec[e+f x]] \tan[e+f x]}{16 a c^2 f \sqrt{a + a \sec[e+f x]} \sqrt{c - c \sec[e+f x]}} + \\ \frac{5 \log[1 + \sec[e+f x]] \tan[e+f x]}{16 a c^2 f \sqrt{a + a \sec[e+f x]} \sqrt{c - c \sec[e+f x]}} - \\ \frac{\tan[e+f x]}{8 a c^2 f (1 - \sec[e+f x])^2 \sqrt{a + a \sec[e+f x]} \sqrt{c - c \sec[e+f x]}} - \\ \frac{\tan[e+f x]}{2 a c^2 f (1 - \sec[e+f x]) \sqrt{a + a \sec[e+f x]} \sqrt{c - c \sec[e+f x]}} - \\ \frac{\tan[e+f x]}{8 a c^2 f (1 + \sec[e+f x]) \sqrt{a + a \sec[e+f x]} \sqrt{c - c \sec[e+f x]}}$$

Result (type 3, 275 leaves):

$$\left(\left(14 - 16 \frac{1}{2} f x - 8 \frac{1}{2} f x \cos[3(e+f x)] + 22 \log[1 - e^{i(e+f x)}] + 11 \cos[3(e+f x)] \log[1 - e^{i(e+f x)}] + \cos[e+f x] (-12 + 8 \frac{1}{2} f x - 11 \log[1 - e^{i(e+f x)}] - 5 \log[1 + e^{i(e+f x)}]) + 2 \cos[2(e+f x)] (-5 + 8 \frac{1}{2} f x - 11 \log[1 - e^{i(e+f x)}] - 5 \log[1 + e^{i(e+f x)}]) + 10 \log[1 + e^{i(e+f x)}] + 5 \cos[3(e+f x)] \log[1 + e^{i(e+f x)}] \right) \tan[e+f x] \right) \Big/ \\ \left(32 a c^2 f (-1 + \cos[e+f x])^2 (1 + \cos[e+f x]) \sqrt{a (1 + \sec[e+f x])} \sqrt{c - c \sec[e+f x]} \right)$$

Problem 124: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(c - c \sec[e+f x])^{7/2}}{(a + a \sec[e+f x])^{5/2}} dx$$

Optimal (type 3, 220 leaves, 3 steps):

$$\frac{c^4 \log[\cos[e+f x]] \tan[e+f x]}{a^2 f \sqrt{a + a \sec[e+f x]} \sqrt{c - c \sec[e+f x]}} + \frac{2 c^4 \log[1 + \sec[e+f x]] \tan[e+f x]}{a^2 f \sqrt{a + a \sec[e+f x]} \sqrt{c - c \sec[e+f x]}} - \\ \frac{4 c^4 \tan[e+f x]}{a^2 f (1 + \sec[e+f x])^2 \sqrt{a + a \sec[e+f x]} \sqrt{c - c \sec[e+f x]}} + \\ \frac{4 c^4 \tan[e+f x]}{a^2 f (1 + \sec[e+f x]) \sqrt{a + a \sec[e+f x]} \sqrt{c - c \sec[e+f x]}}$$

Result (type 3, 157 leaves) :

$$\left(c^3 \operatorname{Cot} \left[\frac{1}{2} (e + f x) \right] (4 \cos [e + f x] (-2 + i f x - 4 \log [1 + e^{i (e+f x)}] + \log [1 + e^{2 i (e+f x)}]) + (3 + \cos [2 (e + f x)]) (i f x - 4 \log [1 + e^{i (e+f x)}] + \log [1 + e^{2 i (e+f x)}])) \sqrt{c - c \sec [e + f x]} \right) / \right.$$

$$\left. \left(2 a^2 f (1 + \cos [e + f x])^2 \sqrt{a (1 + \sec [e + f x])} \right) \right)$$

Problem 125: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(c - c \sec [e + f x])^{5/2}}{(a + a \sec [e + f x])^{5/2}} dx$$

Optimal (type 3, 98 leaves, 3 steps) :

$$- \frac{2 c^3 \tan [e + f x]}{f (a + a \sec [e + f x])^{5/2} \sqrt{c - c \sec [e + f x]}} + \frac{c^3 \log [1 + \cos [e + f x]] \tan [e + f x]}{a^2 f \sqrt{a + a \sec [e + f x]} \sqrt{c - c \sec [e + f x]}}$$

Result (type 3, 154 leaves) :

$$\left(i c^2 \operatorname{Cot} \left[\frac{1}{2} (e + f x) \right] (4 i + 3 f x + \cos [2 (e + f x)] (f x + 2 i \log [1 + e^{i (e+f x)}]) + 4 \cos [e + f x] (2 i + f x + 2 i \log [1 + e^{i (e+f x)}]) + 6 i \log [1 + e^{i (e+f x)}]) \sqrt{c - c \sec [e + f x]} \right) / \right.$$

$$\left. \left(2 a^2 f (1 + \cos [e + f x])^2 \sqrt{a (1 + \sec [e + f x])} \right) \right)$$

Problem 126: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(c - c \sec [e + f x])^{3/2}}{(a + a \sec [e + f x])^{5/2}} dx$$

Optimal (type 3, 144 leaves, 4 steps) :

$$- \frac{c^2 \tan [e + f x]}{f (a + a \sec [e + f x])^{5/2} \sqrt{c - c \sec [e + f x]}} -$$

$$+ \frac{c^2 \tan [e + f x]}{a f (a + a \sec [e + f x])^{3/2} \sqrt{c - c \sec [e + f x]}} + \frac{c^2 \log [1 + \cos [e + f x]] \tan [e + f x]}{a^2 f \sqrt{a + a \sec [e + f x]} \sqrt{c - c \sec [e + f x]}}$$

Result (type 3, 152 leaves) :

$$\left(i c \operatorname{Cot} \left[\frac{1}{2} (e + f x) \right] (4 i + 3 f x + \cos [2 (e + f x)] (f x + 2 i \log [1 + e^{i (e+f x)}]) + \cos [e + f x] (6 i + 4 f x + 8 i \log [1 + e^{i (e+f x)}]) + 6 i \log [1 + e^{i (e+f x)}]) \sqrt{c - c \sec [e + f x]} \right) / \right.$$

$$\left. \left(2 a^2 f (1 + \cos [e + f x])^2 \sqrt{a (1 + \sec [e + f x])} \right) \right)$$

Problem 127: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{c - c \sec[e + f x]}}{(a + a \sec[e + f x])^{5/2}} dx$$

Optimal (type 3, 140 leaves, 4 steps):

$$-\frac{c \tan[e + f x]}{2 f (a + a \sec[e + f x])^{5/2} \sqrt{c - c \sec[e + f x]}} - \frac{c \tan[e + f x]}{a f (a + a \sec[e + f x])^{3/2} \sqrt{c - c \sec[e + f x]}} + \frac{c \log[1 + \cos[e + f x]] \tan[e + f x]}{a^2 f \sqrt{a + a \sec[e + f x]} \sqrt{c - c \sec[e + f x]}}$$

Result (type 3, 151 leaves):

$$\left(\frac{1}{2} \cot\left[\frac{1}{2} (e + f x)\right] (3 i + 3 f x + \cos[2 (e + f x)] (f x + 2 i \log[1 + e^{i (e+f x)}])) + 4 \cos[e + f x] (i + f x + 2 i \log[1 + e^{i (e+f x)}]) + 6 i \log[1 + e^{i (e+f x)}] \right) \sqrt{c - c \sec[e + f x]} \\ \left(2 a^2 f (1 + \cos[e + f x])^2 \sqrt{a (1 + \sec[e + f x])} \right)$$

Problem 128: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(a + a \sec[e + f x])^{5/2} \sqrt{c - c \sec[e + f x]}} dx$$

Optimal (type 3, 270 leaves, 3 steps):

$$\frac{\log[\cos[e + f x]] \tan[e + f x]}{a^2 f \sqrt{a + a \sec[e + f x]} \sqrt{c - c \sec[e + f x]}} + \frac{\log[1 - \sec[e + f x]] \tan[e + f x]}{8 a^2 f \sqrt{a + a \sec[e + f x]} \sqrt{c - c \sec[e + f x]}} + \frac{7 \log[1 + \sec[e + f x]] \tan[e + f x]}{8 a^2 f \sqrt{a + a \sec[e + f x]} \sqrt{c - c \sec[e + f x]}} - \frac{4 a^2 f (1 + \sec[e + f x])^2 \sqrt{a + a \sec[e + f x]} \sqrt{c - c \sec[e + f x]}}{3 \tan[e + f x]} \\ \frac{4 a^2 f (1 + \sec[e + f x]) \sqrt{a + a \sec[e + f x]} \sqrt{c - c \sec[e + f x]}}{}$$

Result (type 3, 195 leaves):

$$\left((8 - 12 i f x + 3 \log[1 - e^{i (e+f x)}] + 21 \log[1 + e^{i (e+f x)}] + \cos[2 (e + f x)] (-4 i f x + \log[1 - e^{i (e+f x)}] + 7 \log[1 + e^{i (e+f x)}])) + 2 \cos[e + f x] (5 - 8 i f x + 2 \log[1 - e^{i (e+f x)}] + 14 \log[1 + e^{i (e+f x)}]) \right) \tan[e + f x] \\ \left(8 a^2 f (1 + \cos[e + f x])^2 \sqrt{a (1 + \sec[e + f x])} \sqrt{c - c \sec[e + f x]} \right)$$

Problem 129: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(a + a \sec(e + f x))^{5/2} (c - c \sec(e + f x))^{3/2}} dx$$

Optimal (type 3, 345 leaves, 3 steps):

$$\begin{aligned} & \frac{\log[\cos(e + f x)] \tan(e + f x)}{a^2 c f \sqrt{a + a \sec(e + f x)} \sqrt{c - c \sec(e + f x)}} + \frac{5 \log[1 - \sec(e + f x)] \tan(e + f x)}{16 a^2 c f \sqrt{a + a \sec(e + f x)} \sqrt{c - c \sec(e + f x)}} + \\ & \frac{11 \log[1 + \sec(e + f x)] \tan(e + f x)}{16 a^2 c f \sqrt{a + a \sec(e + f x)} \sqrt{c - c \sec(e + f x)}} - \\ & \frac{\tan(e + f x)}{8 a^2 c f (1 - \sec(e + f x)) \sqrt{a + a \sec(e + f x)} \sqrt{c - c \sec(e + f x)}} - \\ & \frac{\tan(e + f x)}{8 a^2 c f (1 + \sec(e + f x))^2 \sqrt{a + a \sec(e + f x)} \sqrt{c - c \sec(e + f x)}} - \\ & \frac{2 a^2 c f (1 + \sec(e + f x)) \sqrt{a + a \sec(e + f x)} \sqrt{c - c \sec(e + f x)}} \end{aligned}$$

Result (type 3, 275 leaves):

$$\begin{aligned} & \left((-14 + 16 i f x - 8 i f x \cos[3(e + f x)] - 10 \log[1 - e^{i(e+f x)}] + 5 \cos[3(e + f x)] \log[1 - e^{i(e+f x)}] + \right. \\ & \cos[e + f x] (-12 + 8 i f x - 5 \log[1 - e^{i(e+f x)}] - 11 \log[1 + e^{i(e+f x)}]) - \\ & 22 \log[1 + e^{i(e+f x)}] + 11 \cos[3(e + f x)] \log[1 + e^{i(e+f x)}] + \\ & \left. 2 \cos[2(e + f x)] (5 - 8 i f x + 5 \log[1 - e^{i(e+f x)}] + 11 \log[1 + e^{i(e+f x)}]) \right) \tan(e + f x) \Bigg) / \\ & \left(32 a^2 c f (-1 + \cos[e + f x]) (1 + \cos[e + f x])^2 \sqrt{a (1 + \sec[e + f x])} \sqrt{c - c \sec[e + f x]} \right) \end{aligned}$$

Problem 130: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(a + a \sec(e + f x))^{5/2} (c - c \sec(e + f x))^{5/2}} dx$$

Optimal (type 3, 151 leaves, 4 steps):

$$\begin{aligned} & \frac{\cot[e + f x]}{2 a^2 c^2 f \sqrt{a + a \sec[e + f x]} \sqrt{c - c \sec[e + f x]}} - \\ & \frac{\cot[e + f x]^3}{4 a^2 c^2 f \sqrt{a + a \sec[e + f x]} \sqrt{c - c \sec[e + f x]}} + \frac{\log[\sin[e + f x]] \tan[e + f x]}{a^2 c^2 f \sqrt{a + a \sec[e + f x]} \sqrt{c - c \sec[e + f x]}} \end{aligned}$$

Result (type 3, 195 leaves):

$$\begin{aligned} & \left(\csc[e + f x]^3 (2 - 3 i f x + 3 \log[1 - e^{i(e+f x)}] + \right. \\ & \cos[2(e + f x)] (-4 + 4 i f x - 4 \log[1 - e^{i(e+f x)}] - 4 \log[1 + e^{i(e+f x)}]) + 3 \log[1 + e^{i(e+f x)}] + \\ & \left. \cos[4(e + f x)] (-i f x + \log[1 - e^{i(e+f x)}] + \log[1 + e^{i(e+f x)}]) \right) \sec[e + f x] \Bigg) / \\ & \left(8 a^2 c^2 f \sqrt{a (1 + \sec[e + f x])} \sqrt{c - c \sec[e + f x]} \right) \end{aligned}$$

Problem 131: Unable to integrate problem.

$$\int (1 + \sec(e + fx))^m (c - c \sec(e + fx))^n dx$$

Optimal (type 6, 92 leaves, 2 steps):

$$\left(\frac{2^{\frac{1}{2}+m}}{f} \text{AppellF1}\left[\frac{1}{2}+n, \frac{1}{2}-m, 1, \frac{3}{2}+n, \frac{1}{2} (1-\sec(e+fx)), 1-\sec(e+fx) \right] \right. \\ \left. (c - c \sec(e+fx))^n \tan(e+fx) \right) / \left(f (1+2n) \sqrt{1+\sec(e+fx)} \right)$$

Result (type 8, 26 leaves):

$$\int (1 + \sec(e + fx))^m (c - c \sec(e + fx))^n dx$$

Problem 132: Unable to integrate problem.

$$\int (a + a \sec(e + fx))^m (c - c \sec(e + fx))^n dx$$

Optimal (type 6, 109 leaves, 3 steps):

$$\frac{1}{f (1+2m)} 2^{\frac{1}{2}+n} c \text{AppellF1}\left[\frac{1}{2}+m, \frac{1}{2}-n, 1, \frac{3}{2}+m, \frac{1}{2} (1+\sec(e+fx)), 1+\sec(e+fx) \right] \\ (1-\sec(e+fx))^{\frac{1}{2}-n} (a + a \sec(e + fx))^m (c - c \sec(e + fx))^{-1+n} \tan(e+fx)$$

Result (type 8, 28 leaves):

$$\int (a + a \sec(e + fx))^m (c - c \sec(e + fx))^n dx$$

Problem 133: Unable to integrate problem.

$$\int (a + a \sec(e + fx))^3 (c - c \sec(e + fx))^n dx$$

Optimal (type 6, 101 leaves, 3 steps):

$$\frac{1}{7f} 2^{\frac{1}{2}+n} c \text{AppellF1}\left[\frac{7}{2}, \frac{1}{2}-n, 1, \frac{9}{2}, \frac{1}{2} (1+\sec(e+fx)), 1+\sec(e+fx) \right] \\ (1-\sec(e+fx))^{\frac{1}{2}-n} (a + a \sec(e + fx))^3 (c - c \sec(e + fx))^{-1+n} \tan(e+fx)$$

Result (type 8, 28 leaves):

$$\int (a + a \sec(e + fx))^3 (c - c \sec(e + fx))^n dx$$

Problem 134: Unable to integrate problem.

$$\int (a + a \sec(e + fx))^2 (c - c \sec(e + fx))^n dx$$

Optimal (type 6, 101 leaves, 3 steps) :

$$\frac{1}{5 f} 2^{\frac{1}{2}+n} c \operatorname{AppellF1}\left[\frac{5}{2}, \frac{1}{2}-n, 1, \frac{7}{2}, \frac{1}{2} (1+\sec[e+f x]), 1+\sec[e+f x]\right] \\ (1-\sec[e+f x])^{\frac{1}{2}-n} (a+a \sec[e+f x])^2 (c-c \sec[e+f x])^{-1+n} \tan[e+f x]$$

Result (type 8, 28 leaves) :

$$\int (a+a \sec[e+f x])^2 (c-c \sec[e+f x])^n dx$$

Problem 135: Unable to integrate problem.

$$\int (a+a \sec[e+f x]) (c-c \sec[e+f x])^n dx$$

Optimal (type 6, 99 leaves, 3 steps) :

$$\frac{1}{3 f} 2^{\frac{1}{2}+n} c \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2}-n, 1, \frac{5}{2}, \frac{1}{2} (1+\sec[e+f x]), 1+\sec[e+f x]\right] \\ (1-\sec[e+f x])^{\frac{1}{2}-n} (a+a \sec[e+f x]) (c-c \sec[e+f x])^{-1+n} \tan[e+f x]$$

Result (type 8, 26 leaves) :

$$\int (a+a \sec[e+f x]) (c-c \sec[e+f x])^n dx$$

Problem 136: Unable to integrate problem.

$$\int \frac{(c-c \sec[e+f x])^n}{a+a \sec[e+f x]} dx$$

Optimal (type 6, 99 leaves, 3 steps) :

$$-\left(\left(2^{\frac{1}{2}+n} c \operatorname{AppellF1}\left[-\frac{1}{2}, \frac{1}{2}-n, 1, \frac{1}{2}, \frac{1}{2} (1+\sec[e+f x]), 1+\sec[e+f x]\right]\right.\right. \\ \left.\left.(1-\sec[e+f x])^{\frac{1}{2}-n} (c-c \sec[e+f x])^{-1+n} \tan[e+f x]\right)\right) / (f (a+a \sec[e+f x]))$$

Result (type 8, 28 leaves) :

$$\int \frac{(c-c \sec[e+f x])^n}{a+a \sec[e+f x]} dx$$

Problem 137: Unable to integrate problem.

$$\int \frac{(c-c \sec[e+f x])^n}{(a+a \sec[e+f x])^2} dx$$

Optimal (type 6, 101 leaves, 3 steps) :

$$-\left(\left(2^{\frac{1}{2}+n} c \text{AppellF1}\left[-\frac{3}{2}, \frac{1}{2}-n, 1, -\frac{1}{2}, \frac{1}{2} (1+\sec[e+f x]), 1+\sec[e+f x]\right] \right. \right. \\ \left. \left. (1-\sec[e+f x])^{\frac{1}{2}-n} (c-c \sec[e+f x])^{-1+n} \tan[e+f x] \right) \right) / \left(3 f (a+a \sec[e+f x])^2 \right)$$

Result (type 8, 28 leaves):

$$\int \frac{(c-c \sec[e+f x])^n}{(a+a \sec[e+f x])^2} dx$$

Problem 138: Unable to integrate problem.

$$\int (a+a \sec[e+f x])^{5/2} (c-c \sec[e+f x])^n dx$$

Optimal (type 5, 172 leaves, 4 steps):

$$\frac{6 a^3 (c-c \sec[e+f x])^n \tan[e+f x]}{f (1+2 n) \sqrt{a+a \sec[e+f x]}} + \\ \left(2 a^3 \text{Hypergeometric2F1}\left[1, \frac{1}{2}+n, \frac{3}{2}+n, 1-\sec[e+f x]\right] (c-c \sec[e+f x])^n \tan[e+f x] \right) / \\ \left(f (1+2 n) \sqrt{a+a \sec[e+f x]} \right) - \frac{2 a^3 (c-c \sec[e+f x])^{1+n} \tan[e+f x]}{c f (3+2 n) \sqrt{a+a \sec[e+f x]}}$$

Result (type 8, 30 leaves):

$$\int (a+a \sec[e+f x])^{5/2} (c-c \sec[e+f x])^n dx$$

Problem 139: Unable to integrate problem.

$$\int (a+a \sec[e+f x])^{3/2} (c-c \sec[e+f x])^n dx$$

Optimal (type 5, 119 leaves, 3 steps):

$$\frac{2 a^2 (c-c \sec[e+f x])^n \tan[e+f x]}{f (1+2 n) \sqrt{a+a \sec[e+f x]}} + \\ \left(2 a^2 \text{Hypergeometric2F1}\left[1, \frac{1}{2}+n, \frac{3}{2}+n, 1-\sec[e+f x]\right] (c-c \sec[e+f x])^n \tan[e+f x] \right) / \\ \left(f (1+2 n) \sqrt{a+a \sec[e+f x]} \right)$$

Result (type 8, 30 leaves):

$$\int (a+a \sec[e+f x])^{3/2} (c-c \sec[e+f x])^n dx$$

Problem 140: Unable to integrate problem.

$$\int \sqrt{a + a \sec[e + f x]} \ (c - c \sec[e + f x])^n dx$$

Optimal (type 5, 68 leaves, 2 steps):

$$\left(2 \text{Hypergeometric2F1}\left[1, \frac{1}{2} + n, \frac{3}{2} + n, 1 - \sec[e + f x]\right] (c - c \sec[e + f x])^n \tan[e + f x] \right) / \\ \left(f (1 + 2 n) \sqrt{a + a \sec[e + f x]} \right)$$

Result (type 8, 30 leaves):

$$\int \sqrt{a + a \sec[e + f x]} \ (c - c \sec[e + f x])^n dx$$

Problem 141: Unable to integrate problem.

$$\int \frac{(c - c \sec[e + f x])^n}{\sqrt{a + a \sec[e + f x]}} dx$$

Optimal (type 5, 139 leaves, 4 steps):

$$- \left(\left(\text{Hypergeometric2F1}\left[1, \frac{1}{2} + n, \frac{3}{2} + n, \frac{1}{2} (1 - \sec[e + f x])\right] (c - c \sec[e + f x])^n \tan[e + f x] \right) / \right. \\ \left. \left(f (1 + 2 n) \sqrt{a + a \sec[e + f x]} \right) \right) + \\ \left(2 \text{Hypergeometric2F1}\left[1, \frac{1}{2} + n, \frac{3}{2} + n, 1 - \sec[e + f x]\right] (c - c \sec[e + f x])^n \tan[e + f x] \right) / \\ \left(f (1 + 2 n) \sqrt{a + a \sec[e + f x]} \right)$$

Result (type 8, 30 leaves):

$$\int \frac{(c - c \sec[e + f x])^n}{\sqrt{a + a \sec[e + f x]}} dx$$

Problem 142: Unable to integrate problem.

$$\int \frac{(c - c \sec[e + f x])^n}{(a + a \sec[e + f x])^{3/2}} dx$$

Optimal (type 5, 205 leaves, 5 steps):

$$\begin{aligned}
& - \left(\left((5 - 2n) \text{Hypergeometric2F1}\left[1, \frac{1}{2} + n, \frac{3}{2} + n, \frac{1}{2} (1 - \text{Sec}[e + fx])\right] \right. \right. \\
& \quad \left. \left. (c - c \text{Sec}[e + fx])^n \tan[e + fx]\right) \Big/ \left(4af (1 + 2n) \sqrt{a + a \text{Sec}[e + fx]} \right) \right) + \\
& \left(2 \text{Hypergeometric2F1}\left[1, \frac{1}{2} + n, \frac{3}{2} + n, 1 - \text{Sec}[e + fx]\right] (c - c \text{Sec}[e + fx])^n \tan[e + fx] \right) \Big/ \\
& \quad \left(af (1 + 2n) \sqrt{a + a \text{Sec}[e + fx]} \right) - \frac{(c - c \text{Sec}[e + fx])^n \tan[e + fx]}{2af (1 + \text{Sec}[e + fx]) \sqrt{a + a \text{Sec}[e + fx]}}
\end{aligned}$$

Result (type 8, 30 leaves) :

$$\int \frac{(c - c \text{Sec}[e + fx])^n}{(a + a \text{Sec}[e + fx])^{3/2}} dx$$

Problem 143: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{a + a \text{Sec}[e + fx]}}{c + c \text{Sec}[e + fx]} dx$$

Optimal (type 3, 91 leaves, 6 steps) :

$$\frac{2\sqrt{a} \text{ArcTan}\left[\frac{\sqrt{a} \tan[e+fx]}{\sqrt{a+a \text{Sec}[e+fx]}}\right]}{c f} - \frac{\sqrt{2} \sqrt{a} \text{ArcTan}\left[\frac{\sqrt{a} \tan[e+fx]}{\sqrt{2} \sqrt{a+a \text{Sec}[e+fx]}}\right]}{c f}$$

Result (type 3, 168 leaves) :

$$\begin{aligned}
& \frac{1}{c (1 + e^{\pm (e+fx)}) f} \\
& \sqrt{1 + e^{2 \pm (e+fx)}} \left(f x - \pm \text{ArcSinh}\left[e^{\pm (e+fx)}\right] + \pm \sqrt{2} \log\left[1 + e^{\pm (e+fx)}\right] + \pm \log\left[1 + \sqrt{1 + e^{2 \pm (e+fx)}}\right] - \right. \\
& \quad \left. \pm \sqrt{2} \log\left[1 - e^{\pm (e+fx)} + \sqrt{2} \sqrt{1 + e^{2 \pm (e+fx)}}\right] \right) \sqrt{a (1 + \text{Sec}[e + fx])}
\end{aligned}$$

Problem 146: Unable to integrate problem.

$$\int \frac{1}{(a + a \text{Sec}[e + fx]) \sqrt{c + d \text{Sec}[e + fx]}} dx$$

Optimal (type 4, 319 leaves, 5 steps) :

$$\begin{aligned}
& \frac{1}{a(c-d)f} 2\sqrt{c+d} \cot[e+f x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{c+d} \sec[e+f x]}{\sqrt{c+d}}\right], \frac{c+d}{c-d}\right] \\
& \sqrt{\frac{d(1-\sec[e+f x])}{c+d}} - \sqrt{-\frac{d(1+\sec[e+f x])}{c-d}} - \frac{1}{a c f} \\
& 2\sqrt{c+d} \cot[e+f x] \operatorname{EllipticPi}\left[\frac{c+d}{c}, \operatorname{ArcSin}\left[\frac{\sqrt{c+d} \sec[e+f x]}{\sqrt{c+d}}\right], \frac{c+d}{c-d}\right] \\
& \sqrt{\frac{d(1-\sec[e+f x])}{c+d}} - \sqrt{-\frac{d(1+\sec[e+f x])}{c-d}} - \\
& \left(\operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\tan[e+f x]}{1+\sec[e+f x]}\right], \frac{c-d}{c+d}\right] \sqrt{\frac{1}{1+\sec[e+f x]}} \sqrt{c+d \sec[e+f x]}\right) / \\
& \left(a(c-d)f \sqrt{\frac{c+d \sec[e+f x]}{(c+d)(1+\sec[e+f x])}}\right)
\end{aligned}$$

Result (type 8, 29 leaves):

$$\int \frac{1}{(a+a \sec[e+f x]) \sqrt{c+d \sec[e+f x]}} dx$$

Problem 147: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \sqrt{a+a \sec[e+f x]} (c+d \sec[e+f x])^4 dx$$

Optimal (type 3, 271 leaves, 5 steps):

$$\begin{aligned}
& \frac{2 a d (2 c + d) (2 c^2 + 2 c d + d^2) \tan[e+f x]}{f \sqrt{a+a \sec[e+f x]}} + \frac{2 a^{3/2} c^4 \operatorname{ArcTanh}\left[\frac{\sqrt{a-a \sec[e+f x]}}{\sqrt{a}}\right] \tan[e+f x]}{f \sqrt{a-a \sec[e+f x]} \sqrt{a+a \sec[e+f x]}} - \\
& \frac{2 d^2 (6 c^2 + 8 c d + 3 d^2) (a - a \sec[e+f x]) \tan[e+f x]}{3 f \sqrt{a+a \sec[e+f x]}} + \\
& \frac{2 d^3 (4 c + 3 d) (a - a \sec[e+f x])^2 \tan[e+f x]}{5 a f \sqrt{a+a \sec[e+f x]}} - \frac{2 d^4 (a - a \sec[e+f x])^3 \tan[e+f x]}{7 a^2 f \sqrt{a+a \sec[e+f x]}}
\end{aligned}$$

Result (type 4, 589 leaves):

$$\begin{aligned}
& \frac{1}{f (d + c \cos[e + f x])^4} \cos[e + f x]^4 \sec[\frac{1}{2} (e + f x)] \sqrt{a (1 + \sec[e + f x])} (c + d \sec[e + f x])^4 \\
& \left(\frac{8}{105} d (105 c^3 + 105 c^2 d + 56 c d^2 + 12 d^3) \sin[\frac{1}{2} (e + f x)] + \frac{2}{7} d^4 \sec[e + f x]^3 \sin[\frac{1}{2} (e + f x)] + \right. \\
& \frac{4}{35} \sec[e + f x]^2 \left(14 c d^3 \sin[\frac{1}{2} (e + f x)] + 3 d^4 \sin[\frac{1}{2} (e + f x)] \right) + \frac{4}{105} \sec[e + f x] \\
& \left. \left(105 c^2 d^2 \sin[\frac{1}{2} (e + f x)] + 56 c d^3 \sin[\frac{1}{2} (e + f x)] + 12 d^4 \sin[\frac{1}{2} (e + f x)] \right) \right) - \\
& \frac{1}{f (d + c \cos[e + f x])^4} 8 (-3 - 2 \sqrt{2}) c^4 \cos[\frac{1}{4} (e + f x)]^4 \\
& \sqrt{\frac{7 - 5 \sqrt{2} + (10 - 7 \sqrt{2}) \cos[\frac{1}{2} (e + f x)]}{1 + \cos[\frac{1}{2} (e + f x)]}} \sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \cos[\frac{1}{2} (e + f x)]}{1 + \cos[\frac{1}{2} (e + f x)]}} \\
& \left(1 - \sqrt{2} + (-2 + \sqrt{2}) \cos[\frac{1}{2} (e + f x)] \right) \cos[e + f x]^3 \\
& \left(\text{EllipticF}[\text{ArcSin}[\frac{\tan[\frac{1}{4} (e + f x)]}{\sqrt{3 - 2 \sqrt{2}}}], 17 - 12 \sqrt{2}] + \right. \\
& \left. 2 \text{EllipticPi}[-3 + 2 \sqrt{2}, -\text{ArcSin}[\frac{\tan[\frac{1}{4} (e + f x)]}{\sqrt{3 - 2 \sqrt{2}}}], 17 - 12 \sqrt{2}] \right) \\
& \sqrt{\left(-1 - \sqrt{2} + (2 + \sqrt{2}) \cos[\frac{1}{2} (e + f x)] \right) \sec[\frac{1}{4} (e + f x)]^2 \sec[\frac{1}{2} (e + f x)]} \\
& \sqrt{a (1 + \sec[e + f x])} (c + d \sec[e + f x])^4 \sqrt{3 - 2 \sqrt{2} - \tan[\frac{1}{4} (e + f x)]^2}
\end{aligned}$$

Problem 148: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \sqrt{a + a \sec[e + f x]} (c + d \sec[e + f x])^3 dx$$

Optimal (type 3, 205 leaves, 5 steps) :

$$\begin{aligned}
& \frac{2 a d (3 c^2 + 3 c d + d^2) \tan[e + f x]}{f \sqrt{a + a \sec[e + f x]}} + \frac{2 a^{3/2} c^3 \operatorname{ArcTanh}[\frac{\sqrt{a - a \sec[e + f x]}}{\sqrt{a}}] \tan[e + f x]}{f \sqrt{a - a \sec[e + f x]} \sqrt{a + a \sec[e + f x]}} - \\
& \frac{2 d^2 (3 c + 2 d) (a - a \sec[e + f x]) \tan[e + f x]}{3 f \sqrt{a + a \sec[e + f x]}} + \frac{2 d^3 (a - a \sec[e + f x])^2 \tan[e + f x]}{5 a f \sqrt{a + a \sec[e + f x]}}
\end{aligned}$$

Result (type 4, 519 leaves) :

$$\begin{aligned}
& \left(\cos[e+f x]^3 \sec[\frac{1}{2} (e+f x)] \sqrt{a (1 + \sec[e+f x])} (c+d \sec[e+f x])^3 \right. \\
& \left. \left(\frac{2}{15} d (45 c^2 + 30 c d + 8 d^2) \sin[\frac{1}{2} (e+f x)] + \frac{2}{5} d^3 \sec[e+f x]^2 \sin[\frac{1}{2} (e+f x)] + \right. \right. \\
& \left. \left. \frac{2}{15} \sec[e+f x] \left(15 c d^2 \sin[\frac{1}{2} (e+f x)] + 4 d^3 \sin[\frac{1}{2} (e+f x)] \right) \right) \right) / \\
& \left(f (d+c \cos[e+f x])^3 \right) - \frac{1}{f (d+c \cos[e+f x])^3} 8 (-3 - 2 \sqrt{2}) c^3 \cos[\frac{1}{4} (e+f x)]^4 \\
& \sqrt{\frac{7 - 5 \sqrt{2} + (10 - 7 \sqrt{2}) \cos[\frac{1}{2} (e+f x)]}{1 + \cos[\frac{1}{2} (e+f x)]}} \sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \cos[\frac{1}{2} (e+f x)]}{1 + \cos[\frac{1}{2} (e+f x)]}} \\
& \left(1 - \sqrt{2} + (-2 + \sqrt{2}) \cos[\frac{1}{2} (e+f x)] \right) \cos[e+f x]^2 \\
& \left(\text{EllipticF}[\text{ArcSin}[\frac{\tan[\frac{1}{4} (e+f x)]}{\sqrt{3 - 2 \sqrt{2}}}], 17 - 12 \sqrt{2}] + \right. \\
& \left. 2 \text{EllipticPi}[-3 + 2 \sqrt{2}, -\text{ArcSin}[\frac{\tan[\frac{1}{4} (e+f x)]}{\sqrt{3 - 2 \sqrt{2}}}], 17 - 12 \sqrt{2}] \right) \\
& \sqrt{\left(-1 - \sqrt{2} + (2 + \sqrt{2}) \cos[\frac{1}{2} (e+f x)] \right) \sec[\frac{1}{4} (e+f x)]^2 \sec[\frac{1}{2} (e+f x)]} \\
& \sqrt{a (1 + \sec[e+f x])} (c+d \sec[e+f x])^3 \sqrt{3 - 2 \sqrt{2} - \tan[\frac{1}{4} (e+f x)]^2}
\end{aligned}$$

Problem 149: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \sqrt{a + a \sec[e+f x]} (c+d \sec[e+f x])^2 dx$$

Optimal (type 3, 144 leaves, 5 steps) :

$$\begin{aligned}
& \frac{2 a d (2 c + d) \tan[e+f x]}{f \sqrt{a + a \sec[e+f x]}} + \\
& \frac{2 a^{3/2} c^2 \operatorname{ArcTanh}\left[\frac{\sqrt{a - a \sec[e+f x]}}{\sqrt{a}}\right] \tan[e+f x]}{f \sqrt{a - a \sec[e+f x]} \sqrt{a + a \sec[e+f x]}} - \frac{2 d^2 (a - a \sec[e+f x]) \tan[e+f x]}{3 f \sqrt{a + a \sec[e+f x]}}
\end{aligned}$$

Result (type 4, 463 leaves) :

$$\begin{aligned}
& \left(\cos[e+f x]^2 \sec\left[\frac{1}{2}(e+f x)\right] \sqrt{a(1+\sec[e+f x])} (c+d \sec[e+f x])^2 \right. \\
& \quad \left. \left(\frac{4}{3} d (3 c + d) \sin\left[\frac{1}{2}(e+f x)\right] + \frac{2}{3} d^2 \sec[e+f x] \sin\left[\frac{1}{2}(e+f x)\right] \right) \right) / \\
& \quad \left(f (d + c \cos[e+f x])^2 \right) - \frac{1}{f (d + c \cos[e+f x])^2} \\
& 8 (-3 - 2 \sqrt{2}) c^2 \cos\left[\frac{1}{4}(e+f x)\right]^4 \sqrt{\frac{7 - 5 \sqrt{2} + (10 - 7 \sqrt{2}) \cos\left[\frac{1}{2}(e+f x)\right]}{1 + \cos\left[\frac{1}{2}(e+f x)\right]}} \\
& \sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(e+f x)\right]}{1 + \cos\left[\frac{1}{2}(e+f x)\right]}} \left(1 - \sqrt{2} + (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(e+f x)\right] \right) \\
& \cos[e+f x] \left(\text{EllipticF}[\text{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(e+f x)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}] + \right. \\
& \quad \left. 2 \text{EllipticPi}\left[-3+2\sqrt{2}, -\text{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(e+f x)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] \right) \\
& \sqrt{\left(-1 - \sqrt{2} + (2 + \sqrt{2}) \cos\left[\frac{1}{2}(e+f x)\right]\right) \sec\left[\frac{1}{4}(e+f x)\right]^2 \sec\left[\frac{1}{2}(e+f x)\right]} \\
& \sqrt{a(1+\sec[e+f x])} (c+d \sec[e+f x])^2 \sqrt{3-2\sqrt{2}-\tan\left[\frac{1}{4}(e+f x)\right]^2}
\end{aligned}$$

Problem 150: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \sqrt{a + a \sec[e+f x]} (c + d \sec[e+f x]) dx$$

Optimal (type 3, 66 leaves, 4 steps):

$$\frac{2 \sqrt{a} c \text{ArcTan}\left[\frac{\sqrt{a} \tan[e+f x]}{\sqrt{a+a \sec[e+f x]}}\right]}{f} + \frac{2 a d \tan[e+f x]}{f \sqrt{a+a \sec[e+f x]}}$$

Result (type 4, 407 leaves):

$$\begin{aligned}
& -\frac{1}{f(d + c \cos[e + f x])} 8 \left(-3 - 2\sqrt{2} \right) c \cos\left[\frac{1}{4}(e + f x)\right]^4 \\
& \sqrt{\frac{7 - 5\sqrt{2} + (10 - 7\sqrt{2}) \cos\left[\frac{1}{2}(e + f x)\right]}{1 + \cos\left[\frac{1}{2}(e + f x)\right]}} \sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(e + f x)\right]}{1 + \cos\left[\frac{1}{2}(e + f x)\right]}} \\
& \left(1 - \sqrt{2} + (-2 + \sqrt{2}) \cos\left[\frac{1}{2}(e + f x)\right] \right) \left(\text{EllipticF}\left[\text{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(e + f x)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right] + \right. \\
& \left. 2 \text{EllipticPi}\left[-3 + 2\sqrt{2}, -\text{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(e + f x)\right]}{\sqrt{3 - 2\sqrt{2}}}\right], 17 - 12\sqrt{2}\right]\right) \\
& \sqrt{\left(-1 - \sqrt{2} + (2 + \sqrt{2}) \cos\left[\frac{1}{2}(e + f x)\right] \right) \sec\left[\frac{1}{4}(e + f x)\right]^2 \sec\left[\frac{1}{2}(e + f x)\right]} \\
& \sqrt{a(1 + \sec[e + f x])} (c + d \sec[e + f x]) \sqrt{3 - 2\sqrt{2} - \tan\left[\frac{1}{4}(e + f x)\right]^2} + \\
& \left. \left(2d \cos[e + f x] \sqrt{a(1 + \sec[e + f x])} (c + d \sec[e + f x]) \tan\left[\frac{1}{2}(e + f x)\right] \right) \right/ \\
& (f(d + c \cos[e + f x]))
\end{aligned}$$

Problem 154: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int (a + a \sec[e + f x])^{3/2} (c + d \sec[e + f x])^3 dx$$

Optimal (type 3, 241 leaves, 6 steps):

$$\begin{aligned}
& \frac{2 a^{5/2} c^3 \text{ArcTanh}\left[\frac{\sqrt{a - a \sec[e + f x]}}{\sqrt{a}}\right] \tan[e + f x]}{f \sqrt{a - a \sec[e + f x]} \sqrt{a + a \sec[e + f x]}} + \\
& \frac{2 a^2 (6 c + 13 d) (c + d \sec[e + f x])^2 \tan[e + f x]}{35 f \sqrt{a + a \sec[e + f x]}} + \frac{2 a^2 (c + d \sec[e + f x])^3 \tan[e + f x]}{7 f \sqrt{a + a \sec[e + f x]}} + \\
& \left(\frac{2 a^2 (2 (36 c^3 + 243 c^2 d + 189 c d^2 + 52 d^3) + d (24 c^2 + 111 c d + 52 d^2) \sec[e + f x]) \tan[e + f x]}{105 f \sqrt{a + a \sec[e + f x]}} \right)
\end{aligned}$$

Result (type 4, 590 leaves):

$$\begin{aligned}
& \frac{1}{f (d + c \cos[e + f x])^3} \cos[e + f x]^4 \sec[\frac{1}{2} (e + f x)]^3 (a (1 + \sec[e + f x]))^{3/2} (c + d \sec[e + f x])^3 \\
& \left(\frac{1}{105} (105 c^3 + 525 c^2 d + 378 c d^2 + 104 d^3) \sin[\frac{1}{2} (e + f x)] + \frac{1}{7} d^3 \sec[e + f x]^3 \sin[\frac{1}{2} (e + f x)] + \right. \\
& \frac{1}{35} \sec[e + f x]^2 \left(21 c d^2 \sin[\frac{1}{2} (e + f x)] + 13 d^3 \sin[\frac{1}{2} (e + f x)] \right) + \frac{1}{105} \sec[e + f x] \\
& \left. \left(105 c^2 d \sin[\frac{1}{2} (e + f x)] + 189 c d^2 \sin[\frac{1}{2} (e + f x)] + 52 d^3 \sin[\frac{1}{2} (e + f x)] \right) \right) - \\
& \frac{1}{f (d + c \cos[e + f x])^3} 4 (-3 - 2 \sqrt{2}) c^3 \cos[\frac{1}{4} (e + f x)]^4 \\
& \sqrt{\frac{7 - 5 \sqrt{2} + (10 - 7 \sqrt{2}) \cos[\frac{1}{2} (e + f x)]}{1 + \cos[\frac{1}{2} (e + f x)]}} \sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \cos[\frac{1}{2} (e + f x)]}{1 + \cos[\frac{1}{2} (e + f x)]}} \\
& \left(1 - \sqrt{2} + (-2 + \sqrt{2}) \cos[\frac{1}{2} (e + f x)] \right) \cos[e + f x]^3 \\
& \left(\text{EllipticF}[\text{ArcSin}[\frac{\tan[\frac{1}{4} (e + f x)]}{\sqrt{3 - 2 \sqrt{2}}}], 17 - 12 \sqrt{2}] + \right. \\
& \left. 2 \text{EllipticPi}[-3 + 2 \sqrt{2}, -\text{ArcSin}[\frac{\tan[\frac{1}{4} (e + f x)]}{\sqrt{3 - 2 \sqrt{2}}}], 17 - 12 \sqrt{2}] \right) \\
& \sqrt{\left(-1 - \sqrt{2} + (2 + \sqrt{2}) \cos[\frac{1}{2} (e + f x)] \right) \sec[\frac{1}{4} (e + f x)]^2 \sec[\frac{1}{2} (e + f x)]^3} \\
& (a (1 + \sec[e + f x]))^{3/2} (c + d \sec[e + f x])^3 \sqrt{3 - 2 \sqrt{2} - \tan[\frac{1}{4} (e + f x)]^2}
\end{aligned}$$

Problem 155: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int (a + a \sec[e + f x])^{3/2} (c + d \sec[e + f x])^2 dx$$

Optimal (type 3, 176 leaves, 5 steps) :

$$\begin{aligned}
& \frac{2 a^{5/2} c^2 \operatorname{ArcTanh}[\frac{\sqrt{a - a \sec[e + f x]}}{\sqrt{a}}] \tan[e + f x]}{f \sqrt{a - a \sec[e + f x]} \sqrt{a + a \sec[e + f x]}} + \frac{2 a^2 (c + d \sec[e + f x])^2 \tan[e + f x]}{5 f \sqrt{a + a \sec[e + f x]}} + \\
& \frac{2 a^2 (2 (6 c^2 + 25 c d + 9 d^2) + d (4 c + 9 d) \sec[e + f x]) \tan[e + f x]}{15 f \sqrt{a + a \sec[e + f x]}}
\end{aligned}$$

Result (type 4, 520 leaves) :

$$\begin{aligned}
& \left(\cos[e+f x]^3 \sec[\frac{1}{2}(e+f x)]^3 (a(1+\sec[e+f x]))^{3/2} (c+d \sec[e+f x])^2 \right. \\
& \left. \left(\frac{1}{15} (15 c^2 + 50 c d + 18 d^2) \sin[\frac{1}{2}(e+f x)] + \frac{1}{5} d^2 \sec[e+f x]^2 \sin[\frac{1}{2}(e+f x)] + \right. \right. \\
& \left. \left. \frac{1}{15} \sec[e+f x] \left(10 c d \sin[\frac{1}{2}(e+f x)] + 9 d^2 \sin[\frac{1}{2}(e+f x)] \right) \right) \right) / \\
& \left(f(d+c \cos[e+f x])^2 \right) - \frac{1}{f(d+c \cos[e+f x])^2} 4 \left(-3 - 2\sqrt{2} \right) c^2 \cos[\frac{1}{4}(e+f x)]^4 \\
& \sqrt{\frac{7 - 5\sqrt{2} + (10 - 7\sqrt{2}) \cos[\frac{1}{2}(e+f x)]}{1 + \cos[\frac{1}{2}(e+f x)]}} \sqrt{\frac{-1 + \sqrt{2} - (-2 + \sqrt{2}) \cos[\frac{1}{2}(e+f x)]}{1 + \cos[\frac{1}{2}(e+f x)]}} \\
& \left(1 - \sqrt{2} + (-2 + \sqrt{2}) \cos[\frac{1}{2}(e+f x)] \right) \cos[e+f x]^2 \\
& \left(\text{EllipticF}[\text{ArcSin}[\frac{\tan[\frac{1}{4}(e+f x)]}{\sqrt{3-2\sqrt{2}}}], 17 - 12\sqrt{2}] + \right. \\
& \left. 2 \text{EllipticPi}[-3 + 2\sqrt{2}, -\text{ArcSin}[\frac{\tan[\frac{1}{4}(e+f x)]}{\sqrt{3-2\sqrt{2}}}], 17 - 12\sqrt{2}] \right) \\
& \sqrt{\left(-1 - \sqrt{2} + (2 + \sqrt{2}) \cos[\frac{1}{2}(e+f x)] \right) \sec[\frac{1}{4}(e+f x)]^2 \sec[\frac{1}{2}(e+f x)]^3} \\
& (a(1+\sec[e+f x]))^{3/2} (c+d \sec[e+f x])^2 \sqrt{3 - 2\sqrt{2} - \tan[\frac{1}{4}(e+f x)]^2}
\end{aligned}$$

Problem 156: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int (a + a \sec[e+f x])^{3/2} (c + d \sec[e+f x]) dx$$

Optimal (type 3, 105 leaves, 5 steps) :

$$\frac{2 a^{3/2} c \operatorname{ArcTan}\left[\frac{\sqrt{a} \tan[e+f x]}{\sqrt{a+a \sec[e+f x]}}\right]}{f} + \frac{2 a^2 (3 c + 4 d) \tan[e+f x]}{3 f \sqrt{a+a \sec[e+f x]}} + \frac{2 a d \sqrt{a+a \sec[e+f x]} \tan[e+f x]}{3 f}$$

Result (type 4, 460 leaves) :

$$\begin{aligned}
& \left(\cos[e+f x]^2 \sec\left[\frac{1}{2}(e+f x)\right]^3 (a(1+\sec[e+f x]))^{3/2} (c+d \sec[e+f x]) \right. \\
& \left. \left(\frac{1}{3}(3c+5d) \sin\left[\frac{1}{2}(e+f x)\right] + \frac{1}{3}d \sec[e+f x] \sin\left[\frac{1}{2}(e+f x)\right] \right) \right) / (\tan[(d+c \cos[e+f x])]) - \\
& \frac{1}{f(d+c \cos[e+f x])} 4(-3-2\sqrt{2}) c \cos\left[\frac{1}{4}(e+f x)\right]^4 \sqrt{\frac{7-5\sqrt{2}+(10-7\sqrt{2}) \cos\left[\frac{1}{2}(e+f x)\right]}{1+\cos\left[\frac{1}{2}(e+f x)\right]}} \\
& \sqrt{\frac{-1+\sqrt{2}+(-2+\sqrt{2}) \cos\left[\frac{1}{2}(e+f x)\right]}{1+\cos\left[\frac{1}{2}(e+f x)\right]}} \left(1-\sqrt{2}+(-2+\sqrt{2}) \cos\left[\frac{1}{2}(e+f x)\right]\right) \\
& \cos[e+f x] \left(\text{EllipticF}\left[\text{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(e+f x)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] + \right. \\
& \left. 2 \text{EllipticPi}\left[-3+2\sqrt{2}, -\text{ArcSin}\left[\frac{\tan\left[\frac{1}{4}(e+f x)\right]}{\sqrt{3-2\sqrt{2}}}\right], 17-12\sqrt{2}\right] \right) \\
& \sqrt{\left(-1-\sqrt{2}+(2+\sqrt{2}) \cos\left[\frac{1}{2}(e+f x)\right]\right) \sec\left[\frac{1}{4}(e+f x)\right]^2 \sec\left[\frac{1}{2}(e+f x)\right]^3} \\
& (a(1+\sec[e+f x]))^{3/2} (c+d \sec[e+f x]) \sqrt{3-2\sqrt{2}-\tan\left[\frac{1}{4}(e+f x)\right]^2}
\end{aligned}$$

Problem 160: Result unnecessarily involves higher level functions.

$$\int (a+a \sec[e+f x])^{5/2} (c+d \sec[e+f x])^3 d x$$

Optimal (type 3, 336 leaves, 5 steps) :

$$\begin{aligned}
& \frac{2 a^3 (3 c^3 + 12 c^2 d + 12 c d^2 + 4 d^3) \tan[e+f x]}{f \sqrt{a+a \sec[e+f x]}} + \frac{2 a^{7/2} c^3 \operatorname{ArcTanh}\left[\frac{\sqrt{a-a \sec[e+f x]}}{\sqrt{a}}\right] \tan[e+f x]}{f \sqrt{a-a \sec[e+f x]} \sqrt{a+a \sec[e+f x]}} + \\
& \frac{2 a d (3 c^2 + 15 c d + 13 d^2) (a-a \sec[e+f x])^2 \tan[e+f x]}{5 f \sqrt{a+a \sec[e+f x]}} - \\
& \frac{6 d^2 (c+2 d) (a-a \sec[e+f x])^3 \tan[e+f x]}{7 f \sqrt{a+a \sec[e+f x]}} + \frac{2 d^3 (a-a \sec[e+f x])^4 \tan[e+f x]}{9 a f \sqrt{a+a \sec[e+f x]}} - \\
& \frac{2 (c^3 + 12 c^2 d + 24 c d^2 + 12 d^3) (a^3 - a^3 \sec[e+f x]) \tan[e+f x]}{3 f \sqrt{a+a \sec[e+f x]}}
\end{aligned}$$

Result (type 4, 665 leaves) :

$$\begin{aligned}
& \frac{1}{f(d+c \cos[e+f x])^3} \cos[e+f x]^5 \sec[\frac{1}{2}(e+f x)]^5 (a(1+\sec[e+f x]))^{5/2} (c+d \sec[e+f x])^3 \\
& \left(\frac{1}{630} (840 c^3 + 2709 c^2 d + 2070 c d^2 + 584 d^3) \sin[\frac{1}{2}(e+f x)] + \frac{1}{18} d^3 \sec[e+f x]^4 \right. \\
& \sin[\frac{1}{2}(e+f x)] + \frac{1}{126} \sec[e+f x]^3 \left(27 c d^2 \sin[\frac{1}{2}(e+f x)] + 26 d^3 \sin[\frac{1}{2}(e+f x)] \right) + \frac{1}{210} \\
& \sec[e+f x]^2 \left(63 c^2 d \sin[\frac{1}{2}(e+f x)] + 180 c d^2 \sin[\frac{1}{2}(e+f x)] + 73 d^3 \sin[\frac{1}{2}(e+f x)] \right) + \\
& \frac{1}{630} \sec[e+f x] \left(105 c^3 \sin[\frac{1}{2}(e+f x)] + 882 c^2 d \sin[\frac{1}{2}(e+f x)] + \right. \\
& \left. 1035 c d^2 \sin[\frac{1}{2}(e+f x)] + 292 d^3 \sin[\frac{1}{2}(e+f x)] \right) \Big) - \\
& \frac{1}{f(d+c \cos[e+f x])^3} 2(-3-2\sqrt{2}) c^3 \cos[\frac{1}{4}(e+f x)]^4 \\
& \sqrt{\frac{7-5\sqrt{2}+(10-7\sqrt{2}) \cos[\frac{1}{2}(e+f x)]}{1+\cos[\frac{1}{2}(e+f x)]}} \\
& \sqrt{\frac{-1+\sqrt{2}+(-2+\sqrt{2}) \cos[\frac{1}{2}(e+f x)]}{1+\cos[\frac{1}{2}(e+f x)]}} \\
& \left(1-\sqrt{2}+(-2+\sqrt{2}) \cos[\frac{1}{2}(e+f x)] \right) \cos[e+f x]^4 \\
& \left(\text{EllipticF}[\text{ArcSin}[\frac{\tan[\frac{1}{4}(e+f x)]}{\sqrt{3-2\sqrt{2}}}], 17-12\sqrt{2}] + \right. \\
& \left. 2 \text{EllipticPi}[-3+2\sqrt{2}, -\text{ArcSin}[\frac{\tan[\frac{1}{4}(e+f x)]}{\sqrt{3-2\sqrt{2}}}], 17-12\sqrt{2}] \right) \\
& \sqrt{\left(-1-\sqrt{2}+(2+\sqrt{2}) \cos[\frac{1}{2}(e+f x)] \right) \sec[\frac{1}{4}(e+f x)]^2 \sec[\frac{1}{2}(e+f x)]^5} \\
& (a(1+\sec[e+f x]))^{5/2} (c+d \sec[e+f x])^3 \sqrt{3-2\sqrt{2}-\tan[\frac{1}{4}(e+f x)]^2}
\end{aligned}$$

Problem 161: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int (a+a \sec[e+f x])^{5/2} (c+d \sec[e+f x])^2 dx$$

Optimal (type 3, 258 leaves, 5 steps):

$$\begin{aligned} & \frac{2 a^3 (c+2 d) (3 c+2 d) \tan [e+f x]}{f \sqrt{a+a \sec [e+f x]}} + \frac{2 a^{7/2} c^2 \operatorname{ArcTanh}\left[\frac{\sqrt{a-a \sec [e+f x]}}{\sqrt{a}}\right] \tan [e+f x]}{f \sqrt{a-a \sec [e+f x]} \sqrt{a+a \sec [e+f x]}} + \\ & \frac{2 a d (2 c+5 d) (a-a \sec [e+f x])^2 \tan [e+f x]}{5 f \sqrt{a+a \sec [e+f x]}} - \frac{2 d^2 (a-a \sec [e+f x])^3 \tan [e+f x]}{7 f \sqrt{a+a \sec [e+f x]}} - \\ & \frac{2 (c^2+8 c d+8 d^2) (a^3-a^3 \sec [e+f x]) \tan [e+f x]}{3 f \sqrt{a+a \sec [e+f x]}} \end{aligned}$$

Result (type 4, 577 leaves):

$$\begin{aligned} & \frac{1}{f (d+c \cos [e+f x])^2} \cos [e+f x]^4 \sec \left[\frac{1}{2} (e+f x)\right]^5 (a (1+\sec [e+f x]))^{5/2} (c+d \sec [e+f x])^2 \\ & \left(\frac{1}{105} (140 c^2+301 c d+115 d^2) \sin \left[\frac{1}{2} (e+f x)\right] + \frac{1}{14} d^2 \sec [e+f x]^3 \sin \left[\frac{1}{2} (e+f x)\right] + \right. \\ & \frac{1}{35} \sec [e+f x]^2 \left(7 c d \sin \left[\frac{1}{2} (e+f x)\right] + 10 d^2 \sin \left[\frac{1}{2} (e+f x)\right] \right) + \\ & \left. \frac{1}{210} \sec [e+f x] \left(35 c^2 \sin \left[\frac{1}{2} (e+f x)\right] + 196 c d \sin \left[\frac{1}{2} (e+f x)\right] + 115 d^2 \sin \left[\frac{1}{2} (e+f x)\right] \right) \right) - \\ & \frac{1}{f (d+c \cos [e+f x])^2} 2 (-3-2 \sqrt{2}) c^2 \cos \left[\frac{1}{4} (e+f x)\right]^4 \\ & \sqrt{\frac{7-5 \sqrt{2}+\left(10-7 \sqrt{2}\right) \cos \left[\frac{1}{2} (e+f x)\right]}{1+\cos \left[\frac{1}{2} (e+f x)\right]}} \sqrt{\frac{-1+\sqrt{2}-\left(-2+\sqrt{2}\right) \cos \left[\frac{1}{2} (e+f x)\right]}{1+\cos \left[\frac{1}{2} (e+f x)\right]}} \\ & \left(1-\sqrt{2}+\left(-2+\sqrt{2}\right) \cos \left[\frac{1}{2} (e+f x)\right] \right) \cos [e+f x]^3 \\ & \left(\text{EllipticF} \left[\operatorname{ArcSin} \left[\frac{\tan \left[\frac{1}{4} (e+f x)\right]}{\sqrt{3-2 \sqrt{2}}} \right], 17-12 \sqrt{2} \right] + \right. \\ & \left. 2 \text{EllipticPi} \left[-3+2 \sqrt{2}, -\operatorname{ArcSin} \left[\frac{\tan \left[\frac{1}{4} (e+f x)\right]}{\sqrt{3-2 \sqrt{2}}} \right], 17-12 \sqrt{2} \right] \right) \\ & \sqrt{\left(-1-\sqrt{2}+\left(2+\sqrt{2}\right) \cos \left[\frac{1}{2} (e+f x)\right] \right) \sec \left[\frac{1}{4} (e+f x)\right]^2 \sec \left[\frac{1}{2} (e+f x)\right]^5} \\ & (a (1+\sec [e+f x]))^{5/2} (c+d \sec [e+f x])^2 \sqrt{3-2 \sqrt{2}-\tan \left[\frac{1}{4} (e+f x)\right]^2} \end{aligned}$$

Problem 162: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int (a+a \sec [e+f x])^{5/2} (c+d \sec [e+f x]) dx$$

Optimal (type 3, 142 leaves, 6 steps) :

$$\frac{2 a^{5/2} c \operatorname{ArcTan}\left[\frac{\sqrt{a} \tan [e+f x]}{\sqrt{a+a \sec [e+f x]}}\right]}{f}+\frac{2 a^3 (35 c+32 d) \tan [e+f x]}{15 f \sqrt{a+a \sec [e+f x]}}+$$

$$\frac{2 a^2 (5 c+8 d) \sqrt{a+a \sec [e+f x]} \tan [e+f x]}{15 f}+\frac{2 a d (a+a \sec [e+f x])^{3/2} \tan [e+f x]}{5 f}$$

Result (type 4, 501 leaves) :

$$\left(\cos [e+f x]^3 \sec \left[\frac{1}{2} (e+f x)\right]^5 (a (1+\sec [e+f x]))^{5/2} (c+d \sec [e+f x])\right.$$

$$\left.\left(\frac{1}{30} (40 c+43 d) \sin \left[\frac{1}{2} (e+f x)\right]+\frac{1}{10} d \sec [e+f x]^2 \sin \left[\frac{1}{2} (e+f x)\right]+\right.\right.$$

$$\left.\left.\frac{1}{30} \sec [e+f x] \left(5 c \sin \left[\frac{1}{2} (e+f x)\right]+14 d \sin \left[\frac{1}{2} (e+f x)\right]\right)\right)\right) / (f (d+c \cos [e+f x])) -$$

$$\frac{1}{f (d+c \cos [e+f x])} 2 (-3-2 \sqrt{2}) c \cos \left[\frac{1}{4} (e+f x)\right]^4 \sqrt{\frac{7-5 \sqrt{2}+(10-7 \sqrt{2}) \cos \left[\frac{1}{2} (e+f x)\right]}{1+\cos \left[\frac{1}{2} (e+f x)\right]}}$$

$$\sqrt{\frac{-1+\sqrt{2}-(-2+\sqrt{2}) \cos \left[\frac{1}{2} (e+f x)\right]}{1+\cos \left[\frac{1}{2} (e+f x)\right]}} \left(1-\sqrt{2}+(-2+\sqrt{2}) \cos \left[\frac{1}{2} (e+f x)\right]\right)$$

$$\cos [e+f x]^2 \left(\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\tan \left[\frac{1}{4} (e+f x)\right]}{\sqrt{3-2 \sqrt{2}}}\right], 17-12 \sqrt{2}\right]+\right.$$

$$\left.2 \operatorname{EllipticPi}\left[-3+2 \sqrt{2},-\operatorname{ArcSin}\left[\frac{\tan \left[\frac{1}{4} (e+f x)\right]}{\sqrt{3-2 \sqrt{2}}}\right], 17-12 \sqrt{2}\right]\right)$$

$$\sqrt{\left(-1-\sqrt{2}+(2+\sqrt{2}) \cos \left[\frac{1}{2} (e+f x)\right]\right) \sec \left[\frac{1}{4} (e+f x)\right]^2 \sec \left[\frac{1}{2} (e+f x)\right]^5}$$

$$(a (1+\sec [e+f x]))^{5/2} (c+d \sec [e+f x]) \sqrt{3-2 \sqrt{2}-\tan \left[\frac{1}{4} (e+f x)\right]^2}$$

Problem 170: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{a+a \sec [e+f x]} (c+d \sec [e+f x])^2} dx$$

Optimal (type 3, 416 leaves, 12 steps) :

$$\begin{aligned}
& \frac{2 \sqrt{a} \operatorname{ArcTanh}\left[\frac{\sqrt{a-a \sec [e+f x]}}{\sqrt{a}}\right] \tan [e+f x]}{c^2 f \sqrt{a-a \sec [e+f x]} \sqrt{a+a \sec [e+f x]}} - \frac{\sqrt{2} \sqrt{a} \operatorname{ArcTanh}\left[\frac{\sqrt{a-a \sec [e+f x]}}{\sqrt{2} \sqrt{a}}\right] \tan [e+f x]}{(c-d)^2 f \sqrt{a-a \sec [e+f x]} \sqrt{a+a \sec [e+f x]}} + \\
& \frac{\sqrt{a} d^{3/2} \operatorname{ArcTanh}\left[\frac{\sqrt{d} \sqrt{a-a \sec [e+f x]}}{\sqrt{a} \sqrt{c+d}}\right] \tan [e+f x]}{c (c-d) (c+d)^{3/2} f \sqrt{a-a \sec [e+f x]} \sqrt{a+a \sec [e+f x]}} + \\
& \frac{2 \sqrt{a} (2 c-d) d^{3/2} \operatorname{ArcTanh}\left[\frac{\sqrt{d} \sqrt{a-a \sec [e+f x]}}{\sqrt{a} \sqrt{c+d}}\right] \tan [e+f x]}{c^2 (c-d)^2 \sqrt{c+d} f \sqrt{a-a \sec [e+f x]} \sqrt{a+a \sec [e+f x]}} + \\
& \frac{d^2 \tan [e+f x]}{c (c^2-d^2) f \sqrt{a+a \sec [e+f x]} (c+d \sec [e+f x])}
\end{aligned}$$

Result (type 3, 2477 leaves):

$$\begin{aligned}
& \left(\cos \left[\frac{1}{2} (e+f x) \right] (d+c \cos [e+f x])^2 \sec [e+f x]^3 \right. \\
& \left. \left(-\frac{2 d^2 \sin \left[\frac{1}{2} (e+f x) \right]}{c^2 (-c+d) (c+d)} + \frac{2 d^3 \sin \left[\frac{1}{2} (e+f x) \right]}{c^2 (-c+d) (c+d) (d+c \cos [e+f x])} \right) \right) / \\
& \left(f \sqrt{a (1+\sec [e+f x])} (c+d \sec [e+f x])^2 \right) - \left(\cos \left[\frac{1}{2} (e+f x) \right] (d+c \cos [e+f x])^2 \right. \\
& \left. \left(2 \sqrt{2} d^{3/2} (5 c^2+c d-2 d^2) \operatorname{ArcTan}\left[\frac{\sqrt{d} \tan \left[\frac{1}{2} (e+f x) \right]}{\sqrt{-c-d} \sqrt{\frac{\cos [e+f x]}{1+\cos [e+f x]}}}\right] \right. \right. \\
& \left. \left. \left. -\sqrt{2} (c^2-d^2) \log [\sec \left[\frac{1}{2} (e+f x) \right]]^2 \left(-1+2 \cos [e+f x]-2 \sqrt{-\frac{\cos [e+f x]}{1+\cos [e+f x]}} \sin [e+f x] \right) \right] + \sqrt{2} \right. \\
& \left. \left. (c^2-d^2) \log [\sec \left[\frac{1}{2} (e+f x) \right]]^2 \left(-1+2 \cos [e+f x]+2 \sqrt{-\frac{\cos [e+f x]}{1+\cos [e+f x]}} \sin [e+f x] \right) \right] + \right. \\
& \left. \left. \frac{4 c^2 (c+d) \log [\tan \left[\frac{1}{2} (e+f x) \right]]+\sqrt{-1+\tan \left[\frac{1}{2} (e+f x) \right]^2}}{c-d} \right)
\end{aligned}$$

$$\begin{aligned}
& \left(\frac{d \sec \left[\frac{1}{2} (e + f x) \right]}{(-c + d) (c + d) (d + c \cos [e + f x]) \sqrt{\sec [e + f x]}} + \right. \\
& \quad \frac{d^2 \sec \left[\frac{1}{2} (e + f x) \right]}{2 c (-c + d) (c + d) (d + c \cos [e + f x]) \sqrt{\sec [e + f x]}} - \\
& \quad \frac{c \sec \left[\frac{1}{2} (e + f x) \right] \sqrt{\sec [e + f x]}}{2 (-c + d) (c + d) (d + c \cos [e + f x])} - \frac{c \cos [2 (e + f x)] \sec \left[\frac{1}{2} (e + f x) \right] \sqrt{\sec [e + f x]}}{2 (-c + d) (c + d) (d + c \cos [e + f x])} + \\
& \quad \left. \frac{d^2 \cos [2 (e + f x)] \sec \left[\frac{1}{2} (e + f x) \right] \sqrt{\sec [e + f x]}}{2 c (-c + d) (c + d) (d + c \cos [e + f x])} \right) \\
& \sec [e + f x]^{5/2} \sqrt{\cos \left[\frac{1}{2} (e + f x) \right]^2 \sec [e + f x]} \sqrt{-1 + \tan \left[\frac{1}{2} (e + f x) \right]^2} \Bigg) / \\
& \left(2 c^2 (c - d) (c + d) f \sqrt{a (1 + \sec [e + f x])} (c + d \sec [e + f x])^2 \right. \\
& \left. - \frac{1}{4 c^2 (c - d) (c + d) \sqrt{-1 + \tan \left[\frac{1}{2} (e + f x) \right]^2}} \right. \\
& \left. \frac{2 \sqrt{2} d^{3/2} (5 c^2 + c d - 2 d^2) \operatorname{ArcTan} \left[\frac{\sqrt{d} \tan \left[\frac{1}{2} (e + f x) \right]}{\sqrt{-c - d} \sqrt{-\frac{\cos [e + f x]}{1 + \cos [e + f x]}}} \right]}{\sqrt{-c - d} (c - d)} - \sqrt{2} (c^2 - d^2) \right. \\
& \left. \log \left[\sec \left[\frac{1}{2} (e + f x) \right] \right]^2 \left(-1 + 2 \cos [e + f x] - 2 \sqrt{-\frac{\cos [e + f x]}{1 + \cos [e + f x]}} \sin [e + f x] \right) \right] + \\
& \sqrt{2} (c^2 - d^2) \log \left[\sec \left[\frac{1}{2} (e + f x) \right] \right]^2 \left(-1 + 2 \cos [e + f x] + 2 \sqrt{-\frac{\cos [e + f x]}{1 + \cos [e + f x]}} \sin [e + f x] \right)
\end{aligned}$$

$$\begin{aligned}
& \left. \left(f x] \right) + \frac{1}{c-d} 4 c^2 (c+d) \operatorname{Log}[\operatorname{Tan}\left[\frac{1}{2} (e+f x)\right]] + \sqrt{-1 + \operatorname{Tan}\left[\frac{1}{2} (e+f x)\right]^2} \right) \\
& \operatorname{Sec}\left[\frac{1}{2} (e+f x)\right]^2 \sqrt{\cos\left[\frac{1}{2} (e+f x)\right]^2 \operatorname{Sec}[e+f x]} \operatorname{Tan}\left[\frac{1}{2} (e+f x)\right] - \\
& \frac{1}{2 c^2 (c-d) (c+d)} \sqrt{\cos\left[\frac{1}{2} (e+f x)\right]^2 \operatorname{Sec}[e+f x]} \sqrt{-1 + \operatorname{Tan}\left[\frac{1}{2} (e+f x)\right]^2} \\
& \left(- \left(\sqrt{2} (c^2 - d^2) \cos\left[\frac{1}{2} (e+f x)\right]^2 \right. \right. \\
& \left. \left. \operatorname{Sec}\left[\frac{1}{2} (e+f x)\right]^2 \left(-2 \cos[e+f x] \sqrt{-\frac{\cos[e+f x]}{1+\cos[e+f x]}} - 2 \sin[e+f x] - \right. \right. \right. \\
& \left. \left. \left. \frac{\sin[e+f x] \left(-\frac{\cos[e+f x] \sin[e+f x]}{(1+\cos[e+f x])^2} + \frac{\sin[e+f x]}{1+\cos[e+f x]} \right)}{\sqrt{-\frac{\cos[e+f x]}{1+\cos[e+f x]}}} \right) + \operatorname{Sec}\left[\frac{1}{2} (e+f x)\right]^2 \right. \right. \\
& \left. \left. \left(-1 + 2 \cos[e+f x] - 2 \sqrt{-\frac{\cos[e+f x]}{1+\cos[e+f x]}} \sin[e+f x] \right) \operatorname{Tan}\left[\frac{1}{2} (e+f x)\right] \right) \right) / \\
& \left(-1 + 2 \cos[e+f x] - 2 \sqrt{-\frac{\cos[e+f x]}{1+\cos[e+f x]}} \sin[e+f x] \right) + \sqrt{2} (c^2 - d^2) \\
& \cos\left[\frac{1}{2} (e+f x)\right]^2 \operatorname{Sec}\left[\frac{1}{2} (e+f x)\right]^2 \left(2 \cos[e+f x] \sqrt{-\frac{\cos[e+f x]}{1+\cos[e+f x]}} - 2 \sin[e+f x] \right)
\end{aligned}$$

$$\begin{aligned}
& \left. \left(\frac{\sin[e+f x]}{e+f x} + \frac{\sin[e+f x] \left(-\frac{\cos[e+f x] \sin[e+f x]}{(1+\cos[e+f x])^2} + \frac{\sin[e+f x]}{1+\cos[e+f x]} \right)}{\sqrt{-\frac{\cos[e+f x]}{1+\cos[e+f x]}}} \right) + \sec\left[\frac{1}{2}(e+f x)\right]^2 \right. \\
& \left. \left. \left(-1 + 2 \cos[e+f x] + 2 \sqrt{-\frac{\cos[e+f x]}{1+\cos[e+f x]}} \sin[e+f x] \right) \tan\left[\frac{1}{2}(e+f x)\right] \right) \right\} \\
& \left. \left(-1 + 2 \cos[e+f x] + 2 \sqrt{-\frac{\cos[e+f x]}{1+\cos[e+f x]}} \sin[e+f x] \right) + \left(2 \sqrt{2} d^{3/2} (5 c^2 + c d - 2 d^2) \right. \right. \\
& \left. \left. \left(\frac{\sqrt{d} \sec\left[\frac{1}{2}(e+f x)\right]^2}{2 \sqrt{-c-d} \sqrt{-\frac{\cos[e+f x]}{1+\cos[e+f x]}}} - \left(\sqrt{d} \left(-\frac{\cos[e+f x] \sin[e+f x]}{(1+\cos[e+f x])^2} + \frac{\sin[e+f x]}{1+\cos[e+f x]} \right) \right. \right. \right. \right. \\
& \left. \left. \left. \left. \left. \tan\left[\frac{1}{2}(e+f x)\right] \right) \right) \right) \right) \right\} \\
& \left. \left(\sqrt{-c-d} (c-d) \left(1 - \frac{d (1+\cos[e+f x]) \sec[e+f x] \tan\left[\frac{1}{2}(e+f x)\right]^2}{-c-d} \right) \right) + \right. \\
& \left. \left. \left(4 c^2 (c+d) \left(\frac{1}{2} \sec\left[\frac{1}{2}(e+f x)\right]^2 + \frac{\sec\left[\frac{1}{2}(e+f x)\right]^2 \tan\left[\frac{1}{2}(e+f x)\right]}{2 \sqrt{-1 + \tan\left[\frac{1}{2}(e+f x)\right]^2}} \right) \right) \right) \right\} \\
& \left. \left((c-d) \left(\tan\left[\frac{1}{2}(e+f x)\right] + \sqrt{-1 + \tan\left[\frac{1}{2}(e+f x)\right]^2} \right) \right) \right) - \\
& \frac{1}{4 c^2 (c-d) (c+d) \sqrt{\cos\left[\frac{1}{2}(e+f x)\right]^2 \sec[e+f x]}}
\end{aligned}$$

$$\begin{aligned}
& \left(\frac{2 \sqrt{2} d^{3/2} (5 c^2 + c d - 2 d^2) \operatorname{ArcTan} \left[\frac{\sqrt{d} \tan \left[\frac{1}{2} (e + f x) \right]}{\sqrt{-c - d} \sqrt{-\frac{\cos [e + f x]}{1 + \cos [e + f x]}}} \right]}{\sqrt{-c - d} (c - d)} - \sqrt{2} (c^2 - d^2) \right. \\
& \left. \log \left[\sec \left[\frac{1}{2} (e + f x) \right] \right]^2 \left(-1 + 2 \cos [e + f x] - 2 \sqrt{-\frac{\cos [e + f x]}{1 + \cos [e + f x]}} \sin [e + f x] \right) \right] + \\
& \sqrt{2} (c^2 - d^2) \log \left[\sec \left[\frac{1}{2} (e + f x) \right] \right]^2 \left(-1 + 2 \cos [e + f x] + 2 \sqrt{-\frac{\cos [e + f x]}{1 + \cos [e + f x]}} \sin [e + f x] \right) \\
& \left. \left. \left. + \frac{1}{c - d} 4 c^2 (c + d) \log \left[\tan \left[\frac{1}{2} (e + f x) \right] \right] + \sqrt{-1 + \tan \left[\frac{1}{2} (e + f x) \right]^2} \right) \right] \\
& \left. \left. \left. \left. \sqrt{-1 + \tan \left[\frac{1}{2} (e + f x) \right]^2} \left(-\cos \left[\frac{1}{2} (e + f x) \right] \sec [e + f x] \sin \left[\frac{1}{2} (e + f x) \right] + \right. \right. \right. \right. \\
& \left. \left. \left. \left. \left. \cos \left[\frac{1}{2} (e + f x) \right]^2 \sec [e + f x] \tan [e + f x] \right) \right) \right) \right)
\end{aligned}$$

Problem 171: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{a + a \operatorname{Sec}[e + f x]} \cdot (c + d \operatorname{Sec}[e + f x])^3} dx$$

Optimal (type 3, 653 leaves, 16 steps):

$$\begin{aligned}
& \frac{2 \sqrt{a} \operatorname{ArcTanh}\left[\frac{\sqrt{a-a \sec [e+f x]}}{\sqrt{a}}\right] \tan [e+f x]}{c^3 f \sqrt{a-a \sec [e+f x]} \sqrt{a+a \sec [e+f x]}} - \frac{\sqrt{2} \sqrt{a} \operatorname{ArcTanh}\left[\frac{\sqrt{a-a \sec [e+f x]}}{\sqrt{2} \sqrt{a}}\right] \tan [e+f x]}{(c-d)^3 f \sqrt{a-a \sec [e+f x]} \sqrt{a+a \sec [e+f x]}} + \\
& \frac{3 \sqrt{a} d^{3/2} \operatorname{ArcTanh}\left[\frac{\sqrt{d} \sqrt{a-a \sec [e+f x]}}{\sqrt{a} \sqrt{c+d}}\right] \tan [e+f x]}{4 c (c-d) (c+d)^{5/2} f \sqrt{a-a \sec [e+f x]} \sqrt{a+a \sec [e+f x]}} + \\
& \frac{\sqrt{a} (2 c-d) d^{3/2} \operatorname{ArcTanh}\left[\frac{\sqrt{d} \sqrt{a-a \sec [e+f x]}}{\sqrt{a} \sqrt{c+d}}\right] \tan [e+f x]}{c^2 (c-d)^2 (c+d)^{3/2} f \sqrt{a-a \sec [e+f x]} \sqrt{a+a \sec [e+f x]}} + \\
& \frac{2 \sqrt{a} d^{3/2} (3 c^2-3 c d+d^2) \operatorname{ArcTanh}\left[\frac{\sqrt{d} \sqrt{a-a \sec [e+f x]}}{\sqrt{a} \sqrt{c+d}}\right] \tan [e+f x]}{c^3 (c-d)^3 \sqrt{c+d} f \sqrt{a-a \sec [e+f x]} \sqrt{a+a \sec [e+f x]}} + \\
& \frac{d^2 \tan [e+f x]}{2 c (c^2-d^2) f \sqrt{a+a \sec [e+f x]} (c+d \sec [e+f x])^2} + \\
& \frac{3 d^2 \tan [e+f x]}{4 c (c-d) (c+d)^2 f \sqrt{a+a \sec [e+f x]} (c+d \sec [e+f x])} + \\
& \frac{(2 c-d) d^2 \tan [e+f x]}{c^2 (c-d)^2 (c+d) f \sqrt{a+a \sec [e+f x]} (c+d \sec [e+f x])}
\end{aligned}$$

Result (type 3, 2940 leaves) :

$$\begin{aligned}
& \left(\cos \left[\frac{1}{2} (e+f x) \right] (d+c \cos [e+f x])^3 \sec [e+f x]^4 \right. \\
& \left. - \frac{d^2 (-13 c^2 - c d + 6 d^2) \sin \left[\frac{1}{2} (e+f x) \right]}{2 c^3 (-c+d)^2 (c+d)^2} - \frac{d^4 \sin \left[\frac{1}{2} (e+f x) \right]}{c^3 (-c+d) (c+d) (d+c \cos [e+f x])^2} + \right. \\
& \left. \left(-15 c^2 d^3 \sin \left[\frac{1}{2} (e+f x) \right] - c d^4 \sin \left[\frac{1}{2} (e+f x) \right] + 8 d^5 \sin \left[\frac{1}{2} (e+f x) \right] \right) \right. \\
& \left. \left(2 c^3 (-c+d)^2 (c+d)^2 (d+c \cos [e+f x]) \right) \right) / \\
& \left(f \sqrt{a (1+\sec [e+f x])} (c+d \sec [e+f x])^3 \right) - \left(\cos \left[\frac{1}{2} (e+f x) \right] (d+c \cos [e+f x])^3 \right. \\
& \left. \left(\sqrt{2} d^{3/2} (35 c^4 + 14 c^3 d - 21 c^2 d^2 - 4 c d^3 + 8 d^4) \operatorname{ArcTanh} \left[\frac{\sqrt{d} \tan \left[\frac{1}{2} (e+f x) \right]}{\sqrt{-c-d} \sqrt{-\frac{\cos [e+f x]}{1+\cos [e+f x]}}} \right] \right) \right) /
\end{aligned}$$

$$\begin{aligned}
& \left. \left(\begin{aligned}
& \text{Log}[\sec[\frac{1}{2}(e+f x)]^2 \left(-1 + 2 \cos[e+f x] - 2 \sqrt{-\frac{\cos[e+f x]}{1+\cos[e+f x]} \sin[e+f x]} \right)] + 2\sqrt{2} \\
& (c^2 - d^2)^2 \text{Log}[\sec[\frac{1}{2}(e+f x)]^2 \left(-1 + 2 \cos[e+f x] + 2 \sqrt{-\frac{\cos[e+f x]}{1+\cos[e+f x]} \sin[e+f x]} \right)] + \\
& \frac{8 c^3 (c+d)^2 \text{Log}[\tan[\frac{1}{2}(e+f x)] + \sqrt{-1 + \tan[\frac{1}{2}(e+f x)]^2}]}{c-d} \\
& - \frac{2 c d \sec[\frac{1}{2}(e+f x)]}{(-c+d)^2 (c+d)^2 (d+c \cos[e+f x]) \sqrt{\sec[e+f x]}} - \\
& \frac{13 d^2 \sec[\frac{1}{2}(e+f x)]}{8 (-c+d)^2 (c+d)^2 (d+c \cos[e+f x]) \sqrt{\sec[e+f x]}} + \\
& \frac{d^3 \sec[\frac{1}{2}(e+f x)]}{8 c (-c+d)^2 (c+d)^2 (d+c \cos[e+f x]) \sqrt{\sec[e+f x]}} + \\
& \frac{d^4 \sec[\frac{1}{2}(e+f x)]}{2 c^2 (-c+d)^2 (c+d)^2 (d+c \cos[e+f x]) \sqrt{\sec[e+f x]}} + \\
& \frac{c^2 \sec[\frac{1}{2}(e+f x)] \sqrt{\sec[e+f x]}}{2 (-c+d)^2 (c+d)^2 (d+c \cos[e+f x])} + \frac{3 d^2 \sec[\frac{1}{2}(e+f x)] \sqrt{\sec[e+f x]}}{8 (-c+d)^2 (c+d)^2 (d+c \cos[e+f x])} + \\
& \frac{d^3 \sec[\frac{1}{2}(e+f x)] \sqrt{\sec[e+f x]}}{8 c (-c+d)^2 (c+d)^2 (d+c \cos[e+f x])} + \\
& \frac{c^2 \cos[2(e+f x)] \sec[\frac{1}{2}(e+f x)] \sqrt{\sec[e+f x]}}{2 (-c+d)^2 (c+d)^2 (d+c \cos[e+f x])} - \\
& \frac{d^2 \cos[2(e+f x)] \sec[\frac{1}{2}(e+f x)] \sqrt{\sec[e+f x]}}{(-c+d)^2 (c+d)^2 (d+c \cos[e+f x])} + \\
& \frac{d^4 \cos[2(e+f x)] \sec[\frac{1}{2}(e+f x)] \sqrt{\sec[e+f x]}}{2 c^2 (-c+d)^2 (c+d)^2 (d+c \cos[e+f x])} \end{aligned} \right) \\
& \sec[e+f x]^{7/2} \sqrt{\cos[\frac{1}{2}(e+f x)]^2 \sec[e+f x]} \sqrt{-1 + \tan[\frac{1}{2}(e+f x)]^2} \Bigg) /
\end{aligned}$$

$$\begin{aligned}
& \left(4 c^3 (c - d)^2 (c + d)^2 f \sqrt{a (1 + \text{Sec}[e + f x])} (c + d \text{Sec}[e + f x])^3 \right. \\
& \left. - \frac{1}{8 c^3 (c - d)^2 (c + d)^2 \sqrt{-1 + \tan[\frac{1}{2} (e + f x)]^2}} \right. \\
& \left. \left(\left(\sqrt{2} d^{3/2} (35 c^4 + 14 c^3 d - 21 c^2 d^2 - 4 c d^3 + 8 d^4) \text{ArcTan}\left[\frac{\sqrt{d} \tan[\frac{1}{2} (e + f x)]}{\sqrt{-c - d} \sqrt{-\frac{\cos[e + f x]}{1 + \cos[e + f x]}}} \right] \right) \right. \\
& \left. \left(\sqrt{-c - d} (c - d) \right) - 2 \sqrt{2} (c^2 - d^2)^2 \text{Log}[\text{Sec}[\frac{1}{2} (e + f x)]^2 \right. \\
& \left. \left(-1 + 2 \cos[e + f x] - 2 \sqrt{-\frac{\cos[e + f x]}{1 + \cos[e + f x]}} \sin[e + f x] \right)] + 2 \sqrt{2} (c^2 - d^2)^2 \right. \\
& \left. \text{Log}[\text{Sec}[\frac{1}{2} (e + f x)]^2 \left(-1 + 2 \cos[e + f x] + 2 \sqrt{-\frac{\cos[e + f x]}{1 + \cos[e + f x]}} \sin[e + f x] \right)] + \right. \\
& \left. \left. \frac{1}{c - d} 8 c^3 (c + d)^2 \text{Log}[\tan[\frac{1}{2} (e + f x)] + \sqrt{-1 + \tan[\frac{1}{2} (e + f x)]^2}] \right) \right. \\
& \left. \left(\text{Sec}[\frac{1}{2} (e + f x)]^2 \sqrt{\cos[\frac{1}{2} (e + f x)]^2 \text{Sec}[e + f x] \tan[\frac{1}{2} (e + f x)]} - \right. \right. \\
& \left. \left. \frac{1}{4 c^3 (c - d)^2 (c + d)^2} \sqrt{\cos[\frac{1}{2} (e + f x)]^2 \text{Sec}[e + f x] \sqrt{-1 + \tan[\frac{1}{2} (e + f x)]^2}} \right. \right. \\
& \left. \left. \left(- \left(\left(2 \sqrt{2} (c^2 - d^2)^2 \cos[\frac{1}{2} (e + f x)]^2 \right. \right. \right. \right. \right. \right. \\
& \left. \left. \left. \left. \left. \left. \right) \right) \right) \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left(\operatorname{Sec} \left[\frac{1}{2} (e + f x) \right]^2 \left(-2 \cos[e + f x] \sqrt{-\frac{\cos[e + f x]}{1 + \cos[e + f x]}} - 2 \sin[e + f x] - \right. \right. \\
& \quad \left. \left. \frac{\sin[e + f x] \left(-\frac{\cos[e + f x] \sin[e + f x]}{(1 + \cos[e + f x])^2} + \frac{\sin[e + f x]}{1 + \cos[e + f x]} \right)}{\sqrt{-\frac{\cos[e + f x]}{1 + \cos[e + f x]}}} \right) + \operatorname{Sec} \left[\frac{1}{2} (e + f x) \right]^2 \right. \\
& \quad \left. \left(-1 + 2 \cos[e + f x] - 2 \sqrt{-\frac{\cos[e + f x]}{1 + \cos[e + f x]}} \sin[e + f x] \right) \tan \left[\frac{1}{2} (e + f x) \right] \right) / \\
& \quad \left. \left(-1 + 2 \cos[e + f x] - 2 \sqrt{-\frac{\cos[e + f x]}{1 + \cos[e + f x]}} \sin[e + f x] \right) \right) + \left(2 \sqrt{2} (c^2 - d^2)^2 \right. \\
& \quad \left. \left(\cos \left[\frac{1}{2} (e + f x) \right]^2 \operatorname{Sec} \left[\frac{1}{2} (e + f x) \right]^2 \left(2 \cos[e + f x] \sqrt{-\frac{\cos[e + f x]}{1 + \cos[e + f x]}} - 2 \sin[e + f x] - \right. \right. \right. \\
& \quad \left. \left. \left. \frac{\sin[e + f x] \left(-\frac{\cos[e + f x] \sin[e + f x]}{(1 + \cos[e + f x])^2} + \frac{\sin[e + f x]}{1 + \cos[e + f x]} \right)}{\sqrt{-\frac{\cos[e + f x]}{1 + \cos[e + f x]}}} \right) + \operatorname{Sec} \left[\frac{1}{2} (e + f x) \right]^2 \right. \\
& \quad \left. \left. \left(-1 + 2 \cos[e + f x] + 2 \sqrt{-\frac{\cos[e + f x]}{1 + \cos[e + f x]}} \sin[e + f x] \right) \tan \left[\frac{1}{2} (e + f x) \right] \right) \right) / \\
& \quad \left. \left(-1 + 2 \cos[e + f x] + 2 \sqrt{-\frac{\cos[e + f x]}{1 + \cos[e + f x]}} \sin[e + f x] \right) \right) + \\
& \quad \left(\sqrt{2} d^{3/2} (35 c^4 + 14 c^3 d - 21 c^2 d^2 - 4 c d^3 + 8 d^4) \right)
\end{aligned}$$

$$\begin{aligned}
& \left(\frac{\sqrt{d} \sec \left[\frac{1}{2} (e + f x) \right]^2}{2 \sqrt{-c - d} \sqrt{-\frac{\cos [e+f x]}{1+\cos [e+f x]}}} - \left(\sqrt{d} \left(-\frac{\cos [e+f x] \sin [e+f x]}{(1 + \cos [e+f x])^2} + \frac{\sin [e+f x]}{1 + \cos [e+f x]} \right) \right. \right. \\
& \left. \left. \left. \frac{\tan \left[\frac{1}{2} (e + f x) \right]}{\left(2 \sqrt{-c - d} \left(-\frac{\cos [e+f x]}{1 + \cos [e+f x]} \right)^{3/2} \right)} \right) \right) \right. \\
& \left(\sqrt{-c - d} (c - d) \left(1 - \frac{d (1 + \cos [e + f x]) \sec [e + f x] \tan \left[\frac{1}{2} (e + f x) \right]^2}{-c - d} \right) \right) + \\
& \left. \left(8 c^3 (c + d)^2 \left(\frac{1}{2} \sec \left[\frac{1}{2} (e + f x) \right]^2 + \frac{\sec \left[\frac{1}{2} (e + f x) \right]^2 \tan \left[\frac{1}{2} (e + f x) \right]}{2 \sqrt{-1 + \tan \left[\frac{1}{2} (e + f x) \right]^2}} \right) \right) \right) / \\
& \left. \left((c - d) \left(\tan \left[\frac{1}{2} (e + f x) \right] + \sqrt{-1 + \tan \left[\frac{1}{2} (e + f x) \right]^2} \right) \right) \right) - \\
& \frac{1}{8 c^3 (c - d)^2 (c + d)^2 \sqrt{\cos \left[\frac{1}{2} (e + f x) \right]^2 \sec [e + f x]}} \\
& \left(\sqrt{2} d^{3/2} (35 c^4 + 14 c^3 d - 21 c^2 d^2 - 4 c d^3 + 8 d^4) \operatorname{ArcTan} \left[\frac{\sqrt{d} \tan \left[\frac{1}{2} (e + f x) \right]}{\sqrt{-c - d} \sqrt{-\frac{\cos [e+f x]}{1+\cos [e+f x]}}} \right] \right) / \\
& \left(\sqrt{-c - d} (c - d) \right) - 2 \sqrt{2} (c^2 - d^2)^2 \log [\sec \left[\frac{1}{2} (e + f x) \right]^2 \\
& \left. \left(-1 + 2 \cos [e + f x] - 2 \sqrt{-\frac{\cos [e + f x]}{1 + \cos [e + f x]}} \sin [e + f x] \right) \right] + 2 \sqrt{2} (c^2 - d^2)^2 \\
& \log [\sec \left[\frac{1}{2} (e + f x) \right]^2 \left(-1 + 2 \cos [e + f x] + 2 \sqrt{-\frac{\cos [e + f x]}{1 + \cos [e + f x]}} \sin [e + f x] \right)] +
\end{aligned}$$

$$\left. \begin{aligned} & \frac{1}{c-d} 8 c^3 (c+d)^2 \log \left[\tan \left[\frac{1}{2} (e+f x) \right] + \sqrt{-1 + \tan \left[\frac{1}{2} (e+f x) \right]^2} \right] \\ & \sqrt{-1 + \tan \left[\frac{1}{2} (e+f x) \right]^2} \left(-\cos \left[\frac{1}{2} (e+f x) \right] \sec [e+f x] \sin \left[\frac{1}{2} (e+f x) \right] + \right. \\ & \left. \left. \cos \left[\frac{1}{2} (e+f x) \right]^2 \sec [e+f x] \tan [e+f x] \right) \right) \end{aligned} \right|$$

Problem 175: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{(a+a \sec [e+f x])^{3/2} (c+d \sec [e+f x])} dx$$

Optimal (type 3, 394 leaves, 12 steps):

$$\begin{aligned} & -\frac{\tan [e+f x]}{2 a (c-d) f (1+\sec [e+f x]) \sqrt{a+a \sec [e+f x]}} + \\ & -\frac{2 \operatorname{ArcTanh} \left[\frac{\sqrt{a-a \sec [e+f x]}}{\sqrt{a}} \right] \tan [e+f x]}{\sqrt{a} c f \sqrt{a-a \sec [e+f x]} \sqrt{a+a \sec [e+f x]}} - \\ & -\frac{\sqrt{2} (c-2 d) \operatorname{ArcTanh} \left[\frac{\sqrt{a-a \sec [e+f x]}}{\sqrt{2} \sqrt{a}} \right] \tan [e+f x]}{\sqrt{a} (c-d)^2 f \sqrt{a-a \sec [e+f x]} \sqrt{a+a \sec [e+f x]}} - \\ & -\frac{\operatorname{ArcTanh} \left[\frac{\sqrt{a-a \sec [e+f x]}}{\sqrt{2} \sqrt{a}} \right] \tan [e+f x]}{2 \sqrt{2} \sqrt{a} (c-d) f \sqrt{a-a \sec [e+f x]} \sqrt{a+a \sec [e+f x]}} - \\ & -\frac{2 d^{5/2} \operatorname{ArcTanh} \left[\frac{\sqrt{d} \sqrt{a-a \sec [e+f x]}}{\sqrt{a} \sqrt{c+d}} \right] \tan [e+f x]}{\sqrt{a} c (c-d)^2 \sqrt{c+d} f \sqrt{a-a \sec [e+f x]} \sqrt{a+a \sec [e+f x]}} \end{aligned}$$

Result (type 3, 1574 leaves):

$$\begin{aligned} & \left(\cos \left[\frac{1}{2} (e+f x) \right]^3 (d+c \cos [e+f x]) \right. \\ & \left. \sec [e+f x]^3 \left(\frac{2 \sin \left[\frac{1}{2} (e+f x) \right]}{-c+d} - \frac{\sec \left[\frac{1}{2} (e+f x) \right] \tan \left[\frac{1}{2} (e+f x) \right]}{-c+d} \right) \right) / \\ & \left(f (a (1+\sec [e+f x]))^{3/2} (c+d \sec [e+f x]) \right) + \end{aligned}$$

$$\begin{aligned}
& \left(\sqrt{-c-d} \left(-c (5c-9d) \operatorname{ArcSin}[\tan[\frac{1}{2}(e+f x)]] + 4\sqrt{2} (c-d)^2 \operatorname{ArcTan}\left[\frac{\tan[\frac{1}{2}(e+f x)]}{\sqrt{\frac{\cos[e+f x]}{1+\cos[e+f x]}}}\right] \right. \right. + \\
& \quad \left. \left. 4\sqrt{2} d^{5/2} \operatorname{ArcTanh}\left[\frac{\sqrt{d} \tan[\frac{1}{2}(e+f x)]}{\sqrt{-c-d} \sqrt{\frac{\cos[e+f x]}{1+\cos[e+f x]}}}\right] \cos[\frac{1}{2}(e+f x)]^3 \right. \right. \\
& \quad \left. \sqrt{\frac{\cos[e+f x]}{1+\cos[e+f x]}} (d+c \cos[e+f x]) \left(\frac{c \sec[\frac{1}{2}(e+f x)]}{2(-c+d)(d+c \cos[e+f x]) \sqrt{\sec[e+f x]}} - \right. \right. \\
& \quad \left. \left. \frac{2d \sec[\frac{1}{2}(e+f x)]}{(-c+d)(d+c \cos[e+f x]) \sqrt{\sec[e+f x]}} - \frac{c \sec[\frac{1}{2}(e+f x)] \sqrt{\sec[e+f x]}}{(-c+d)(d+c \cos[e+f x])} \right. \right. \\
& \quad \left. \left. \frac{3d \sec[\frac{1}{2}(e+f x)] \sqrt{\sec[e+f x]}}{2(-c+d)(d+c \cos[e+f x])} - \frac{c \cos[2(e+f x)] \sec[\frac{1}{2}(e+f x)] \sqrt{\sec[e+f x]}}{(-c+d)(d+c \cos[e+f x])} \right. \right. \\
& \quad \left. \left. \frac{d \cos[2(e+f x)] \sec[\frac{1}{2}(e+f x)] \sqrt{\sec[e+f x]}}{(-c+d)(d+c \cos[e+f x])} \right) \sec[e+f x]^{5/2} \sqrt{1+\sec[e+f x]} \right) / \\
& \quad \left(c \sqrt{-c-d} (c-d)^2 f (a (1+\sec[e+f x]))^{3/2} (c+d \sec[e+f x]) \right. \\
& \quad \left(\left(\sqrt{-c-d} \left(-c (5c-9d) \operatorname{ArcSin}[\tan[\frac{1}{2}(e+f x)]] + 4\sqrt{2} (c-d)^2 \operatorname{ArcTan}\left[\frac{\tan[\frac{1}{2}(e+f x)]}{\sqrt{\frac{\cos[e+f x]}{1+\cos[e+f x]}}}\right] \right. \right. \right. \\
& \quad \left. \left. \left. \frac{\tan[\frac{1}{2}(e+f x)]}{\sqrt{\frac{\cos[e+f x]}{1+\cos[e+f x]}}}\right] + 4\sqrt{2} d^{5/2} \operatorname{ArcTanh}\left[\frac{\sqrt{d} \tan[\frac{1}{2}(e+f x)]}{\sqrt{-c-d} \sqrt{\frac{\cos[e+f x]}{1+\cos[e+f x]}}}\right] \right) \right. \\
& \quad \left. \left. \sqrt{1+\sec[e+f x]} \left(\frac{\cos[e+f x] \sin[e+f x]}{(1+\cos[e+f x])^2} - \frac{\sin[e+f x]}{1+\cos[e+f x]} \right) \right) \right) /
\end{aligned}$$

$$\begin{aligned}
& \left(2 c \sqrt{-c-d} (c-d)^2 \sqrt{\frac{\cos[e+f x]}{1+\cos[e+f x]}} \right) + \left(\sqrt{\frac{\cos[e+f x]}{1+\cos[e+f x]}} \sqrt{1+\sec[e+f x]} \right) \left(4 \sqrt{2} \right. \\
& d^{5/2} \left(\frac{\sqrt{d} \sec[\frac{1}{2}(e+f x)]^2}{2 \sqrt{-c-d} \sqrt{\frac{\cos[e+f x]}{1+\cos[e+f x]}}} - \left(\sqrt{d} \left(\frac{\cos[e+f x] \sin[e+f x]}{(1+\cos[e+f x])^2} - \frac{\sin[e+f x]}{1+\cos[e+f x]} \right) \right. \right. \\
& \left. \left. \tan[\frac{1}{2}(e+f x)] \right) \right) \Bigg/ \left(2 \sqrt{-c-d} \left(\frac{\cos[e+f x]}{1+\cos[e+f x]} \right)^{3/2} \right) \Bigg) \Bigg/ \\
& \left(1 - \frac{d (1+\cos[e+f x]) \sec[e+f x] \tan[\frac{1}{2}(e+f x)]^2}{-c-d} \right) + \\
& \sqrt{-c-d} \left(-\frac{c (5c-9d) \sec[\frac{1}{2}(e+f x)]^2}{2 \sqrt{1-\tan[\frac{1}{2}(e+f x)]^2}} + \left(4 \sqrt{2} (c-d)^2 \right. \right. \\
& \left. \left. \frac{\sec[\frac{1}{2}(e+f x)]^2 - \left(\frac{\cos[e+f x] \sin[e+f x]}{(1+\cos[e+f x])^2} - \frac{\sin[e+f x]}{1+\cos[e+f x]} \right) \tan[\frac{1}{2}(e+f x)]}{2 \left(\frac{\cos[e+f x]}{1+\cos[e+f x]} \right)^{3/2}} \right) \right) \Bigg) \Bigg/ \\
& \left(1 + (1+\cos[e+f x]) \sec[e+f x] \tan[\frac{1}{2}(e+f x)]^2 \right) \Bigg) \Bigg/ \left(c \sqrt{-c-d} (c-d)^2 \right) + \\
& \left(\left(\sqrt{-c-d} \left(-c (5c-9d) \text{ArcSin}[\tan[\frac{1}{2}(e+f x)]] + 4 \sqrt{2} (c-d)^2 \text{ArcTan} \right. \right. \right. \\
& \left. \left. \left. \frac{\tan[\frac{1}{2}(e+f x)]}{\sqrt{\frac{\cos[e+f x]}{1+\cos[e+f x]}}} \right) \right) + 4 \sqrt{2} d^{5/2} \text{ArcTanh} \left(\frac{\sqrt{d} \tan[\frac{1}{2}(e+f x)]}{\sqrt{-c-d} \sqrt{\frac{\cos[e+f x]}{1+\cos[e+f x]}}} \right) \right)
\end{aligned}$$

$$\left. \begin{aligned} & \sqrt{\frac{\cos[e+fx]}{1+\cos[e+fx]}} \sec[e+fx] \tan[e+fx] \\ & \sqrt{1+\sec[e+fx]} \end{aligned} \right) \Bigg/ \left(2c\sqrt{-c-d} (c-d)^2 \right)$$

Problem 176: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{(a + a \operatorname{Sec}[e + f x])^{3/2} (c + d \operatorname{Sec}[e + f x])^2} dx$$

Optimal (type 3, 560 leaves, 15 steps):

$$\begin{aligned}
 & \frac{\operatorname{Tan}[e + fx]}{2 a (c - d)^2 f (1 + \operatorname{Sec}[e + fx]) \sqrt{a + a \operatorname{Sec}[e + fx]}} + \\
 & \frac{2 \operatorname{ArcTanh}\left[\frac{\sqrt{a - a \operatorname{Sec}[e + fx]}}{\sqrt{a}}\right] \operatorname{Tan}[e + fx]}{\sqrt{a} c^2 f \sqrt{a - a \operatorname{Sec}[e + fx]} \sqrt{a + a \operatorname{Sec}[e + fx]}} - \\
 & \frac{\sqrt{2} (c - 3d) \operatorname{ArcTanh}\left[\frac{\sqrt{a - a \operatorname{Sec}[e + fx]}}{\sqrt{2} \sqrt{a}}\right] \operatorname{Tan}[e + fx]}{\sqrt{a} (c - d)^3 f \sqrt{a - a \operatorname{Sec}[e + fx]} \sqrt{a + a \operatorname{Sec}[e + fx]}} - \\
 & \frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a - a \operatorname{Sec}[e + fx]}}{\sqrt{2} \sqrt{a}}\right] \operatorname{Tan}[e + fx]}{2 \sqrt{2} \sqrt{a} (c - d)^2 f \sqrt{a - a \operatorname{Sec}[e + fx]} \sqrt{a + a \operatorname{Sec}[e + fx]}} - \\
 & \frac{d^{5/2} \operatorname{ArcTanh}\left[\frac{\sqrt{d} \sqrt{a - a \operatorname{Sec}[e + fx]}}{\sqrt{a} \sqrt{c + d}}\right] \operatorname{Tan}[e + fx]}{\sqrt{a} c (c - d)^2 (c + d)^{3/2} f \sqrt{a - a \operatorname{Sec}[e + fx]} \sqrt{a + a \operatorname{Sec}[e + fx]}} - \\
 & \frac{2 (3c - d) d^{5/2} \operatorname{ArcTanh}\left[\frac{\sqrt{d} \sqrt{a - a \operatorname{Sec}[e + fx]}}{\sqrt{a} \sqrt{c + d}}\right] \operatorname{Tan}[e + fx]}{\sqrt{a} c^2 (c - d)^3 \sqrt{c + d} f \sqrt{a - a \operatorname{Sec}[e + fx]} \sqrt{a + a \operatorname{Sec}[e + fx]}} - \\
 & \frac{d^3 \operatorname{Tan}[e + fx]}{a c (c - d)^2 (c + d) f \sqrt{a + a \operatorname{Sec}[e + fx]} (c + d \operatorname{Sec}[e + fx])}
 \end{aligned}$$

Result (type 3, 2118 leaves):

$$\left(\cos \left[\frac{1}{2} (e + f x) \right]^3 (d + c \cos [e + f x])^2 \sec [e + f x]^4 \left(-\frac{2 (c^3 + c^2 d + 2 d^3) \sin \left[\frac{1}{2} (e + f x) \right]}{c^2 (-c + d)^2 (c + d)} + \right. \right.$$

$$\begin{aligned}
& \left. \frac{4 d^4 \sin \left[\frac{1}{2} (e + f x) \right]}{c^2 (-c + d)^2 (c + d) (d + c \cos [e + f x])} + \frac{\sec \left[\frac{1}{2} (e + f x) \right] \tan \left[\frac{1}{2} (e + f x) \right]}{(-c + d)^2} \right) \Bigg) \\
& \left(f (a (1 + \sec [e + f x]))^{3/2} (c + d \sec [e + f x])^2 \right) + \left(\begin{array}{l} (-c - d)^{3/2} \\ \left(-c^2 (5c - 13d) \operatorname{ArcSin} [\tan \left[\frac{1}{2} (e + f x) \right]] + 4\sqrt{2} (c - d)^3 \operatorname{ArcTan} \left[\frac{\tan \left[\frac{1}{2} (e + f x) \right]}{\sqrt{\frac{\cos [e + f x]}{1 + \cos [e + f x]}}} \right] \right) + \right. \right. \\
\left. \left. 2\sqrt{2} d^{5/2} (-7c^2 - 3cd + 2d^2) \operatorname{ArcTanh} \left[\frac{\sqrt{d} \tan \left[\frac{1}{2} (e + f x) \right]}{\sqrt{-c - d} \sqrt{\frac{\cos [e + f x]}{1 + \cos [e + f x]}}} \right] \right) \right. \\
\left. \cos \left[\frac{1}{2} (e + f x) \right]^3 (d + c \cos [e + f x])^2 \sqrt{\cos [e + f x] \sec \left[\frac{1}{2} (e + f x) \right]^2} \right. \\
\left. - \frac{c^2 \sec \left[\frac{1}{2} (e + f x) \right]}{2 (-c + d)^2 (c + d) (d + c \cos [e + f x]) \sqrt{\sec [e + f x]}} + \right. \\
\left. \frac{3cd \sec \left[\frac{1}{2} (e + f x) \right]}{2 (-c + d)^2 (c + d) (d + c \cos [e + f x]) \sqrt{\sec [e + f x]}} + \right. \\
\left. \frac{4d^2 \sec \left[\frac{1}{2} (e + f x) \right]}{(-c + d)^2 (c + d) (d + c \cos [e + f x]) \sqrt{\sec [e + f x]}} + \right. \\
\left. \frac{d^3 \sec \left[\frac{1}{2} (e + f x) \right]}{c (-c + d)^2 (c + d) (d + c \cos [e + f x]) \sqrt{\sec [e + f x]}} - \right. \\
\left. \frac{c^2 \sec \left[\frac{1}{2} (e + f x) \right] \sqrt{\sec [e + f x]}}{(-c + d)^2 (c + d) (d + c \cos [e + f x])} - \frac{3cd \sec \left[\frac{1}{2} (e + f x) \right] \sqrt{\sec [e + f x]}}{2 (-c + d)^2 (c + d) (d + c \cos [e + f x])} - \right. \\
\left. \frac{3d^2 \sec \left[\frac{1}{2} (e + f x) \right] \sqrt{\sec [e + f x]}}{2 (-c + d)^2 (c + d) (d + c \cos [e + f x])} + \frac{c^2 \cos [2(e + f x)] \sec \left[\frac{1}{2} (e + f x) \right] \sqrt{\sec [e + f x]}}{(-c + d)^2 (c + d) (d + c \cos [e + f x])} - \right. \\
\left. \frac{cd \cos [2(e + f x)] \sec \left[\frac{1}{2} (e + f x) \right] \sqrt{\sec [e + f x]}}{(-c + d)^2 (c + d) (d + c \cos [e + f x])} - \right. \\
\left. \frac{d^2 \cos [2(e + f x)] \sec \left[\frac{1}{2} (e + f x) \right] \sqrt{\sec [e + f x]}}{(-c + d)^2 (c + d) (d + c \cos [e + f x])} \right)
\end{aligned}$$

$$\begin{aligned}
& \left(\sqrt{d} \left(\frac{\cos[e+f x] \sin[e+f x]}{(1+\cos[e+f x])^2} - \frac{\sin[e+f x]}{1+\cos[e+f x]} \right) \tan\left[\frac{1}{2}(e+f x)\right] \right) / \\
& \left(2\sqrt{-c-d} \left(\frac{\cos[e+f x]}{1+\cos[e+f x]} \right)^{3/2} \right) \Bigg) / \\
& \left(1 - \frac{d(1+\cos[e+f x]) \sec[e+f x] \tan\left[\frac{1}{2}(e+f x)\right]^2}{-c-d} \right) + \\
& (-c-d)^{3/2} \left(-\frac{c^2(5c-13d) \sec\left[\frac{1}{2}(e+f x)\right]^2}{2\sqrt{1-\tan\left[\frac{1}{2}(e+f x)\right]^2}} + \frac{4\sqrt{2}(c-d)^3}{2\sqrt{\frac{\cos[e+f x]}{1+\cos[e+f x]}}} \right. \\
& \left. \left(\frac{\sec\left[\frac{1}{2}(e+f x)\right]^2}{2\sqrt{\frac{\cos[e+f x]}{1+\cos[e+f x]}}} - \frac{\left(\frac{\cos[e+f x] \sin[e+f x]}{(1+\cos[e+f x])^2} - \frac{\sin[e+f x]}{1+\cos[e+f x]}\right) \tan\left[\frac{1}{2}(e+f x)\right]}{2\left(\frac{\cos[e+f x]}{1+\cos[e+f x]}\right)^{3/2}} \right) \right) \Bigg) / \\
& \left(1 + (1+\cos[e+f x]) \sec[e+f x] \tan\left[\frac{1}{2}(e+f x)\right]^2 \right) \Bigg) \Bigg) / \\
& \left(c^2(-c-d)^{3/2}(c-d)^3 + \frac{1}{2c^2(-c-d)^{3/2}(c-d)^3\sqrt{\cos\left[\frac{1}{2}(e+f x)\right]^2 \sec[e+f x]}} \right. \\
& \left. \left((-c-d)^{3/2} \left(-c^2(5c-13d) \text{ArcSin}[\tan\left[\frac{1}{2}(e+f x)\right]] + \right. \right. \right. \\
& \left. \left. \left. 4\sqrt{2}(c-d)^3 \text{ArcTan}\left[\frac{\tan\left[\frac{1}{2}(e+f x)\right]}{\sqrt{\frac{\cos[e+f x]}{1+\cos[e+f x]}}}\right] \right) \right) \right)
\end{aligned}$$

$$\begin{aligned}
& 2 \sqrt{2} d^{5/2} (-7 c^2 - 3 c d + 2 d^2) \operatorname{ArcTanh}\left[\frac{\sqrt{d} \tan\left[\frac{1}{2} (e + f x)\right]}{\sqrt{-c - d} \sqrt{\frac{\cos[e+f x]}{1+\cos[e+f x]}}}\right] \\
& \sqrt{\cos[e+f x] \sec^2\left[\frac{1}{2} (e + f x)\right]^2} \left(-\cos\left[\frac{1}{2} (e + f x)\right] \sec[e+f x] \sin\left[\frac{1}{2} (e + f x)\right] + \right. \\
& \left. \cos^2\left[\frac{1}{2} (e + f x)\right] \sec[e+f x] \tan[e+f x] \right)
\end{aligned}$$

Problem 177: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{(a + a \sec[e + f x])^{3/2} (c + d \sec[e + f x])^3} dx$$

Optimal (type 3, 802 leaves, 19 steps):

$$\begin{aligned}
& - \frac{\tan[e + f x]}{2 a (c - d)^3 f (1 + \sec[e + f x]) \sqrt{a + a \sec[e + f x]}} + \\
& \frac{2 \operatorname{ArcTanh}\left[\frac{\sqrt{a-a \sec[e+f x]}}{\sqrt{a}}\right] \tan[e + f x]}{\sqrt{a} c^3 f \sqrt{a-a \sec[e+f x]} \sqrt{a+a \sec[e+f x]}} - \\
& \frac{\sqrt{2} (c-4 d) \operatorname{ArcTanh}\left[\frac{\sqrt{a-a \sec[e+f x]}}{\sqrt{2} \sqrt{a}}\right] \tan[e + f x]}{\sqrt{a} (c-d)^4 f \sqrt{a-a \sec[e+f x]} \sqrt{a+a \sec[e+f x]}} - \\
& \frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a-a \sec[e+f x]}}{\sqrt{2} \sqrt{a}}\right] \tan[e + f x]}{2 \sqrt{2} \sqrt{a} (c-d)^3 f \sqrt{a-a \sec[e+f x]} \sqrt{a+a \sec[e+f x]}} - \\
& \frac{3 d^{5/2} \operatorname{ArcTanh}\left[\frac{\sqrt{d} \sqrt{a-a \sec[e+f x]}}{\sqrt{a} \sqrt{c+d}}\right] \tan[e + f x]}{4 \sqrt{a} c (c-d)^2 (c+d)^{5/2} f \sqrt{a-a \sec[e+f x]} \sqrt{a+a \sec[e+f x]}} - \\
& \frac{(3 c-d) d^{5/2} \operatorname{ArcTanh}\left[\frac{\sqrt{d} \sqrt{a-a \sec[e+f x]}}{\sqrt{a} \sqrt{c+d}}\right] \tan[e + f x]}{\sqrt{a} c^2 (c-d)^3 (c+d)^{3/2} f \sqrt{a-a \sec[e+f x]} \sqrt{a+a \sec[e+f x]}} - \\
& \frac{2 d^{5/2} (6 c^2 - 4 c d + d^2) \operatorname{ArcTanh}\left[\frac{\sqrt{d} \sqrt{a-a \sec[e+f x]}}{\sqrt{a} \sqrt{c+d}}\right] \tan[e + f x]}{\sqrt{a} c^3 (c-d)^4 \sqrt{c+d} f \sqrt{a-a \sec[e+f x]} \sqrt{a+a \sec[e+f x]}} - \\
& \frac{d^3 \tan[e + f x]}{2 a c (c-d)^2 (c+d) f \sqrt{a+a \sec[e+f x]} (c+d \sec[e+f x])^2} - \\
& \frac{(3 c-d) d^3 \tan[e + f x]}{a c^2 (c-d)^3 (c+d) f \sqrt{a+a \sec[e+f x]} (c+d \sec[e+f x])} - \\
& \frac{3 d^3 \tan[e + f x]}{4 a c (c^2 - d^2)^2 f \sqrt{a+a \sec[e+f x]} (c+d \sec[e+f x])}
\end{aligned}$$

Result (type 3, 2632 leaves):

$$\begin{aligned}
& \left(\cos\left[\frac{1}{2} (e + f x)\right]^3 (d + c \cos[e + f x])^3 \sec[e + f x]^5 \right. \\
& \left. - \left(\left((-2 c^5 - 4 c^4 d - 2 c^3 d^2 - 17 c^2 d^3 - 5 c d^4 + 6 d^5) \sin\left[\frac{1}{2} (e + f x)\right]\right) \right. \right. \\
& \left. \left. \left(c^3 (-c + d)^3 (c + d)^2 \right) - \frac{2 d^5 \sin\left[\frac{1}{2} (e + f x)\right]}{c^3 (-c + d)^2 (c + d) (d + c \cos[e + f x])^2} + \right. \right. \\
& \left. \left. \left(-19 c^2 d^4 \sin\left[\frac{1}{2} (e + f x)\right] - 5 c d^5 \sin\left[\frac{1}{2} (e + f x)\right] + 8 d^6 \sin\left[\frac{1}{2} (e + f x)\right] \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left(\frac{\sec[\frac{1}{2}(e+f x)] \tan[\frac{1}{2}(e+f x)]}{(-c+d)^3} \right) \Bigg) \Bigg) \\
& \left(f(a(1+\sec[e+f x]))^{3/2} (c+d \sec[e+f x])^3 \right) - \\
& \left(\left(2 c^3 (5 c - 17 d) (c + d)^2 \operatorname{ArcSin}[\tan[\frac{1}{2}(e+f x)]] \right) - \right. \\
& \left. \left(8 \sqrt{2} (c - d)^4 (c + d)^2 \operatorname{ArcTan}\left[\frac{\tan[\frac{1}{2}(e+f x)]}{\sqrt{\frac{\cos[e+f x]}{1+\cos[e+f x]}}}\right] - \frac{1}{\sqrt{-c - d}} \right. \right. \\
& \left. \left. \sqrt{2} d^{5/2} (63 c^4 + 54 c^3 d - 17 c^2 d^2 - 12 c d^3 + 8 d^4) \operatorname{ArcTanh}\left[\frac{\sqrt{d} \tan[\frac{1}{2}(e+f x)]}{\sqrt{-c - d} \sqrt{\frac{\cos[e+f x]}{1+\cos[e+f x]}}}\right] \right) \right) \\
& \cos[\frac{1}{2}(e+f x)]^3 (d + c \cos[e+f x])^3 \sqrt{\cos[e+f x] \sec[\frac{1}{2}(e+f x)]^2} \\
& \left(\frac{c^3 \sec[\frac{1}{2}(e+f x)]}{2 (-c+d)^3 (c+d)^2 (d+c \cos[e+f x]) \sqrt{\sec[e+f x]}} - \right. \\
& \left. \frac{c^2 d \sec[\frac{1}{2}(e+f x)]}{(-c+d)^3 (c+d)^2 (d+c \cos[e+f x]) \sqrt{\sec[e+f x]}} - \right. \\
& \left. \frac{19 c d^2 \sec[\frac{1}{2}(e+f x)]}{2 (-c+d)^3 (c+d)^2 (d+c \cos[e+f x]) \sqrt{\sec[e+f x]}} - \right. \\
& \left. \frac{33 d^3 \sec[\frac{1}{2}(e+f x)]}{4 (-c+d)^3 (c+d)^2 (d+c \cos[e+f x]) \sqrt{\sec[e+f x]}} - \right. \\
& \left. \frac{3 d^4 \sec[\frac{1}{2}(e+f x)]}{4 c (-c+d)^3 (c+d)^2 (d+c \cos[e+f x]) \sqrt{\sec[e+f x]}} + \right. \\
& \left. \frac{d^5 \sec[\frac{1}{2}(e+f x)]}{c^2 (-c+d)^3 (c+d)^2 (d+c \cos[e+f x]) \sqrt{\sec[e+f x]}} - \right. \\
& \left. \frac{c^3 \sec[\frac{1}{2}(e+f x)] \sqrt{\sec[e+f x]}}{(-c+d)^3 (c+d)^2 (d+c \cos[e+f x])} + \frac{3 c^2 d \sec[\frac{1}{2}(e+f x)] \sqrt{\sec[e+f x]}}{2 (-c+d)^3 (c+d)^2 (d+c \cos[e+f x])} + \right. \\
& \left. \frac{3 c d^2 \sec[\frac{1}{2}(e+f x)] \sqrt{\sec[e+f x]}}{(-c+d)^3 (c+d)^2 (d+c \cos[e+f x])} + \frac{9 d^3 \sec[\frac{1}{2}(e+f x)] \sqrt{\sec[e+f x]}}{4 (-c+d)^3 (c+d)^2 (d+c \cos[e+f x])} + \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{d^4 \sec \left[\frac{1}{2} (e+f x)\right] \sqrt{\sec [e+f x]}}{4 c (-c+d)^3 (c+d)^2 (d+c \cos [e+f x])} - \\
& \frac{c^3 \cos [2 (e+f x)] \sec \left[\frac{1}{2} (e+f x)\right] \sqrt{\sec [e+f x]}}{(-c+d)^3 (c+d)^2 (d+c \cos [e+f x])} + \\
& \frac{c^2 d \cos [2 (e+f x)] \sec \left[\frac{1}{2} (e+f x)\right] \sqrt{\sec [e+f x]}}{(-c+d)^3 (c+d)^2 (d+c \cos [e+f x])} + \\
& \frac{2 c d^2 \cos [2 (e+f x)] \sec \left[\frac{1}{2} (e+f x)\right] \sqrt{\sec [e+f x]}}{(-c+d)^3 (c+d)^2 (d+c \cos [e+f x])} - \\
& \frac{2 d^3 \cos [2 (e+f x)] \sec \left[\frac{1}{2} (e+f x)\right] \sqrt{\sec [e+f x]}}{(-c+d)^3 (c+d)^2 (d+c \cos [e+f x])} - \\
& \frac{d^4 \cos [2 (e+f x)] \sec \left[\frac{1}{2} (e+f x)\right] \sqrt{\sec [e+f x]}}{c (-c+d)^3 (c+d)^2 (d+c \cos [e+f x])} + \\
& \frac{d^5 \cos [2 (e+f x)] \sec \left[\frac{1}{2} (e+f x)\right] \sqrt{\sec [e+f x]}}{c^2 (-c+d)^3 (c+d)^2 (d+c \cos [e+f x])} \Bigg) \\
& \sec [e+f x]^{9/2} \sqrt{\cos \left[\frac{1}{2} (e+f x)\right]^2 \sec [e+f x]} \Bigg) / \\
& \left(2 c^3 (c-d)^4 (c+d)^2 f (a (1+\sec [e+f x]))^{3/2} (c+d \sec [e+f x])^3 \right. \\
& \left. - \frac{1}{4 c^3 (c-d)^4 (c+d)^2} \left(2 c^3 (5 c - 17 d) (c+d)^2 \text{ArcSin} [\tan \left[\frac{1}{2} (e+f x)\right]] - \right. \right. \\
& \left. \left. 8 \sqrt{2} (c-d)^4 (c+d)^2 \text{ArcTan} \left[\frac{\tan \left[\frac{1}{2} (e+f x)\right]}{\sqrt{\frac{\cos [e+f x]}{1+\cos [e+f x]}}} \right] - \frac{1}{\sqrt{-c-d}} \right. \right. \\
& \left. \left. \sqrt{2} d^{5/2} (63 c^4 + 54 c^3 d - 17 c^2 d^2 - 12 c d^3 + 8 d^4) \text{ArcTanh} \left[\frac{\sqrt{d} \tan \left[\frac{1}{2} (e+f x)\right]}{\sqrt{-c-d} \sqrt{\frac{\cos [e+f x]}{1+\cos [e+f x]}}} \right] \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \sqrt{\cos[e+f x] \sec[\frac{1}{2} (e+f x)]^2} \left(\cos[\frac{1}{2} (e+f x)]^2 \sec[e+f x] \right)^{3/2} \\
& \left(-\sec[\frac{1}{2} (e+f x)]^2 \sin[e+f x] + \cos[e+f x] \sec[\frac{1}{2} (e+f x)]^2 \tan[\frac{1}{2} (e+f x)] \right) - \\
& \left(\sqrt{\cos[e+f x] \sec[\frac{1}{2} (e+f x)]^2} \sqrt{\cos[\frac{1}{2} (e+f x)]^2 \sec[e+f x]} \right. \\
& \left. \left(\frac{c^3 (5 c - 17 d) (c + d)^2 \sec[\frac{1}{2} (e+f x)]^2}{\sqrt{1 - \tan[\frac{1}{2} (e+f x)]^2}} - \left(8 \sqrt{2} (c - d)^4 (c + d)^2 \left(\frac{\sec[\frac{1}{2} (e+f x)]^2}{2 \sqrt{\frac{\cos[e+f x]}{1 + \cos[e+f x]}}} - \right. \right. \right. \right. \\
& \left. \left. \left. \left. \left(\frac{\cos[e+f x] \sin[e+f x]}{(1 + \cos[e+f x])^2} - \frac{\sin[e+f x]}{1 + \cos[e+f x]} \right) \tan[\frac{1}{2} (e+f x)] \right) \right)^{3/2} \right) \right) / \left(1 + (1 + \cos[e+f x]) \sec[e+f x] \right) \\
& e + f x] \tan[\frac{1}{2} (e+f x)]^2 \right) - \left(\sqrt{2} d^{5/2} (63 c^4 + 54 c^3 d - 17 c^2 d^2 - 12 c d^3 + 8 d^4) \right. \\
& \left(\frac{\sqrt{d} \sec[\frac{1}{2} (e+f x)]^2}{2 \sqrt{-c - d} \sqrt{\frac{\cos[e+f x]}{1 + \cos[e+f x]}}} - \left(\sqrt{d} \left(\frac{\cos[e+f x] \sin[e+f x]}{(1 + \cos[e+f x])^2} - \frac{\sin[e+f x]}{1 + \cos[e+f x]} \right) \right. \right. \\
& \left. \left. \left. \left. \left. \tan[\frac{1}{2} (e+f x)] \right) \right)^{3/2} \right) \right) / \left(2 \sqrt{-c - d} \left(\frac{\cos[e+f x]}{1 + \cos[e+f x]} \right)^{3/2} \right) \right) \\
& \left(\sqrt{-c - d} \left(1 - \frac{d (1 + \cos[e+f x]) \sec[e+f x] \tan[\frac{1}{2} (e+f x)]^2}{-c - d} \right) \right) \right) / \\
& \left(2 c^3 (c - d)^4 (c + d)^2 - \frac{1}{4 c^3 (c - d)^4 (c + d)^2 \sqrt{\cos[\frac{1}{2} (e+f x)]^2 \sec[e+f x]}} \right)
\end{aligned}$$

$$\begin{aligned}
& \left(2 c^3 (5 c - 17 d) (c + d)^2 \operatorname{ArcSin}[\tan[\frac{1}{2} (e + f x)]] - \right. \\
& \quad 8 \sqrt{2} (c - d)^4 (c + d)^2 \operatorname{ArcTan}\left[\frac{\tan[\frac{1}{2} (e + f x)]}{\sqrt{\frac{\cos[e + f x]}{1 + \cos[e + f x]}}}\right] - \frac{1}{\sqrt{-c - d}} \\
& \quad \left. \sqrt{2} d^{5/2} (63 c^4 + 54 c^3 d - 17 c^2 d^2 - 12 c d^3 + 8 d^4) \operatorname{ArcTanh}\left[\frac{\sqrt{d} \tan[\frac{1}{2} (e + f x)]}{\sqrt{-c - d} \sqrt{\frac{\cos[e + f x]}{1 + \cos[e + f x]}}}\right] \right) \\
& \quad \sqrt{\cos[e + f x] \sec[\frac{1}{2} (e + f x)]^2} \left(-\cos[\frac{1}{2} (e + f x)] \sec[e + f x] \sin[\frac{1}{2} (e + f x)] + \right. \\
& \quad \left. \left. \cos[\frac{1}{2} (e + f x)]^2 \sec[e + f x] \tan[e + f x] \right) \right)
\end{aligned}$$

Problem 180: Result more than twice size of optimal antiderivative.

$$\int \frac{c + d \sec[e + f x]}{(a + a \sec[e + f x])^{5/2}} dx$$

Optimal (type 3, 164 leaves, 7 steps):

$$\begin{aligned}
& \frac{2 c \operatorname{ArcTan}\left[\frac{\sqrt{a} \tan[e + f x]}{\sqrt{a+a \sec[e + f x]}}\right]}{a^{5/2} f} - \frac{(43 c - 3 d) \operatorname{ArcTan}\left[\frac{\sqrt{a} \tan[e + f x]}{\sqrt{2} \sqrt{a+a \sec[e + f x]}}\right]}{16 \sqrt{2} a^{5/2} f} - \\
& \quad \frac{(c - d) \tan[e + f x]}{4 f (a + a \sec[e + f x])^{5/2}} - \frac{(11 c - 3 d) \tan[e + f x]}{16 a f (a + a \sec[e + f x])^{3/2}}
\end{aligned}$$

Result (type 3, 343 leaves):

$$\begin{aligned}
& \left(\left(-43 c + 3 d \right) \operatorname{ArcSin} \left[\tan \left[\frac{1}{2} (e + f x) \right] \right] + 32 \sqrt{2} c \operatorname{ArcTan} \left[\frac{\tan \left[\frac{1}{2} (e + f x) \right]}{\sqrt{\frac{\cos [e+f x]}{1+\cos [e+f x]}}} \right] \right) \cos \left[\frac{1}{2} (e + f x) \right]^4 \\
& \left. \left(\sqrt{\frac{\cos [e+f x]}{1+\cos [e+f x]}} \sec [e+f x]^{3/2} \sqrt{1+\sec [e+f x]} (c+d \sec [e+f x]) \right) \right) / \\
& \left(4 f (d+c \cos [e+f x]) \sqrt{\sec \left[\frac{1}{2} (e+f x) \right]^2 (a (1+\sec [e+f x]))^{5/2}} \right. + \\
& \left(\cos \left[\frac{1}{2} (e+f x) \right]^5 \sec [e+f x]^2 (c+d \sec [e+f x]) \left(\frac{1}{2} (-15 c + 7 d) \sin \left[\frac{1}{2} (e+f x) \right] + \right. \right. \\
& \left. \left. \frac{1}{4} \sec \left[\frac{1}{2} (e+f x) \right]^2 \left(19 c \sin \left[\frac{1}{2} (e+f x) \right] - 11 d \sin \left[\frac{1}{2} (e+f x) \right] \right) + \right. \\
& \left. \left. \frac{1}{2} \sec \left[\frac{1}{2} (e+f x) \right]^4 \left(-c \sin \left[\frac{1}{2} (e+f x) \right] + d \sin \left[\frac{1}{2} (e+f x) \right] \right) \right) \right) / \\
& \left(f (d+c \cos [e+f x]) (a (1+\sec [e+f x]))^{5/2} \right)
\end{aligned}$$

Problem 181: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{(a + a \sec [e + f x])^{5/2} (c + d \sec [e + f x])} dx$$

Optimal (type 3, 592 leaves, 16 steps):

$$\begin{aligned}
& - \frac{\tan[e + fx]}{4 a^2 (c - d) f (1 + \sec[e + fx])^2 \sqrt{a + a \sec[e + fx]}} - \\
& \frac{(c - 2 d) \tan[e + fx]}{2 a^2 (c - d)^2 f (1 + \sec[e + fx]) \sqrt{a + a \sec[e + fx]}} - \\
& \frac{3 \tan[e + fx]}{16 a^2 (c - d) f (1 + \sec[e + fx]) \sqrt{a + a \sec[e + fx]}} + \\
& \frac{2 \operatorname{ArcTanh}\left[\frac{\sqrt{a-a \sec[e+f x]}}{\sqrt{a}}\right] \tan[e+f x]}{a^{3/2} c f \sqrt{a-a \sec[e+f x]} \sqrt{a+a \sec[e+f x]}} - \\
& \frac{(c-2 d) \operatorname{ArcTanh}\left[\frac{\sqrt{a-a \sec[e+f x]}}{\sqrt{2} \sqrt{a}}\right] \tan[e+f x]}{2 \sqrt{2} a^{3/2} (c-d)^2 f \sqrt{a-a \sec[e+f x]} \sqrt{a+a \sec[e+f x]}} - \\
& \frac{3 \operatorname{ArcTanh}\left[\frac{\sqrt{a-a \sec[e+f x]}}{\sqrt{2} \sqrt{a}}\right] \tan[e+f x]}{16 \sqrt{2} a^{3/2} (c-d) f \sqrt{a-a \sec[e+f x]} \sqrt{a+a \sec[e+f x]}} - \\
& \frac{\sqrt{2} (c^2 - 3 c d + 3 d^2) \operatorname{ArcTanh}\left[\frac{\sqrt{a-a \sec[e+f x]}}{\sqrt{2} \sqrt{a}}\right] \tan[e+f x]}{a^{3/2} (c-d)^3 f \sqrt{a-a \sec[e+f x]} \sqrt{a+a \sec[e+f x]}} + \\
& \frac{2 d^{7/2} \operatorname{ArcTanh}\left[\frac{\sqrt{d} \sqrt{a-a \sec[e+f x]}}{\sqrt{a} \sqrt{c+d}}\right] \tan[e+f x]}{a^{3/2} c (c-d)^3 \sqrt{c+d} f \sqrt{a-a \sec[e+f x]} \sqrt{a+a \sec[e+f x]}}
\end{aligned}$$

Result (type 3, 1826 leaves):

$$\begin{aligned}
& \left(\cos\left[\frac{1}{2}(e+fx)\right]^5 (d+c \cos[e+fx]) \sec[e+fx]^4 \right. \\
& \left(\frac{(-15 c + 23 d) \sin\left[\frac{1}{2}(e+fx)\right]}{2 (-c+d)^2} + \frac{1}{4 (-c+d)^2} \sec\left[\frac{1}{2}(e+fx)\right]^2 \right. \\
& \left. \left. \left(19 c \sin\left[\frac{1}{2}(e+fx)\right] - 27 d \sin\left[\frac{1}{2}(e+fx)\right] \right) + \frac{\sec\left[\frac{1}{2}(e+fx)\right]^3 \tan\left[\frac{1}{2}(e+fx)\right]}{2 (-c+d)} \right) \right) / \\
& \left(f (a (1 + \sec[e + fx]))^{5/2} (c + d \sec[e + fx]) \right) - \\
& \left(\sqrt{-c-d} \left(c (43 c^2 - 126 c d + 115 d^2) \operatorname{ArcSin}\left[\tan\left[\frac{1}{2}(e+fx)\right]\right] - \right. \right.
\end{aligned}$$

Problem 182: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{(a + a \operatorname{Sec}[e + f x])^{5/2} (c + d \operatorname{Sec}[e + f x])^2} dx$$

Optimal (type 3, 756 leaves, 19 steps):

$$\begin{aligned}
& - \frac{\tan[e + f x]}{4 a^2 (c - d)^2 f (1 + \sec[e + f x])^2 \sqrt{a + a \sec[e + f x]}} - \\
& \frac{(c - 3 d) \tan[e + f x]}{2 a^2 (c - d)^3 f (1 + \sec[e + f x]) \sqrt{a + a \sec[e + f x]}} - \\
& \frac{3 \tan[e + f x]}{16 a^2 (c - d)^2 f (1 + \sec[e + f x]) \sqrt{a + a \sec[e + f x]}} + \\
& \frac{2 \operatorname{ArcTanh}\left[\frac{\sqrt{a-a \sec[e+f x]}}{\sqrt{a}}\right] \tan[e+f x]}{a^{3/2} c^2 f \sqrt{a-a \sec[e+f x]} \sqrt{a+a \sec[e+f x]}} - \\
& \frac{(c-3 d) \operatorname{ArcTanh}\left[\frac{\sqrt{a-a \sec[e+f x]}}{\sqrt{2} \sqrt{a}}\right] \tan[e+f x]}{2 \sqrt{2} a^{3/2} (c-d)^3 f \sqrt{a-a \sec[e+f x]} \sqrt{a+a \sec[e+f x]}} - \\
& \frac{3 \operatorname{ArcTanh}\left[\frac{\sqrt{a-a \sec[e+f x]}}{\sqrt{2} \sqrt{a}}\right] \tan[e+f x]}{16 \sqrt{2} a^{3/2} (c-d)^2 f \sqrt{a-a \sec[e+f x]} \sqrt{a+a \sec[e+f x]}} - \\
& \frac{\sqrt{2} (c^2 - 4 c d + 6 d^2) \operatorname{ArcTanh}\left[\frac{\sqrt{a-a \sec[e+f x]}}{\sqrt{2} \sqrt{a}}\right] \tan[e+f x]}{a^{3/2} (c-d)^4 f \sqrt{a-a \sec[e+f x]} \sqrt{a+a \sec[e+f x]}} + \\
& \frac{d^{7/2} \operatorname{ArcTanh}\left[\frac{\sqrt{d} \sqrt{a-a \sec[e+f x]}}{\sqrt{a} \sqrt{c+d}}\right] \tan[e+f x]}{a^{3/2} c (c-d)^3 (c+d)^{3/2} f \sqrt{a-a \sec[e+f x]} \sqrt{a+a \sec[e+f x]}} + \\
& \frac{2 (4 c - d) d^{7/2} \operatorname{ArcTanh}\left[\frac{\sqrt{d} \sqrt{a-a \sec[e+f x]}}{\sqrt{a} \sqrt{c+d}}\right] \tan[e+f x]}{a^{3/2} c^2 (c-d)^4 \sqrt{c+d} f \sqrt{a-a \sec[e+f x]} \sqrt{a+a \sec[e+f x]}} + \\
& \frac{d^4 \tan[e+f x]}{a^2 c (c-d)^3 (c+d) f \sqrt{a+a \sec[e+f x]} (c+d \sec[e+f x])}
\end{aligned}$$

Result (type 3, 2343 leaves):

$$\begin{aligned}
& \left(\cos\left[\frac{1}{2} (e + f x)\right]^5 (d + c \cos[e + f x])^2 \right. \\
& \left. \sec[e + f x]^5 \left(-\frac{(-15 c^4 + 16 c^3 d + 31 c^2 d^2 + 16 d^4) \sin\left[\frac{1}{2} (e + f x)\right]}{2 c^2 (-c + d)^3 (c + d)} + \right. \right. \\
& \left. \left. \frac{8 d^5 \sin\left[\frac{1}{2} (e + f x)\right]}{c^2 (-c + d)^3 (c + d) (d + c \cos[e + f x])} + \frac{1}{4 (-c + d)^3} \sec\left[\frac{1}{2} (e + f x)\right]^2 \right. \right. \\
& \left. \left. \left(-19 c \sin\left[\frac{1}{2} (e + f x)\right] + 35 d \sin\left[\frac{1}{2} (e + f x)\right] \right) - \frac{\sec\left[\frac{1}{2} (e + f x)\right]^3 \tan\left[\frac{1}{2} (e + f x)\right]}{2 (-c + d)^2} \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left(f \left(a \left(1 + \sec(e + fx) \right) \right)^{5/2} \left(c + d \sec(e + fx) \right)^2 \right) - \\
& \left(\left(c^2 (43 c^3 - 123 c^2 d + 53 c d^2 + 219 d^3) \operatorname{ArcSin}[\tan(\frac{1}{2}(e + fx))] - 32 \sqrt{2} (c - d)^4 (c + d) \right. \right. \\
& \left. \left. \operatorname{ArcTan}\left[\frac{\tan(\frac{1}{2}(e + fx))}{\sqrt{\frac{\cos(e + fx)}{1 + \cos(e + fx)}}} \right] + \frac{16 \sqrt{2} d^{7/2} (9 c^2 + 5 c d - 2 d^2) \operatorname{ArcTanh}\left[\frac{\sqrt{d} \tan(\frac{1}{2}(e + fx))}{\sqrt{-c - d} \sqrt{\frac{\cos(e + fx)}{1 + \cos(e + fx)}}} \right]}{\sqrt{-c - d}} \right) \right. \\
& \left. \left(\frac{\cos(\frac{1}{2}(e + fx))^5 (d + c \cos(e + fx))^2 \sqrt{\cos(e + fx) \sec(\frac{1}{2}(e + fx))^2}}{8 (-c + d)^3 (c + d) (d + c \cos(e + fx)) \sqrt{\sec(e + fx)}} - \right. \right. \\
& \left. \left. \frac{6 c^2 d \sec(\frac{1}{2}(e + fx))}{(-c + d)^3 (c + d) (d + c \cos(e + fx)) \sqrt{\sec(e + fx)}} + \right. \right. \\
& \left. \left. \frac{37 c d^2 \sec(\frac{1}{2}(e + fx))}{8 (-c + d)^3 (c + d) (d + c \cos(e + fx)) \sqrt{\sec(e + fx)}} + \right. \right. \\
& \left. \left. \frac{16 d^3 \sec(\frac{1}{2}(e + fx))}{(-c + d)^3 (c + d) (d + c \cos(e + fx)) \sqrt{\sec(e + fx)}} + \right. \right. \\
& \left. \left. \frac{2 d^4 \sec(\frac{1}{2}(e + fx))}{c (-c + d)^3 (c + d) (d + c \cos(e + fx)) \sqrt{\sec(e + fx)}} - \right. \right. \\
& \left. \left. \frac{2 c^3 \sec(\frac{1}{2}(e + fx)) \sqrt{\sec(e + fx)}}{(-c + d)^3 (c + d) (d + c \cos(e + fx))} + \frac{43 c^2 d \sec(\frac{1}{2}(e + fx)) \sqrt{\sec(e + fx)}}{8 (-c + d)^3 (c + d) (d + c \cos(e + fx))} - \right. \right. \\
& \left. \left. \frac{2 c d^2 \sec(\frac{1}{2}(e + fx)) \sqrt{\sec(e + fx)}}{(-c + d)^3 (c + d) (d + c \cos(e + fx))} - \frac{59 d^3 \sec(\frac{1}{2}(e + fx)) \sqrt{\sec(e + fx)}}{8 (-c + d)^3 (c + d) (d + c \cos(e + fx))} - \right. \right. \\
& \left. \left. \frac{2 c^3 \cos(2(e + fx)) \sec(\frac{1}{2}(e + fx)) \sqrt{\sec(e + fx)}}{(-c + d)^3 (c + d) (d + c \cos(e + fx))} + \right. \right. \\
& \left. \left. \frac{4 c^2 d \cos(2(e + fx)) \sec(\frac{1}{2}(e + fx)) \sqrt{\sec(e + fx)}}{(-c + d)^3 (c + d) (d + c \cos(e + fx))} \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left(\frac{c^2 (43 c^3 - 123 c^2 d + 53 c d^2 + 219 d^3) \sec^2 \left[\frac{1}{2} (e + f x) \right]}{2 \sqrt{1 - \tan^2 \left[\frac{1}{2} (e + f x) \right]}} - \frac{32 \sqrt{2} (c - d)^4 (c + d)}{\right. \\
& \left. \left(\frac{\sec^2 \left[\frac{1}{2} (e + f x) \right]^2}{2 \sqrt{\frac{\cos [e+f x]}{1+\cos [e+f x]}}} - \frac{\left(\frac{\cos [e+f x] \sin [e+f x]}{(1+\cos [e+f x])^2} - \frac{\sin [e+f x]}{1+\cos [e+f x]} \right) \tan \left[\frac{1}{2} (e + f x) \right]}{2 \left(\frac{\cos [e+f x]}{1+\cos [e+f x]} \right)^{3/2}} \right) \right) / \left(1 + \right. \\
& \left. \left(1 + \cos [e + f x] \right) \sec [e + f x] \tan \left[\frac{1}{2} (e + f x) \right]^2 \right) + \frac{16 \sqrt{2} d^{7/2} (9 c^2 + 5 c d - 2 d^2)}{\left. \left(\frac{\sqrt{d} \sec^2 \left[\frac{1}{2} (e + f x) \right]^2}{2 \sqrt{-c - d} \sqrt{\frac{\cos [e+f x]}{1+\cos [e+f x]}}} - \left(\sqrt{d} \left(\frac{\cos [e+f x] \sin [e+f x]}{(1+\cos [e+f x])^2} - \frac{\sin [e+f x]}{1+\cos [e+f x]} \right) \right. \right. \right. \\
& \left. \left. \left. \tan \left[\frac{1}{2} (e + f x) \right] \right) / \left(2 \sqrt{-c - d} \left(\frac{\cos [e+f x]}{1+\cos [e+f x]} \right)^{3/2} \right) \right) \right) / \left. \left(\sqrt{-c - d} \left(1 - \frac{d (1 + \cos [e + f x]) \sec [e + f x] \tan \left[\frac{1}{2} (e + f x) \right]^2}{-c - d} \right) \right) \right) / \\
& \left(4 c^2 (c - d)^4 (c + d) \right) - \frac{1}{8 c^2 (c - d)^4 (c + d) \sqrt{\cos^2 \left[\frac{1}{2} (e + f x) \right]^2 \sec [e + f x]}} \\
& \left(c^2 (43 c^3 - 123 c^2 d + 53 c d^2 + 219 d^3) \text{ArcSin} [\tan \left[\frac{1}{2} (e + f x) \right]] - 32 \sqrt{2} (c - d)^4 (c + d) \right)
\end{aligned}$$

$$\begin{aligned}
& \left. \left(\text{ArcTan} \left[\frac{\tan \left[\frac{1}{2} (e + f x) \right]}{\sqrt{\frac{\cos [e+f x]}{1+\cos [e+f x]}}} \right] + \frac{16 \sqrt{2} d^{7/2} (9 c^2 + 5 c d - 2 d^2) \text{ArcTanh} \left[\frac{\sqrt{d} \tan \left[\frac{1}{2} (e + f x) \right]}{\sqrt{-c-d} \sqrt{\frac{\cos [e+f x]}{1+\cos [e+f x]}}} \right]}{\sqrt{-c-d}} \right) \right. \\
& \left. \left. \sqrt{\cos [e+f x] \sec \left[\frac{1}{2} (e + f x) \right]^2 \left(-\cos \left[\frac{1}{2} (e + f x) \right] \sec [e+f x] \sin \left[\frac{1}{2} (e + f x) \right] + \cos \left[\frac{1}{2} (e + f x) \right]^2 \sec [e+f x] \tan [e+f x] \right)} \right) \right)
\end{aligned}$$

Problem 183: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{(a + a \sec [e + f x])^{5/2} (c + d \sec [e + f x])^3} dx$$

Optimal (type 3, 999 leaves, 23 steps):

$$\begin{aligned}
& - \frac{\tan[e+f x]}{4 a^2 (c-d)^3 f (1+\sec[e+f x])^2 \sqrt{a+a \sec[e+f x]}} - \\
& \frac{(c-4 d) \tan[e+f x]}{2 a^2 (c-d)^4 f (1+\sec[e+f x]) \sqrt{a+a \sec[e+f x]}} - \\
& \frac{3 \tan[e+f x]}{16 a^2 (c-d)^3 f (1+\sec[e+f x]) \sqrt{a+a \sec[e+f x]}} + \\
& \frac{2 \operatorname{ArcTanh}\left[\frac{\sqrt{a-a \sec[e+f x]}}{\sqrt{a}}\right] \tan[e+f x]}{a^{3/2} c^3 f \sqrt{a-a \sec[e+f x]} \sqrt{a+a \sec[e+f x]}} - \\
& \frac{(c-4 d) \operatorname{ArcTanh}\left[\frac{\sqrt{a-a \sec[e+f x]}}{\sqrt{2} \sqrt{a}}\right] \tan[e+f x]}{2 \sqrt{2} a^{3/2} (c-d)^4 f \sqrt{a-a \sec[e+f x]} \sqrt{a+a \sec[e+f x]}} - \\
& \frac{3 \operatorname{ArcTanh}\left[\frac{\sqrt{a-a \sec[e+f x]}}{\sqrt{2} \sqrt{a}}\right] \tan[e+f x]}{16 \sqrt{2} a^{3/2} (c-d)^3 f \sqrt{a-a \sec[e+f x]} \sqrt{a+a \sec[e+f x]}} - \\
& \frac{\sqrt{2} (c^2-5 c d+10 d^2) \operatorname{ArcTanh}\left[\frac{\sqrt{a-a \sec[e+f x]}}{\sqrt{2} \sqrt{a}}\right] \tan[e+f x]}{a^{3/2} (c-d)^5 f \sqrt{a-a \sec[e+f x]} \sqrt{a+a \sec[e+f x]}} + \\
& \frac{3 d^{7/2} \operatorname{ArcTanh}\left[\frac{\sqrt{d} \sqrt{a-a \sec[e+f x]}}{\sqrt{a} \sqrt{c+d}}\right] \tan[e+f x]}{4 a^{3/2} c (c-d)^3 (c+d)^{5/2} f \sqrt{a-a \sec[e+f x]} \sqrt{a+a \sec[e+f x]}} + \\
& \frac{(4 c-d) d^{7/2} \operatorname{ArcTanh}\left[\frac{\sqrt{d} \sqrt{a-a \sec[e+f x]}}{\sqrt{a} \sqrt{c+d}}\right] \tan[e+f x]}{a^{3/2} c^2 (c-d)^4 (c+d)^{3/2} f \sqrt{a-a \sec[e+f x]} \sqrt{a+a \sec[e+f x]}} + \\
& \frac{2 d^{7/2} (10 c^2-5 c d+d^2) \operatorname{ArcTanh}\left[\frac{\sqrt{d} \sqrt{a-a \sec[e+f x]}}{\sqrt{a} \sqrt{c+d}}\right] \tan[e+f x]}{a^{3/2} c^3 (c-d)^5 \sqrt{c+d} f \sqrt{a-a \sec[e+f x]} \sqrt{a+a \sec[e+f x]}} + \\
& \frac{d^4 \tan[e+f x]}{2 a^2 c (c-d)^3 (c+d) f \sqrt{a+a \sec[e+f x]} (c+d \sec[e+f x])^2} + \\
& \frac{3 d^4 \tan[e+f x]}{4 a^2 c (c-d)^3 (c+d)^2 f \sqrt{a+a \sec[e+f x]} (c+d \sec[e+f x])} + \\
& \frac{(4 c-d) d^4 \tan[e+f x]}{a^2 c^2 (c-d)^4 (c+d) f \sqrt{a+a \sec[e+f x]} (c+d \sec[e+f x])}
\end{aligned}$$

Result (type 3, 2904 leaves):

$$\left(\cos\left[\frac{1}{2} (e+f x)\right]^5 (d+c \cos[e+f x])^3 \sec[e+f x]^6 \right)$$

$$\begin{aligned}
& \left(- \left(\left(3 (5 c^6 - 3 c^5 d - 21 c^4 d^2 - 13 c^3 d^3 - 28 c^2 d^4 - 12 c d^5 + 8 d^6) \sin \left[\frac{1}{2} (e + f x) \right] \right) \right. \right. \\
& \quad \left(2 c^3 (-c + d)^4 (c + d)^2 \right) - \frac{4 d^6 \sin \left[\frac{1}{2} (e + f x) \right]}{c^3 (-c + d)^3 (c + d) (d + c \cos [e + f x])^2} + \\
& \quad \frac{1}{4 (-c + d)^4} \sec \left[\frac{1}{2} (e + f x) \right]^2 \left(19 c \sin \left[\frac{1}{2} (e + f x) \right] - 43 d \sin \left[\frac{1}{2} (e + f x) \right] \right) + \\
& \quad \left. \left. \left(2 \left(-23 c^2 d^5 \sin \left[\frac{1}{2} (e + f x) \right] - 9 c d^6 \sin \left[\frac{1}{2} (e + f x) \right] + 8 d^7 \sin \left[\frac{1}{2} (e + f x) \right] \right) \right) \right) / \\
& \quad \left(c^3 (-c + d)^4 (c + d)^2 (d + c \cos [e + f x]) \right) + \frac{\sec \left[\frac{1}{2} (e + f x) \right]^3 \tan \left[\frac{1}{2} (e + f x) \right]}{2 (-c + d)^3} \right) / \\
& \quad \left(f (a (1 + \sec [e + f x]))^{5/2} (c + d \sec [e + f x])^3 \right) - \\
& \left(\left(c^3 (c + d)^2 (43 c^2 - 206 c d + 355 d^2) \arcsin [\tan \left[\frac{1}{2} (e + f x) \right]] - \right. \right. \\
& \quad \left. \left. 32 \sqrt{2} (c - d)^5 (c + d)^2 \operatorname{ArcTan} \left[\frac{\tan \left[\frac{1}{2} (e + f x) \right]}{\sqrt{\frac{\cos [e + f x]}{1 + \cos [e + f x]}}} \right] + \frac{1}{\sqrt{-c - d}} \right. \right. \\
& \quad \left. \left. 4 \sqrt{2} d^{7/2} (99 c^4 + 110 c^3 d - 5 c^2 d^2 - 20 c d^3 + 8 d^4) \operatorname{ArcTanh} \left[\frac{\sqrt{d} \tan \left[\frac{1}{2} (e + f x) \right]}{\sqrt{-c - d} \sqrt{\frac{\cos [e + f x]}{1 + \cos [e + f x]}}} \right] \right) \right. \\
& \quad \left. \cos \left[\frac{1}{2} (e + f x) \right]^5 (d + c \cos [e + f x])^3 \sqrt{\cos [e + f x] \sec \left[\frac{1}{2} (e + f x) \right]^2} \right. \\
& \quad \left(- \frac{11 c^4 \sec \left[\frac{1}{2} (e + f x) \right]}{8 (-c + d)^4 (c + d)^2 (d + c \cos [e + f x]) \sqrt{\sec [e + f x]}} + \right. \\
& \quad \left. \frac{45 c^3 d \sec \left[\frac{1}{2} (e + f x) \right]}{8 (-c + d)^4 (c + d)^2 (d + c \cos [e + f x]) \sqrt{\sec [e + f x]}} - \right. \\
& \quad \left. \frac{5 c^2 d^2 \sec \left[\frac{1}{2} (e + f x) \right]}{8 (-c + d)^4 (c + d)^2 (d + c \cos [e + f x]) \sqrt{\sec [e + f x]}} - \right. \\
& \quad \left. \frac{317 c d^3 \sec \left[\frac{1}{2} (e + f x) \right]}{8 (-c + d)^4 (c + d)^2 (d + c \cos [e + f x]) \sqrt{\sec [e + f x]}} \right)
\end{aligned}$$

$$\begin{aligned}
& \frac{69 d^4 \operatorname{Sec}\left[\frac{1}{2} (e + f x)\right]}{2 (-c + d)^4 (c + d)^2 (d + c \cos[e + f x]) \sqrt{\operatorname{Sec}[e + f x]}} - \\
& \frac{7 d^5 \operatorname{Sec}\left[\frac{1}{2} (e + f x)\right]}{2 c (-c + d)^4 (c + d)^2 (d + c \cos[e + f x]) \sqrt{\operatorname{Sec}[e + f x]}} + \\
& \frac{2 d^6 \operatorname{Sec}\left[\frac{1}{2} (e + f x)\right]}{c^2 (-c + d)^4 (c + d)^2 (d + c \cos[e + f x]) \sqrt{\operatorname{Sec}[e + f x]}} + \\
& \frac{2 c^4 \operatorname{Sec}\left[\frac{1}{2} (e + f x)\right] \sqrt{\operatorname{Sec}[e + f x]}}{(-c + d)^4 (c + d)^2 (d + c \cos[e + f x])} - \frac{43 c^3 d \operatorname{Sec}\left[\frac{1}{2} (e + f x)\right] \sqrt{\operatorname{Sec}[e + f x]}}{8 (-c + d)^4 (c + d)^2 (d + c \cos[e + f x])} - \\
& \frac{3 c^2 d^2 \operatorname{Sec}\left[\frac{1}{2} (e + f x)\right] \sqrt{\operatorname{Sec}[e + f x]}}{8 (-c + d)^4 (c + d)^2 (d + c \cos[e + f x])} + \frac{123 c d^3 \operatorname{Sec}\left[\frac{1}{2} (e + f x)\right] \sqrt{\operatorname{Sec}[e + f x]}}{8 (-c + d)^4 (c + d)^2 (d + c \cos[e + f x])} + \\
& \frac{95 d^4 \operatorname{Sec}\left[\frac{1}{2} (e + f x)\right] \sqrt{\operatorname{Sec}[e + f x]}}{8 (-c + d)^4 (c + d)^2 (d + c \cos[e + f x])} + \frac{d^5 \operatorname{Sec}\left[\frac{1}{2} (e + f x)\right] \sqrt{\operatorname{Sec}[e + f x]}}{2 c (-c + d)^4 (c + d)^2 (d + c \cos[e + f x])} + \\
& \frac{2 c^4 \cos[2 (e + f x)] \operatorname{Sec}\left[\frac{1}{2} (e + f x)\right] \sqrt{\operatorname{Sec}[e + f x]}}{(-c + d)^4 (c + d)^2 (d + c \cos[e + f x])} - \\
& \frac{4 c^3 d \cos[2 (e + f x)] \operatorname{Sec}\left[\frac{1}{2} (e + f x)\right] \sqrt{\operatorname{Sec}[e + f x]}}{(-c + d)^4 (c + d)^2 (d + c \cos[e + f x])} - \\
& \frac{2 c^2 d^2 \cos[2 (e + f x)] \operatorname{Sec}\left[\frac{1}{2} (e + f x)\right] \sqrt{\operatorname{Sec}[e + f x]}}{(-c + d)^4 (c + d)^2 (d + c \cos[e + f x])} + \\
& \frac{8 c d^3 \cos[2 (e + f x)] \operatorname{Sec}\left[\frac{1}{2} (e + f x)\right] \sqrt{\operatorname{Sec}[e + f x]}}{(-c + d)^4 (c + d)^2 (d + c \cos[e + f x])} - \\
& \frac{2 d^4 \cos[2 (e + f x)] \operatorname{Sec}\left[\frac{1}{2} (e + f x)\right] \sqrt{\operatorname{Sec}[e + f x]}}{(-c + d)^4 (c + d)^2 (d + c \cos[e + f x])} - \\
& \frac{4 d^5 \cos[2 (e + f x)] \operatorname{Sec}\left[\frac{1}{2} (e + f x)\right] \sqrt{\operatorname{Sec}[e + f x]}}{c (-c + d)^4 (c + d)^2 (d + c \cos[e + f x])} + \\
& \frac{2 d^6 \cos[2 (e + f x)] \operatorname{Sec}\left[\frac{1}{2} (e + f x)\right] \sqrt{\operatorname{Sec}[e + f x]}}{c^2 (-c + d)^4 (c + d)^2 (d + c \cos[e + f x])} \Big) \\
& \operatorname{Sec}[e + f x]^{11/2} \sqrt{\cos\left[\frac{1}{2} (e + f x)\right]^2 \operatorname{Sec}[e + f x]} \Bigg)
\end{aligned}$$

$$\begin{aligned}
& \left(4 c^3 (c-d)^5 (c+d)^2 f \left(a \left(1 + \operatorname{Sec}[e+f x] \right) \right)^{5/2} (c+d \operatorname{Sec}[e+f x])^3 \right. \\
& \left. - \frac{1}{8 c^3 (c-d)^5 (c+d)^2} \left(c^3 (c+d)^2 (43 c^2 - 206 c d + 355 d^2) \operatorname{ArcSin}[\operatorname{Tan}\left[\frac{1}{2} (e+f x)\right]] - \right. \right. \\
& \quad \left. 32 \sqrt{2} (c-d)^5 (c+d)^2 \operatorname{ArcTan}\left[\frac{\operatorname{Tan}\left[\frac{1}{2} (e+f x)\right]}{\sqrt{\frac{\cos[e+f x]}{1+\cos[e+f x]}}} \right] + \frac{1}{\sqrt{-c-d}} \right. \\
& \quad \left. \left. 4 \sqrt{2} d^{7/2} (99 c^4 + 110 c^3 d - 5 c^2 d^2 - 20 c d^3 + 8 d^4) \operatorname{ArcTanh}\left[\frac{\sqrt{d} \operatorname{Tan}\left[\frac{1}{2} (e+f x)\right]}{\sqrt{-c-d} \sqrt{\frac{\cos[e+f x]}{1+\cos[e+f x]}}} \right] \right) \right. \\
& \quad \left. \sqrt{\cos[e+f x] \operatorname{Sec}\left[\frac{1}{2} (e+f x)\right]^2} \left(\cos\left[\frac{1}{2} (e+f x)\right]^2 \operatorname{Sec}[e+f x] \right)^{3/2} \right. \\
& \quad \left. - \operatorname{Sec}\left[\frac{1}{2} (e+f x)\right]^2 \sin[e+f x] + \cos[e+f x] \operatorname{Sec}\left[\frac{1}{2} (e+f x)\right]^2 \operatorname{Tan}\left[\frac{1}{2} (e+f x)\right] \right) - \\
& \quad \frac{1}{4 c^3 (c-d)^5 (c+d)^2} \sqrt{\cos[e+f x] \operatorname{Sec}\left[\frac{1}{2} (e+f x)\right]^2} \sqrt{\cos\left[\frac{1}{2} (e+f x)\right]^2 \operatorname{Sec}[e+f x]} \\
& \quad \left. \left(\frac{c^3 (c+d)^2 (43 c^2 - 206 c d + 355 d^2) \operatorname{Sec}\left[\frac{1}{2} (e+f x)\right]^2}{2 \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2} (e+f x)\right]^2}} - \left(32 \sqrt{2} (c-d)^5 (c+d)^2 \right. \right. \right. \\
& \quad \left. \left. \left. \left. \frac{\operatorname{Sec}\left[\frac{1}{2} (e+f x)\right]^2 - \left(\frac{\cos[e+f x] \sin[e+f x]}{(1+\cos[e+f x])^2} - \frac{\sin[e+f x]}{1+\cos[e+f x]} \right) \operatorname{Tan}\left[\frac{1}{2} (e+f x)\right]}{2 \left(\frac{\cos[e+f x]}{1+\cos[e+f x]} \right)^{3/2}} \right) \right) \right) / \\
& \quad \left(1 + (1 + \cos[e+f x]) \operatorname{Sec}[e+f x] \operatorname{Tan}\left[\frac{1}{2} (e+f x)\right]^2 \right) +
\end{aligned}$$

$$\begin{aligned}
& \left(\frac{\sqrt{d} \operatorname{Sec} \left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x) \right]^2}{2 \sqrt{-c - d} \sqrt{\frac{\cos[\mathbf{e}+\mathbf{f} x]}{1+\cos[\mathbf{e}+\mathbf{f} x]}}} - \right. \\
& \left. \frac{\sqrt{d} \left(\frac{\cos[\mathbf{e}+\mathbf{f} x] \sin[\mathbf{e}+\mathbf{f} x]}{(1+\cos[\mathbf{e}+\mathbf{f} x])^2} - \frac{\sin[\mathbf{e}+\mathbf{f} x]}{1+\cos[\mathbf{e}+\mathbf{f} x]} \right) \tan \left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x) \right]}{2 \sqrt{-c - d} \left(\frac{\cos[\mathbf{e}+\mathbf{f} x]}{1+\cos[\mathbf{e}+\mathbf{f} x]} \right)^{3/2}} \right) / \\
& \left. \left(\sqrt{-c - d} \left(1 - \frac{d (1 + \cos[\mathbf{e} + \mathbf{f} x]) \operatorname{Sec}[\mathbf{e} + \mathbf{f} x] \tan \left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x) \right]^2}{-c - d} \right) \right) \right) - \\
& \frac{1}{8 c^3 (c - d)^5 (c + d)^2 \sqrt{\cos \left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x) \right]^2 \operatorname{Sec}[\mathbf{e} + \mathbf{f} x]}} \\
& \left(c^3 (c + d)^2 (43 c^2 - 206 c d + 355 d^2) \operatorname{ArcSin} \left[\tan \left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x) \right] \right] - \right. \\
& \left. 32 \sqrt{2} (c - d)^5 (c + d)^2 \operatorname{ArcTan} \left[\frac{\tan \left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x) \right]}{\sqrt{\frac{\cos[\mathbf{e}+\mathbf{f} x]}{1+\cos[\mathbf{e}+\mathbf{f} x]}}} \right] + \frac{1}{\sqrt{-c - d}} \right. \\
& \left. 4 \sqrt{2} d^{7/2} (99 c^4 + 110 c^3 d - 5 c^2 d^2 - 20 c d^3 + 8 d^4) \operatorname{ArcTanh} \left[\frac{\sqrt{d} \tan \left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x) \right]}{\sqrt{-c - d} \sqrt{\frac{\cos[\mathbf{e}+\mathbf{f} x]}{1+\cos[\mathbf{e}+\mathbf{f} x]}}} \right] \right. \\
& \left. \sqrt{\cos[\mathbf{e} + \mathbf{f} x] \operatorname{Sec} \left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x) \right]^2} \left(-\cos \left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x) \right] \operatorname{Sec}[\mathbf{e} + \mathbf{f} x] \sin \left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x) \right] + \right. \right. \\
& \left. \left. \cos \left[\frac{1}{2} (\mathbf{e} + \mathbf{f} x) \right]^2 \operatorname{Sec}[\mathbf{e} + \mathbf{f} x] \tan[\mathbf{e} + \mathbf{f} x] \right) \right)
\end{aligned}$$

Problem 193: Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b \sec(e + f x))^2}{(c + d \sec(e + f x))^3} dx$$

Optimal (type 3, 237 leaves, 6 steps):

$$\begin{aligned} & \frac{a^2 x}{c^3} - \\ & \left(\left(3 b^2 c^4 d - 2 a b c^3 (2 c^2 + d^2) + a^2 (6 c^4 d - 5 c^2 d^3 + 2 d^5) \right) \operatorname{ArcTanh} \left[\frac{\sqrt{c-d} \tan \left[\frac{1}{2} (e+f x) \right]}{\sqrt{c+d}} \right] \right) / \\ & \left(c^3 (c-d)^{5/2} (c+d)^{5/2} f \right) - \frac{d (b c - a d)^2 \sin(e+f x)}{2 c^2 (c^2 - d^2) f (d + c \cos(e+f x))^2} - \\ & \frac{(b c - a d) (3 a d (2 c^2 - d^2) - b c (2 c^2 + d^2)) \sin(e+f x)}{2 c^2 (c^2 - d^2)^2 f (d + c \cos(e+f x))} \end{aligned}$$

Result (type 3, 493 leaves):

$$\begin{aligned} & \frac{1}{4 c^3 f (b + a \cos(e+f x))^2 (c + d \sec(e+f x))^3} \\ & (d + c \cos(e+f x)) \sec(e+f x) (a + b \sec(e+f x))^2 \left(\frac{1}{(c^2 - d^2)^{5/2}} \right. \\ & 4 (3 b^2 c^4 d - 2 a b c^3 (2 c^2 + d^2) + a^2 (6 c^4 d - 5 c^2 d^3 + 2 d^5)) \operatorname{ArcTanh} \left[\frac{(-c+d) \tan \left[\frac{1}{2} (e+f x) \right]}{\sqrt{c^2 - d^2}} \right] \\ & (d + c \cos(e+f x))^2 + \frac{1}{(c^2 - d^2)^2} (2 a^2 c^6 e - 6 a^2 c^2 d^4 e + 4 a^2 d^6 e + 2 a^2 c^6 f x - \\ & 6 a^2 c^2 d^4 f x + 4 a^2 d^6 f x + 8 a^2 c d (c^2 - d^2)^2 (e + f x) \cos(e + f x) + \\ & 2 a^2 c^2 (c^2 - d^2)^2 (e + f x) \cos[2 (e + f x)] + 2 b^2 c^5 d \sin(e + f x) - 12 a b c^4 d^2 \sin(e + f x) + \\ & 10 a^2 c^3 d^3 \sin(e + f x) + 4 b^2 c^3 d^3 \sin(e + f x) - 4 a^2 c d^5 \sin(e + f x) + \\ & 2 b^2 c^6 \sin[2 (e + f x)] - 8 a b c^5 d \sin[2 (e + f x)] + 6 a^2 c^4 d^2 \sin[2 (e + f x)] + \\ & b^2 c^4 d^2 \sin[2 (e + f x)] + 2 a b c^3 d^3 \sin[2 (e + f x)] - 3 a^2 c^2 d^4 \sin[2 (e + f x)] \left. \right) \end{aligned}$$

Problem 195: Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b \sec(e + f x))^3}{(c + d \sec(e + f x))^3} dx$$

Optimal (type 3, 254 leaves, 6 steps):

$$\frac{a^3 x}{c^3} - \left(\left(b c - a d \right) \left(2 a b c d \left(4 c^2 - d^2 \right) - b^2 c^2 \left(c^2 + 2 d^2 \right) - a^2 \left(6 c^4 - 5 c^2 d^2 + 2 d^4 \right) \right) \right.$$

$$\left. \operatorname{ArcTanh} \left[\frac{\sqrt{c-d} \tan \left[\frac{1}{2} (e+f x) \right]}{\sqrt{c+d}} \right] \right) / \left(c^3 (c-d)^{5/2} (c+d)^{5/2} f \right) +$$

$$\frac{(b c - a d)^2 (b + a \cos[e + f x]) \sin[e + f x]}{2 c (c^2 - d^2) f (d + c \cos[e + f x])^2} + \frac{(b c - a d)^2 (5 a c^2 - 3 b c d - 2 a d^2) \sin[e + f x]}{2 c^2 (c^2 - d^2)^2 f (d + c \cos[e + f x])}$$

Result (type 3, 517 leaves) :

$$\frac{1}{4 c^3 f}$$

$$\left(- \frac{1}{(c^2 - d^2)^{5/2}} 4 (-9 a b^2 c^4 d + 3 a^2 b c^3 (2 c^2 + d^2) + b^3 c^3 (c^2 + 2 d^2) + a^3 (-6 c^4 d + 5 c^2 d^3 - 2 d^5)) \right.$$

$$\operatorname{ArcTanh} \left[\frac{(-c+d) \tan \left[\frac{1}{2} (e+f x) \right]}{\sqrt{c^2 - d^2}} \right] +$$

$$\frac{1}{(c^2 - d^2)^2 (d + c \cos[e + f x])^2} \left(2 a^3 c^6 e - 6 a^3 c^2 d^4 e + 4 a^3 d^6 e + 2 a^3 c^6 f x - 6 a^3 c^2 d^4 f x + \right.$$

$$4 a^3 d^6 f x + 8 a^3 c d (c^2 - d^2)^2 (e + f x) \cos[e + f x] + 2 a^3 (c^3 - c d^2)^2 (e + f x) \cos[2 (e + f x)] +$$

$$2 b^3 c^6 \sin[e + f x] + 6 a b^2 c^5 d \sin[e + f x] - 18 a^2 b c^4 d^2 \sin[e + f x] -$$

$$8 b^3 c^4 d^2 \sin[e + f x] + 10 a^3 c^3 d^3 \sin[e + f x] + 12 a b^2 c^3 d^3 \sin[e + f x] -$$

$$4 a^3 c d^5 \sin[e + f x] + 6 a b^2 c^6 \sin[2 (e + f x)] - 12 a^2 b c^5 d \sin[2 (e + f x)] -$$

$$3 b^3 c^5 d \sin[2 (e + f x)] + 6 a^3 c^4 d^2 \sin[2 (e + f x)] + 3 a b^2 c^4 d^2 \sin[2 (e + f x)] +$$

$$3 a^2 b c^3 d^3 \sin[2 (e + f x)] - 3 a^3 c^2 d^4 \sin[2 (e + f x)] \left. \right)$$

Problem 196: Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b \sec[e + f x])^3}{(c + d \sec[e + f x])^4} dx$$

Optimal (type 3, 412 leaves, 7 steps) :

$$\begin{aligned}
& \frac{a^3 x}{c^4} - \\
& \left(\left(3 a b^2 c^4 d (4 c^2 + d^2) - b^3 c^5 (c^2 + 4 d^2) - a^2 b (6 c^7 + 9 c^5 d^2) + a^3 (8 c^6 d - 8 c^4 d^3 + 7 c^2 d^5 - 2 d^7) \right) \right. \\
& \left. \text{ArcTanh} \left[\frac{\sqrt{c-d} \tan \left[\frac{1}{2} (e+f x) \right]}{\sqrt{c+d}} \right] \right) / \left(c^4 \sqrt{c-d} \sqrt{c+d} (c^2 - d^2)^3 f \right) - \\
& \frac{d (b c - a d) (b + a \cos [e + f x])^2 \sin [e + f x]}{3 c (c^2 - d^2) f (d + c \cos [e + f x])^3} + \\
& \frac{(b c - a d)^2 (3 b c^3 - 8 a c^2 d + 2 b c d^2 + 3 a d^3) \sin [e + f x]}{6 c^3 (c^2 - d^2)^2 f (d + c \cos [e + f x])^2} - \\
& ((b c - a d) (b^2 c^2 d (13 c^2 + 2 d^2) - a b c (18 c^4 + 17 c^2 d^2 - 5 d^4) + a^2 (34 c^4 d - 28 c^2 d^3 + 9 d^5)) \\
& \sin [e + f x]) / (6 c^3 (c^2 - d^2)^3 f (d + c \cos [e + f x]))
\end{aligned}$$

Result (type 3, 885 leaves):

$$\begin{aligned}
& \frac{a^3 (e + f x) (d + c \cos [e + f x])^4 \sec [e + f x] (a + b \sec [e + f x])^3}{c^4 f (b + a \cos [e + f x])^3 (c + d \sec [e + f x])^4} + \\
& \left(6 a^2 b c^7 + b^3 c^7 - 8 a^3 c^6 d - 12 a b^2 c^6 d + \right. \\
& 9 a^2 b c^5 d^2 + 4 b^3 c^5 d^2 + 8 a^3 c^4 d^3 - 3 a b^2 c^4 d^3 - 7 a^3 c^2 d^5 + 2 a^3 d^7) \\
& \left. \text{ArcTanh} \left[\frac{(-c+d) \tan \left[\frac{1}{2} (e+f x) \right]}{\sqrt{c^2 - d^2}} \right] (d + c \cos [e + f x])^4 \sec [e + f x] (a + b \sec [e + f x])^3 \right) / \\
& \left(c^4 \sqrt{c^2 - d^2} (-c^2 + d^2)^3 f (b + a \cos [e + f x])^3 (c + d \sec [e + f x])^4 \right) + \\
& \left((d + c \cos [e + f x]) \sec [e + f x] (a + b \sec [e + f x])^3 \right. \\
& (-b^3 c^3 d \sin [e + f x] + 3 a b^2 c^2 d^2 \sin [e + f x] - 3 a^2 b c d^3 \sin [e + f x] + a^3 d^4 \sin [e + f x]) \Big) / \\
& \left(3 c^3 (c^2 - d^2) f (b + a \cos [e + f x])^3 (c + d \sec [e + f x])^4 \right) + \\
& \left((d + c \cos [e + f x])^2 \sec [e + f x] (a + b \sec [e + f x])^3 (3 b^3 c^5 \sin [e + f x] - \right. \\
& 18 a b^2 c^4 d \sin [e + f x] + 27 a^2 b c^3 d^2 \sin [e + f x] + 2 b^3 c^3 d^2 \sin [e + f x] - 12 a^3 c^2 d^3 \\
& \sin [e + f x] + 3 a b^2 c^2 d^3 \sin [e + f x] - 12 a^2 b c d^4 \sin [e + f x] + 7 a^3 d^5 \sin [e + f x]) \Big) / \\
& \left(6 c^3 (c^2 - d^2)^2 f (b + a \cos [e + f x])^3 (c + d \sec [e + f x])^4 \right) + \\
& \frac{1}{6 c^3 (c^2 - d^2)^3 f (b + a \cos [e + f x])^3 (c + d \sec [e + f x])^4} \\
& \left((d + c \cos [e + f x])^3 \sec [e + f x] (a + b \sec [e + f x])^3 \right. \\
& (18 a b^2 c^6 \sin [e + f x] - 54 a^2 b c^5 d \sin [e + f x] - 13 b^3 c^5 d \sin [e + f x] + 36 a^3 c^4 d^2 \sin [e + f x] + \\
& 30 a b^2 c^4 d^2 \sin [e + f x] + 15 a^2 b c^3 d^3 \sin [e + f x] - 2 b^3 c^3 d^3 \sin [e + f x] - \\
& 32 a^3 c^2 d^4 \sin [e + f x] - 3 a b^2 c^2 d^4 \sin [e + f x] - 6 a^2 b c d^5 \sin [e + f x] + 11 a^3 d^6 \sin [e + f x]) \Big)
\end{aligned}$$

Problem 197: Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b \sec(e + f x))^3}{(c + d \sec(e + f x))^5} dx$$

Optimal (type 3, 622 leaves, 8 steps) :

$$\begin{aligned} \frac{a^3 x}{c^5} - & \left(\left(15 a b^2 c^6 d (4 c^2 + 3 d^2) - 3 a^2 b c^5 (8 c^4 + 24 c^2 d^2 + 3 d^4) - b^3 c^5 (4 c^4 + 27 c^2 d^2 + 4 d^4) + \right. \right. \\ & \left. \left. a^3 (40 c^8 d - 40 c^6 d^3 + 63 c^4 d^5 - 36 c^2 d^7 + 8 d^9) \right) \operatorname{ArcTanh} \left[\frac{\sqrt{c-d} \tan \left[\frac{1}{2} (e+f x) \right]}{\sqrt{c+d}} \right] \right) / \\ & \left(4 c^5 \sqrt{c-d} \sqrt{c+d} (c^2 - d^2)^4 f + \frac{d^2 (b + a \cos(e+f x))^3 \sin(e+f x)}{4 c (c^2 - d^2)^4 f (d + c \cos(e+f x))^4} - \right. \\ & \left. \frac{d (8 b c^3 - 11 a c^2 d - b c d^2 + 4 a d^3) (b + a \cos(e+f x))^2 \sin(e+f x)}{12 c^2 (c^2 - d^2)^2 f (d + c \cos(e+f x))^3} - \right. \\ & \left. ((b c - a d) (2 a b c d (32 c^4 + c^2 d^2 + 2 d^4) - a^2 d^2 (58 c^4 - 35 c^2 d^2 + 12 d^4) - \right. \\ & \left. b^2 (12 c^6 + 25 c^4 d^2 - 2 c^2 d^4)) \sin(e+f x)) / (24 c^4 (c^2 - d^2)^3 f (d + c \cos(e+f x))^2) - \right. \\ & \left. ((b^3 c^3 d (68 c^4 + 39 c^2 d^2 - 2 d^4) + a^2 b c d (272 c^6 + 10 c^4 d^2 + 49 c^2 d^4 - 16 d^6) - \right. \\ & \left. 3 a b^2 c^2 (24 c^6 + 84 c^4 d^2 - 5 c^2 d^4 + 2 d^6) - a^3 (212 c^6 d^2 - 210 c^4 d^4 + 139 c^2 d^6 - 36 d^8)) \right. \\ & \left. \sin(e+f x) \right) / (24 c^4 (c^2 - d^2)^4 f (d + c \cos(e+f x))) \end{aligned}$$

Result (type 3, 1285 leaves) :

$$\begin{aligned}
& \frac{a^3 (e + f x) (d + c \cos[e + f x])^5 \sec[e + f x]^2 (a + b \sec[e + f x])^3}{c^5 f (b + a \cos[e + f x])^3 (c + d \sec[e + f x])^5} + \\
& \left(\begin{aligned}
& (-24 a^2 b c^9 - 4 b^3 c^9 + 40 a^3 c^8 d + 60 a b^2 c^8 d - 72 a^2 b c^7 d^2 - 27 b^3 c^7 d^2 - \\
& 40 a^3 c^6 d^3 + 45 a b^2 c^6 d^3 - 9 a^2 b c^5 d^4 - 4 b^3 c^5 d^4 + 63 a^3 c^4 d^5 - 36 a^3 c^2 d^7 + 8 a^3 d^9) \\
& \operatorname{ArcTanh}\left[\frac{(-c+d) \tan\left[\frac{1}{2}(e+f x)\right]}{\sqrt{c^2-d^2}}\right] (d+c \cos[e+f x])^5 \sec[e+f x]^2 (a+b \sec[e+f x])^3 \Bigg) \Bigg) / \\
& \left(4 c^5 \sqrt{c^2-d^2} (-c^2+d^2)^4 f (b+a \cos[e+f x])^3 (c+d \sec[e+f x])^5 \right) + \\
& \left((d+c \cos[e+f x]) \sec[e+f x]^2 (a+b \sec[e+f x])^3 \right. \\
& \left. (b^3 c^3 d^2 \sin[e+f x] - 3 a b^2 c^2 d^3 \sin[e+f x] + 3 a^2 b c d^4 \sin[e+f x] - a^3 d^5 \sin[e+f x]) \right) / \\
& \left(4 c^4 (c^2-d^2) f (b+a \cos[e+f x])^3 (c+d \sec[e+f x])^5 \right) + \\
& \left((d+c \cos[e+f x])^2 \sec[e+f x]^2 (a+b \sec[e+f x])^3 (-8 b^3 c^5 d \sin[e+f x] + 36 a b^2 c^4 d^2 \right. \\
& \sin[e+f x] - 48 a^2 b c^3 d^3 \sin[e+f x] + b^3 c^3 d^3 \sin[e+f x] + 20 a^3 c^2 d^4 \sin[e+f x] - \\
& 15 a b^2 c^2 d^4 \sin[e+f x] + 27 a^2 b c d^5 \sin[e+f x] - 13 a^3 d^6 \sin[e+f x]) \Bigg) / \\
& \left(12 c^4 (c^2-d^2)^2 f (b+a \cos[e+f x])^3 (c+d \sec[e+f x])^5 \right) + \\
& \frac{1}{24 c^4 (c^2-d^2)^3 f (b+a \cos[e+f x])^3 (c+d \sec[e+f x])^5} \\
& \left((d+c \cos[e+f x])^3 \sec[e+f x]^2 (a+b \sec[e+f x])^3 \right. \\
& \left(12 b^3 c^7 \sin[e+f x] - 108 a b^2 c^6 d \sin[e+f x] + 216 a^2 b c^5 d^2 \sin[e+f x] + \right. \\
& 25 b^3 c^5 d^2 \sin[e+f x] - 120 a^3 c^4 d^3 \sin[e+f x] + 9 a b^2 c^4 d^3 \sin[e+f x] - \\
& 165 a^2 b c^3 d^4 \sin[e+f x] - 2 b^3 c^3 d^4 \sin[e+f x] + 131 a^3 c^2 d^5 \sin[e+f x] - \\
& 6 a b^2 c^2 d^5 \sin[e+f x] + 54 a^2 b c d^6 \sin[e+f x] - 46 a^3 d^7 \sin[e+f x]) + \\
& \frac{1}{24 c^4 (c^2-d^2)^4 f (b+a \cos[e+f x])^3 (c+d \sec[e+f x])^5} \\
& \left((d+c \cos[e+f x])^4 \sec[e+f x]^2 (a+b \sec[e+f x])^3 \right. \\
& \left(72 a b^2 c^8 \sin[e+f x] - 288 a^2 b c^7 d \sin[e+f x] - 68 b^3 c^7 d \sin[e+f x] + \right. \\
& 240 a^3 c^6 d^2 \sin[e+f x] + 252 a b^2 c^6 d^2 \sin[e+f x] + 24 a^2 b c^5 d^3 \sin[e+f x] - \\
& 39 b^3 c^5 d^3 \sin[e+f x] - 280 a^3 c^4 d^4 \sin[e+f x] - 15 a b^2 c^4 d^4 \sin[e+f x] - \\
& 69 a^2 b c^3 d^5 \sin[e+f x] + 2 b^3 c^3 d^5 \sin[e+f x] + 195 a^3 c^2 d^6 \sin[e+f x] + \\
& 6 a b^2 c^2 d^6 \sin[e+f x] + 18 a^2 b c d^7 \sin[e+f x] - 50 a^3 d^8 \sin[e+f x])
\end{aligned}$$

Problem 198: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \sqrt{a+b \sec[e+f x]} (c+d \sec[e+f x]) dx$$

Optimal (type 4, 320 leaves, 5 steps):

$$\begin{aligned}
& -\frac{1}{b f} 2 (a-b) \sqrt{a+b} d \operatorname{Cot}[e+f x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b} \operatorname{Sec}[e+f x]}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \\
& \sqrt{\frac{b(1-\operatorname{Sec}[e+f x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[e+f x])}{a-b}} + \frac{1}{b f} \\
& 2 \sqrt{a+b} (b(c-d)+ad) \operatorname{Cot}[e+f x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b} \operatorname{Sec}[e+f x]}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \\
& \sqrt{\frac{b(1-\operatorname{Sec}[e+f x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[e+f x])}{a-b}} - \frac{1}{f} \\
& 2 \sqrt{a+b} c \operatorname{Cot}[e+f x] \operatorname{EllipticPi}\left[\frac{a+b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b} \operatorname{Sec}[e+f x]}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \\
& \sqrt{\frac{b(1-\operatorname{Sec}[e+f x])}{a+b}} \sqrt{-\frac{b(1+\operatorname{Sec}[e+f x])}{a-b}}
\end{aligned}$$

Result (type 4, 913 leaves):

$$\begin{aligned}
& \frac{2 d \cos[e+f x] \sqrt{a+b \operatorname{Sec}[e+f x]} (c+d \operatorname{Sec}[e+f x]) \sin[e+f x]}{f(d+c \cos[e+f x])} + \\
& \left(2 \sqrt{a+b \operatorname{Sec}[e+f x]} (c+d \operatorname{Sec}[e+f x]) \right. \\
& \left(a \sqrt{\frac{-a+b}{a+b}} d \tan\left[\frac{1}{2}(e+f x)\right] + b \sqrt{\frac{-a+b}{a+b}} d \tan\left[\frac{1}{2}(e+f x)\right] - 2 a \sqrt{\frac{-a+b}{a+b}} d \right. \\
& \left. \tan\left[\frac{1}{2}(e+f x)\right]^3 + a \sqrt{\frac{-a+b}{a+b}} d \tan\left[\frac{1}{2}(e+f x)\right]^5 - b \sqrt{\frac{-a+b}{a+b}} d \tan\left[\frac{1}{2}(e+f x)\right]^5 + \right. \\
& \left. 2 i a c \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \tan\left[\frac{1}{2}(e+f x)\right]\right], \frac{a+b}{a-b}\right] \right. \\
& \left. \sqrt{1 - \tan\left[\frac{1}{2}(e+f x)\right]^2} \sqrt{\frac{a+b - a \tan\left[\frac{1}{2}(e+f x)\right]^2 + b \tan\left[\frac{1}{2}(e+f x)\right]^2}{a+b}} + \right. \\
& \left. 2 i a c \operatorname{EllipticPi}\left[-\frac{a+b}{a-b}, i \operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \tan\left[\frac{1}{2}(e+f x)\right]\right], \frac{a+b}{a-b}\right] \tan\left[\frac{1}{2}(e+f x)\right]^2 \right. \\
& \left. \sqrt{1 - \tan\left[\frac{1}{2}(e+f x)\right]^2} \sqrt{\frac{a+b - a \tan\left[\frac{1}{2}(e+f x)\right]^2 + b \tan\left[\frac{1}{2}(e+f x)\right]^2}{a+b}} - \right)
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{2} (a-b) d \operatorname{EllipticE}\left[\operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \tan\left(\frac{1}{2}(e+f x)\right)\right], \frac{a+b}{a-b}\right] \sqrt{1-\tan^2\left(\frac{1}{2}(e+f x)\right)} \\
& \left(1+\tan^2\left(\frac{1}{2}(e+f x)\right)\right) \sqrt{\frac{a+b-a \tan^2\left(\frac{1}{2}(e+f x)\right)+b \tan^2\left(\frac{1}{2}(e+f x)\right)}{a+b}} - \frac{1}{2} (a-b) \\
& (c-d) \operatorname{EllipticF}\left[\operatorname{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \tan\left(\frac{1}{2}(e+f x)\right)\right], \frac{a+b}{a-b}\right] \sqrt{1-\tan^2\left(\frac{1}{2}(e+f x)\right)} \\
& \left(1+\tan^2\left(\frac{1}{2}(e+f x)\right)\right) \sqrt{\frac{a+b-a \tan^2\left(\frac{1}{2}(e+f x)\right)+b \tan^2\left(\frac{1}{2}(e+f x)\right)}{a+b}}\right) / \\
& \left(\sqrt{\frac{-a+b}{a+b}} f \sqrt{b+a \cos(e+f x)} (d+c \cos(e+f x)) \sec^3(e+f x)\right. \\
& \left.\sqrt{\frac{1}{1-\tan^2\left(\frac{1}{2}(e+f x)\right)}} \left(-1+\tan^2\left(\frac{1}{2}(e+f x)\right)\right) \left(1+\tan^2\left(\frac{1}{2}(e+f x)\right)\right)^{3/2}\right. \\
& \left.\sqrt{\frac{a+b-a \tan^2\left(\frac{1}{2}(e+f x)\right)+b \tan^2\left(\frac{1}{2}(e+f x)\right)}{1+\tan^2\left(\frac{1}{2}(e+f x)\right)}}\right)
\end{aligned}$$

Problem 199: Unable to integrate problem.

$$\int \frac{\sqrt{a+b \sec(e+f x)}}{c+d \sec(e+f x)} dx$$

Optimal (type 4, 220 leaves, 3 steps):

$$\begin{aligned}
& -\frac{1}{c f} 2 \sqrt{a+b} \cot(e+f x) \operatorname{EllipticPi}\left[\frac{a+b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \sec(e+f x)}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \\
& \sqrt{\frac{b(1-\sec(e+f x))}{a+b}} - \sqrt{\frac{b(1+\sec(e+f x))}{a-b}} + \\
& \left(2(b c-a d) \operatorname{EllipticPi}\left[\frac{2 d}{c+d}, \operatorname{ArcSin}\left[\frac{\sqrt{1-\sec(e+f x)}}{\sqrt{2}}\right], \frac{2 b}{a+b}\right] \sqrt{\frac{a+b \sec(e+f x)}{a+b}}\right. \\
& \left.\tan(e+f x)\right) / \left(c(c+d)f \sqrt{a+b \sec(e+f x)} \sqrt{-\tan^2(e+f x)}\right)
\end{aligned}$$

Result (type 8, 29 leaves):

$$\int \frac{\sqrt{a+b \sec[e+f x]}}{c+d \sec[e+f x]} dx$$

Problem 201: Unable to integrate problem.

$$\int \frac{(a+b \sec[e+f x])^{3/2}}{c+d \sec[e+f x]} dx$$

Optimal (type 4, 326 leaves, 5 steps):

$$\begin{aligned} & \frac{1}{d f} 2 b \sqrt{a+b} \cot[e+f x] \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b \sec[e+f x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \\ & \sqrt{\frac{b(1-\sec[e+f x])}{a+b}} - \sqrt{-\frac{b(1+\sec[e+f x])}{a-b}} - \frac{1}{c f} \\ & 2 a \sqrt{a+b} \cot[e+f x] \operatorname{EllipticPi}\left[\frac{a+b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b \sec[e+f x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \\ & \sqrt{\frac{b(1-\sec[e+f x])}{a+b}} - \sqrt{-\frac{b(1+\sec[e+f x])}{a-b}} - \\ & \left. \left(2(b c - a d)^2 \operatorname{EllipticPi}\left[\frac{2 d}{c+d}, \operatorname{ArcSin}\left[\frac{\sqrt{1-\sec[e+f x]}}{\sqrt{2}}\right], \frac{2 b}{a+b}\right] \sqrt{\frac{a+b \sec[e+f x]}{a+b}} \right. \right. \\ & \left. \left. \operatorname{Tan}[e+f x] \right) \middle/ \left(c d (c+d) f \sqrt{a+b \sec[e+f x]} \sqrt{-\operatorname{Tan}[e+f x]^2} \right) \right) \end{aligned}$$

Result (type 8, 29 leaves):

$$\int \frac{(a+b \sec[e+f x])^{3/2}}{c+d \sec[e+f x]} dx$$

Problem 204: Unable to integrate problem.

$$\int \frac{1}{\sqrt{a+b \sec[e+f x]} (c+d \sec[e+f x])} dx$$

Optimal (type 4, 216 leaves, 3 steps):

$$\begin{aligned}
& -\frac{1}{a c f} 2 \sqrt{a+b} \cot[e+f x] \operatorname{EllipticPi}\left[\frac{a+b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b} \sec [e+f x]}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \\
& \sqrt{\frac{b(1-\sec[e+f x])}{a+b}}-\sqrt{-\frac{b(1+\sec[e+f x])}{a-b}}- \\
& \left(2 d \operatorname{EllipticPi}\left[\frac{2 d}{c+d}, \operatorname{ArcSin}\left[\frac{\sqrt{1-\sec[e+f x]}}{\sqrt{2}}\right], \frac{2 b}{a+b}\right] \sqrt{\frac{a+b \sec [e+f x]}{a+b}} \tan [e+f x]\right) / \\
& \left(c(c+d) f \sqrt{a+b \sec [e+f x]} \sqrt{-\tan [e+f x]^2}\right)
\end{aligned}$$

Result (type 8, 29 leaves):

$$\int \frac{1}{\sqrt{a+b \sec [e+f x]} (c+d \sec [e+f x])} dx$$

Problem 205: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{c+d \sec [e+f x]}{(a+b \sec [e+f x])^{3/2}} dx$$

Optimal (type 4, 376 leaves, 6 steps):

$$\begin{aligned}
& \frac{1}{a b \sqrt{a+b} f} 2(b c-a d) \cot[e+f x] \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b} \sec [e+f x]}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \\
& \sqrt{\frac{b(1-\sec[e+f x])}{a+b}}-\sqrt{-\frac{b(1+\sec[e+f x])}{a-b}}-\frac{1}{a b \sqrt{a+b} f} 2(b c-a d) \cot[e+f x] \\
& \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a+b} \sec [e+f x]}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b(1-\sec[e+f x])}{a+b}}-\sqrt{-\frac{b(1+\sec[e+f x])}{a-b}}- \\
& \frac{1}{a^2 f} 2 \sqrt{a+b} c \cot[e+f x] \operatorname{EllipticPi}\left[\frac{a+b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b} \sec [e+f x]}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \\
& \sqrt{\frac{b(1-\sec[e+f x])}{a+b}}-\sqrt{-\frac{b(1+\sec[e+f x])}{a-b}}+\frac{2 b(b c-a d) \tan [e+f x]}{a(a^2-b^2) f \sqrt{a+b \sec [e+f x]}}
\end{aligned}$$

Result (type 4, 1491 leaves):

$$\begin{aligned}
& \left((b+a \cos [e+f x])^2 \sec [e+f x] (c+d \sec [e+f x])\right. \\
& \left.\left(\frac{2(-b c+a d) \sin [e+f x]}{a\left(a^2-b^2\right)}-\frac{2\left(-b^2 c \sin [e+f x]+a b d \sin [e+f x]\right)}{a\left(a^2-b^2\right)(b+a \cos [e+f x])}\right)\right) / \\
& \left(f(d+c \cos [e+f x])(a+b \sec [e+f x])^{3/2}\right) +
\end{aligned}$$

$$\begin{aligned}
& \left(2 \left(b + a \cos[e + f x] \right)^{3/2} \sqrt{\sec[e + f x]} \left(c + d \sec[e + f x] \right) \right. \\
& \quad \left. \sqrt{\frac{a + b - a \tan[\frac{1}{2}(e + f x)]^2 + b \tan[\frac{1}{2}(e + f x)]^2}{1 + \tan[\frac{1}{2}(e + f x)]^2}} \right) \left(a b \sqrt{\frac{-a + b}{a + b}} c \tan[\frac{1}{2}(e + f x)] + \right. \\
& \quad \left. b^2 \sqrt{\frac{-a + b}{a + b}} c \tan[\frac{1}{2}(e + f x)] - a^2 \sqrt{\frac{-a + b}{a + b}} d \tan[\frac{1}{2}(e + f x)] - \right. \\
& \quad \left. a b \sqrt{\frac{-a + b}{a + b}} d \tan[\frac{1}{2}(e + f x)] - 2 a b \sqrt{\frac{-a + b}{a + b}} c \tan[\frac{1}{2}(e + f x)]^3 + 2 a^2 \sqrt{\frac{-a + b}{a + b}} d \right. \\
& \quad \left. \tan[\frac{1}{2}(e + f x)]^3 + a b \sqrt{\frac{-a + b}{a + b}} c \tan[\frac{1}{2}(e + f x)]^5 - b^2 \sqrt{\frac{-a + b}{a + b}} c \tan[\frac{1}{2}(e + f x)]^5 - \right. \\
& \quad \left. a^2 \sqrt{\frac{-a + b}{a + b}} d \tan[\frac{1}{2}(e + f x)]^5 + a b \sqrt{\frac{-a + b}{a + b}} d \tan[\frac{1}{2}(e + f x)]^5 - \right. \\
& \quad \left. 2 \pm a^2 c \text{EllipticPi}\left[-\frac{a + b}{a - b}, \pm \text{ArcSinh}\left[\sqrt{\frac{-a + b}{a + b}} \tan[\frac{1}{2}(e + f x)]\right], \frac{a + b}{a - b}\right] \right. \\
& \quad \left. \sqrt{1 - \tan[\frac{1}{2}(e + f x)]^2} \sqrt{\frac{a + b - a \tan[\frac{1}{2}(e + f x)]^2 + b \tan[\frac{1}{2}(e + f x)]^2}{a + b}} + \right. \\
& \quad \left. 2 \pm b^2 c \text{EllipticPi}\left[-\frac{a + b}{a - b}, \pm \text{ArcSinh}\left[\sqrt{\frac{-a + b}{a + b}} \tan[\frac{1}{2}(e + f x)]\right], \frac{a + b}{a - b}\right] \right. \\
& \quad \left. \sqrt{1 - \tan[\frac{1}{2}(e + f x)]^2} \sqrt{\frac{a + b - a \tan[\frac{1}{2}(e + f x)]^2 + b \tan[\frac{1}{2}(e + f x)]^2}{a + b}} - \right. \\
& \quad \left. 2 \pm a^2 c \text{EllipticPi}\left[-\frac{a + b}{a - b}, \pm \text{ArcSinh}\left[\sqrt{\frac{-a + b}{a + b}} \tan[\frac{1}{2}(e + f x)]\right], \frac{a + b}{a - b}\right] \tan[\frac{1}{2}(e + f x)]^2 \right. \\
& \quad \left. \sqrt{1 - \tan[\frac{1}{2}(e + f x)]^2} \sqrt{\frac{a + b - a \tan[\frac{1}{2}(e + f x)]^2 + b \tan[\frac{1}{2}(e + f x)]^2}{a + b}} + \right. \\
& \quad \left. 2 \pm b^2 c \text{EllipticPi}\left[-\frac{a + b}{a - b}, \pm \text{ArcSinh}\left[\sqrt{\frac{-a + b}{a + b}} \tan[\frac{1}{2}(e + f x)]\right], \frac{a + b}{a - b}\right] \tan[\frac{1}{2}(e + f x)]^2 \right)
\end{aligned}$$

$$\begin{aligned}
& \sqrt{1 - \tan^2\left(\frac{1}{2}(e + f x)\right)} \sqrt{\frac{a + b - a \tan^2\left(\frac{1}{2}(e + f x)\right) + b \tan^2\left(\frac{1}{2}(e + f x)\right)}{a + b}} + \\
& \pm (a - b) (-b c + a d) \operatorname{EllipticE}\left[\pm \operatorname{ArcSinh}\left[\sqrt{\frac{-a + b}{a + b}} \tan\left(\frac{1}{2}(e + f x)\right)\right], \frac{a + b}{a - b}\right] \\
& \sqrt{1 - \tan^2\left(\frac{1}{2}(e + f x)\right)^2} \left(1 + \tan^2\left(\frac{1}{2}(e + f x)\right)^2\right) \\
& \sqrt{\frac{a + b - a \tan^2\left(\frac{1}{2}(e + f x)\right) + b \tan^2\left(\frac{1}{2}(e + f x)\right)}{a + b}} + \pm (a - b) (2 b c + a (c - d)) \\
& \operatorname{EllipticF}\left[\pm \operatorname{ArcSinh}\left[\sqrt{\frac{-a + b}{a + b}} \tan\left(\frac{1}{2}(e + f x)\right)\right], \frac{a + b}{a - b}\right] \sqrt{1 - \tan^2\left(\frac{1}{2}(e + f x)\right)^2} \\
& \left(1 + \tan^2\left(\frac{1}{2}(e + f x)\right)^2\right) \sqrt{\frac{a + b - a \tan^2\left(\frac{1}{2}(e + f x)\right) + b \tan^2\left(\frac{1}{2}(e + f x)\right)}{a + b}} \Bigg) / \\
& \left(a \sqrt{\frac{-a + b}{a + b}} (a^2 - b^2) f (d + c \cos(e + f x)) (a + b \sec(e + f x))^{3/2} \right. \\
& \left. \left(-1 + \tan^2\left(\frac{1}{2}(e + f x)\right)^2\right) \sqrt{\frac{1 + \tan^2\left(\frac{1}{2}(e + f x)\right)^2}{1 - \tan^2\left(\frac{1}{2}(e + f x)\right)^2}} \right. \\
& \left. \left(a \left(-1 + \tan^2\left(\frac{1}{2}(e + f x)\right)^2\right) - b \left(1 + \tan^2\left(\frac{1}{2}(e + f x)\right)^2\right)\right) \right)
\end{aligned}$$

Problem 206: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{c + d \sec(e + f x)}{(a + b \sec(e + f x))^{5/2}} dx$$

Optimal (type 4, 495 leaves, 7 steps):

$$\begin{aligned}
& \left(2 (7 a^2 b c - 3 b^3 c - 4 a^3 d) \operatorname{Cot}[e + f x] \operatorname{EllipticE}[\operatorname{ArcSin}\left[\frac{\sqrt{a+b} \operatorname{Sec}[e+f x]}{\sqrt{a+b}}\right], \frac{a+b}{a-b}] \right. \\
& \quad \left. \sqrt{\frac{b (1 - \operatorname{Sec}[e+f x])}{a+b}} \sqrt{-\frac{b (1 + \operatorname{Sec}[e+f x])}{a-b}} \right) / \left(3 a^2 (a-b) b (a+b)^{3/2} f \right) - \\
& \left(2 (6 a^2 b c - a b^2 c - 3 b^3 c - 3 a^3 d + a^2 b d) \operatorname{Cot}[e + f x] \operatorname{EllipticF}[\operatorname{ArcSin}\left[\frac{\sqrt{a+b} \operatorname{Sec}[e+f x]}{\sqrt{a+b}}\right], \right. \\
& \quad \left. \frac{a+b}{a-b}] \sqrt{\frac{b (1 - \operatorname{Sec}[e+f x])}{a+b}} \sqrt{-\frac{b (1 + \operatorname{Sec}[e+f x])}{a-b}} \right) / \left(3 a^2 (a-b) b (a+b)^{3/2} f \right) - \\
& \frac{1}{a^3 f} 2 \sqrt{a+b} c \operatorname{Cot}[e + f x] \operatorname{EllipticPi}\left[\frac{a+b}{a}, \operatorname{ArcSin}\left[\frac{\sqrt{a+b} \operatorname{Sec}[e+f x]}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \\
& \sqrt{\frac{b (1 - \operatorname{Sec}[e+f x])}{a+b}} \sqrt{-\frac{b (1 + \operatorname{Sec}[e+f x])}{a-b}} + \\
& \frac{2 b (b c - a d) \operatorname{Tan}[e + f x]}{3 a (a^2 - b^2) f (a + b \operatorname{Sec}[e + f x])^{3/2}} + \frac{2 b (7 a^2 b c - 3 b^3 c - 4 a^3 d) \operatorname{Tan}[e + f x]}{3 a^2 (a^2 - b^2)^2 f \sqrt{a+b} \operatorname{Sec}[e+f x]}
\end{aligned}$$

Result (type 4, 2083 leaves):

$$\begin{aligned}
& \left((b + a \operatorname{Cos}[e + f x])^3 \operatorname{Sec}[e + f x]^2 (c + d \operatorname{Sec}[e + f x]) \right. \\
& \quad \left(\frac{2 (-7 a^2 b c + 3 b^3 c + 4 a^3 d) \operatorname{Sin}[e + f x]}{3 a^2 (a^2 - b^2)^2} - \frac{2 (b^3 c \operatorname{Sin}[e + f x] - a b^2 d \operatorname{Sin}[e + f x])}{3 a^2 (a^2 - b^2) (b + a \operatorname{Cos}[e + f x])^2} - \right. \\
& \quad \left. (2 (-8 a^2 b^2 c \operatorname{Sin}[e + f x] + 4 b^4 c \operatorname{Sin}[e + f x] + 5 a^3 b d \operatorname{Sin}[e + f x] - a b^3 d \operatorname{Sin}[e + f x])) \right) / \\
& \quad \left(3 a^2 (a^2 - b^2)^2 (b + a \operatorname{Cos}[e + f x]) \right) \Bigg) / \left(f (d + c \operatorname{Cos}[e + f x]) (a + b \operatorname{Sec}[e + f x])^{5/2} \right) + \\
& \left(2 (b + a \operatorname{Cos}[e + f x])^{5/2} \operatorname{Sec}[e + f x]^{3/2} (c + d \operatorname{Sec}[e + f x]) \right. \\
& \quad \left(\sqrt{\frac{a + b - a \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]^2}{1 + \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]^2}} \right. \\
& \quad \left. \left(7 a^3 b \sqrt{\frac{-a+b}{a+b}} c \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right] + 7 a^2 b^2 \sqrt{\frac{-a+b}{a+b}} c \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right] - 3 a b^3 \sqrt{\frac{-a+b}{a+b}} c \right. \right)
\end{aligned}$$

$$\begin{aligned}
& \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right] - 3 b^4 \sqrt{\frac{-a + b}{a + b}} c \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right] - 4 a^4 \sqrt{\frac{-a + b}{a + b}} d \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right] - \\
& 4 a^3 b \sqrt{\frac{-a + b}{a + b}} d \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right] - 14 a^3 b \sqrt{\frac{-a + b}{a + b}} c \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]^3 + \\
& 6 a b^3 \sqrt{\frac{-a + b}{a + b}} c \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]^3 + 8 a^4 \sqrt{\frac{-a + b}{a + b}} d \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]^3 + \\
& 7 a^3 b \sqrt{\frac{-a + b}{a + b}} c \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]^5 - 7 a^2 b^2 \sqrt{\frac{-a + b}{a + b}} c \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]^5 - \\
& 3 a b^3 \sqrt{\frac{-a + b}{a + b}} c \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]^5 + 3 b^4 \sqrt{\frac{-a + b}{a + b}} c \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]^5 - \\
& 4 a^4 \sqrt{\frac{-a + b}{a + b}} d \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]^5 + 4 a^3 b \sqrt{\frac{-a + b}{a + b}} d \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]^5 - \\
& 6 \pm a^4 c \operatorname{EllipticPi}\left[-\frac{a + b}{a - b}, \pm \operatorname{ArcSinh}\left[\sqrt{\frac{-a + b}{a + b}} \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]\right], \frac{a + b}{a - b}\right] \\
& \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]^2} \sqrt{\frac{a + b - a \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]^2}{a + b}} + \\
& 12 \pm a^2 b^2 c \operatorname{EllipticPi}\left[-\frac{a + b}{a - b}, \pm \operatorname{ArcSinh}\left[\sqrt{\frac{-a + b}{a + b}} \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]\right], \frac{a + b}{a - b}\right] \\
& \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]^2} \sqrt{\frac{a + b - a \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]^2}{a + b}} - \\
& 6 \pm b^4 c \operatorname{EllipticPi}\left[-\frac{a + b}{a - b}, \pm \operatorname{ArcSinh}\left[\sqrt{\frac{-a + b}{a + b}} \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]\right], \frac{a + b}{a - b}\right] \\
& \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]^2} \sqrt{\frac{a + b - a \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]^2}{a + b}} - \\
& 6 \pm a^4 c \operatorname{EllipticPi}\left[-\frac{a + b}{a - b}, \pm \operatorname{ArcSinh}\left[\sqrt{\frac{-a + b}{a + b}} \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]\right], \frac{a + b}{a - b}\right] \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]^2 \\
& \sqrt{1 - \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]^2} \sqrt{\frac{a + b - a \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]^2 + b \operatorname{Tan}\left[\frac{1}{2} (e + f x)\right]^2}{a + b}} + 12 \pm a^2 b^2 c
\end{aligned}$$

$$\begin{aligned}
& \text{EllipticPi}\left[-\frac{a+b}{a-b}, \pm \text{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \tan\left[\frac{1}{2}(e+f x)\right]\right], \frac{a+b}{a-b}\right] \tan\left[\frac{1}{2}(e+f x)\right]^2 \\
& \sqrt{1-\tan\left[\frac{1}{2}(e+f x)\right]^2} \sqrt{\frac{a+b-a \tan\left[\frac{1}{2}(e+f x)\right]^2+b \tan\left[\frac{1}{2}(e+f x)\right]^2}{a+b}} - \\
& 6 \pm b^4 c \text{EllipticPi}\left[-\frac{a+b}{a-b}, \pm \text{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \tan\left[\frac{1}{2}(e+f x)\right]\right], \frac{a+b}{a-b}\right] \tan\left[\frac{1}{2}(e+f x)\right]^2 \\
& \sqrt{1-\tan\left[\frac{1}{2}(e+f x)\right]^2} \sqrt{\frac{a+b-a \tan\left[\frac{1}{2}(e+f x)\right]^2+b \tan\left[\frac{1}{2}(e+f x)\right]^2}{a+b}} + \\
& \pm (a-b) (-7 a^2 b c + 3 b^3 c + 4 a^3 d) \text{EllipticE}\left[\pm \text{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \tan\left[\frac{1}{2}(e+f x)\right]\right], \frac{a+b}{a-b}\right] \\
& \sqrt{1-\tan\left[\frac{1}{2}(e+f x)\right]^2} \left(1+\tan\left[\frac{1}{2}(e+f x)\right]^2\right) \\
& \sqrt{\frac{a+b-a \tan\left[\frac{1}{2}(e+f x)\right]^2+b \tan\left[\frac{1}{2}(e+f x)\right]^2}{a+b}} + \\
& \pm (a-b) (-4 a b^2 c - 6 b^3 c + 3 a^3 (c-d) + a^2 b (9 c + d)) \\
& \text{EllipticF}\left[\pm \text{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \tan\left[\frac{1}{2}(e+f x)\right]\right], \frac{a+b}{a-b}\right] \sqrt{1-\tan\left[\frac{1}{2}(e+f x)\right]^2} \\
& \left(1+\tan\left[\frac{1}{2}(e+f x)\right]^2\right) \sqrt{\frac{a+b-a \tan\left[\frac{1}{2}(e+f x)\right]^2+b \tan\left[\frac{1}{2}(e+f x)\right]^2}{a+b}} \Bigg) \Bigg) / \\
& \left(3 a^2 \sqrt{\frac{-a+b}{a+b}} (a^2-b^2)^2 f (d+c \cos[e+f x]) (a+b \sec[e+f x])^{5/2}\right. \\
& \left. \left(-1+\tan\left[\frac{1}{2}(e+f x)\right]^2\right) \sqrt{\frac{1+\tan\left[\frac{1}{2}(e+f x)\right]^2}{1-\tan\left[\frac{1}{2}(e+f x)\right]^2}} \right. \\
& \left. \left(a \left(-1+\tan\left[\frac{1}{2}(e+f x)\right]^2\right) - b \left(1+\tan\left[\frac{1}{2}(e+f x)\right]^2\right)\right) \right)
\end{aligned}$$

Problem 207: Unable to integrate problem.

$$\int \sqrt{a+b \sec[e+f x]} \sqrt{c+d \sec[e+f x]} dx$$

Optimal (type 4, 389 leaves, 3 steps):

$$\begin{aligned}
& -\frac{1}{\sqrt{a+b} f} 2 \sqrt{c+d} \cot[e+f x] \\
& \text{EllipticPi}\left[\frac{a(c+d)}{(a+b)c}, \text{ArcSin}\left[\frac{\sqrt{a+b} \sqrt{c+d} \sec[e+f x]}{\sqrt{c+d} \sqrt{a+b} \sec[e+f x]}\right], \frac{(a-b)(c+d)}{(a+b)(c-d)}\right] \\
& \sqrt{-\frac{(b c-a d) (1-\sec[e+f x])}{(c+d) (a+b \sec[e+f x])}} \sqrt{\frac{(b c-a d) (1+\sec[e+f x])}{(c-d) (a+b \sec[e+f x])}} (a+b \sec[e+f x]) + \frac{1}{\sqrt{\frac{a+b}{c+d}} f} \\
& 2 \cot[e+f x] \text{EllipticPi}\left[\frac{b(c+d)}{(a+b)d}, \text{ArcSin}\left[\frac{\sqrt{\frac{a+b}{c+d}} \sqrt{c+d} \sec[e+f x]}{\sqrt{a+b} \sec[e+f x]}\right], \frac{(a-b)(c+d)}{(a+b)(c-d)}\right] \\
& \sqrt{-\frac{(b c-a d) (1-\sec[e+f x])}{(c+d) (a+b \sec[e+f x])}} \sqrt{\frac{(b c-a d) (1+\sec[e+f x])}{(c-d) (a+b \sec[e+f x])}} (a+b \sec[e+f x])
\end{aligned}$$

Result (type 8, 31 leaves):

$$\int \sqrt{a+b \sec[e+f x]} \sqrt{c+d \sec[e+f x]} dx$$

Problem 208: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{a+b \sec[e+f x]}}{\sqrt{c+d \sec[e+f x]}} dx$$

Optimal (type 4, 198 leaves, 1 step):

$$\begin{aligned}
& -\frac{1}{\sqrt{a+b} c f} \\
& 2 \sqrt{c+d} \cot[e+f x] \text{EllipticPi}\left[\frac{a(c+d)}{(a+b)c}, \text{ArcSin}\left[\frac{\sqrt{a+b} \sqrt{c+d} \sec[e+f x]}{\sqrt{c+d} \sqrt{a+b} \sec[e+f x]}\right], \frac{(a-b)(c+d)}{(a+b)(c-d)}\right] \\
& \sqrt{-\frac{(b c-a d) (1-\sec[e+f x])}{(c+d) (a+b \sec[e+f x])}} \sqrt{\frac{(b c-a d) (1+\sec[e+f x])}{(c-d) (a+b \sec[e+f x])}} (a+b \sec[e+f x])
\end{aligned}$$

Result (type 4, 554 leaves):

$$\begin{aligned}
& \frac{1}{f \sqrt{b+a \cos[e+f x]} \sqrt{c+d \sec[e+f x]}} 4 (b c - a d) \sqrt{d+c \cos[e+f x]} \sqrt{a+b \sec[e+f x]} \\
& \left(\left(\sqrt{\frac{(c+d) \cot[\frac{1}{2}(e+f x)]^2}{c-d}} \sqrt{\frac{(c+d) (b+a \cos[e+f x]) \csc[\frac{1}{2}(e+f x)]^2}{b c - a d}} \right. \right. \\
& \left. \left. \sqrt{-\frac{(a+b) (d+c \cos[e+f x]) \csc[\frac{1}{2}(e+f x)]^2}{b c - a d}} \csc[e+f x] \right. \right. \\
& \text{EllipticF} \left[\text{ArcSin} \left[\frac{\sqrt{-\frac{(a+b) (d+c \cos[e+f x]) \csc[\frac{1}{2}(e+f x)]^2}{b c - a d}}}{\sqrt{2}} \right], \frac{2 (b c - a d)}{(a+b) (c-d)} \right] \\
& \left. \left. \sin[\frac{1}{2}(e+f x)]^4 \right) \right/ \left((c+d) \sqrt{b+a \cos[e+f x]} \sqrt{d+c \cos[e+f x]} \right) - \\
& \left(a \sqrt{\frac{(c+d) \cot[\frac{1}{2}(e+f x)]^2}{c-d}} \sqrt{\frac{(c+d) (b+a \cos[e+f x]) \csc[\frac{1}{2}(e+f x)]^2}{b c - a d}} \right. \\
& \left. \sqrt{-\frac{(a+b) (d+c \cos[e+f x]) \csc[\frac{1}{2}(e+f x)]^2}{b c - a d}} \csc[e+f x] \right. \\
& \text{EllipticPi} \left[\frac{b c - a d}{(a+b) c}, \text{ArcSin} \left[\frac{\sqrt{-\frac{(a+b) (d+c \cos[e+f x]) \csc[\frac{1}{2}(e+f x)]^2}{b c - a d}}}{\sqrt{2}} \right], \frac{2 (b c - a d)}{(a+b) (c-d)} \right] \\
& \left. \left. \sin[\frac{1}{2}(e+f x)]^4 \right) \right/ \left((a+b) c \sqrt{b+a \cos[e+f x]} \sqrt{d+c \cos[e+f x]} \right)
\end{aligned}$$

Problem 209: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{a+b \sec[e+f x]}}{(c+d \sec[e+f x])^{3/2}} dx$$

Optimal (type 4, 598 leaves, 5 steps):

$$\begin{aligned}
& -\frac{1}{\sqrt{a+b} c^2 f} 2 \sqrt{c+d} \cot[e+f x] \\
& \text{EllipticPi}\left[\frac{a(c+d)}{(a+b)c}, \text{ArcSin}\left[\frac{\sqrt{a+b} \sqrt{c+d} \sec[e+f x]}{\sqrt{c+d} \sqrt{a+b} \sec[e+f x]}\right], \frac{(a-b)(c+d)}{(a+b)(c-d)}\right] \\
& \sqrt{-\frac{(b c-a d) (1-\sec[e+f x])}{(c+d) (a+b \sec[e+f x])}} \sqrt{\frac{(b c-a d) (1+\sec[e+f x])}{(c-d) (a+b \sec[e+f x])}} (a+b \sec[e+f x])- \\
& \left(2 \sqrt{a+b} d \cot[e+f x] \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{c+d} \sqrt{a+b} \sec[e+f x]}{\sqrt{a+b} \sqrt{c+d} \sec[e+f x]}\right], \frac{(a+b)(c-d)}{(a-b)(c+d)}\right]\right. \\
& \left.(1+\sec[e+f x]) \sqrt{\frac{(b c-a d) (1-\sec[e+f x])}{(a+b) (c+d \sec[e+f x])}}\right)/ \\
& \left(c (c-d) \sqrt{c+d} f \sqrt{-\frac{(b c-a d) (1+\sec[e+f x])}{(a-b) (c+d \sec[e+f x])}}\right)- \\
& \left(2 (a-b) \sqrt{a+b} d \cot[e+f x] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{c+d} \sqrt{a+b} \sec[e+f x]}{\sqrt{a+b} \sqrt{c+d} \sec[e+f x]}\right], \frac{(a+b)(c-d)}{(a-b)(c+d)}\right]\right. \\
& \left.\sqrt{\frac{(b c-a d) (1-\sec[e+f x])}{(a+b) (c+d \sec[e+f x])}} \sqrt{-\frac{(b c-a d) (1+\sec[e+f x])}{(a-b) (c+d \sec[e+f x])}}\right. \\
& \left.(c+d \sec[e+f x])\right) / \left(c (c-d) \sqrt{c+d} (b c-a d) f\right)
\end{aligned}$$

Result (type 4, 1678 leaves):

$$\begin{aligned}
& \frac{1}{(c-d)(c+d)f\sqrt{b+a \cos[e+f x]}(c+d \sec[e+f x])^{3/2}} \\
& \frac{(d+c \cos[e+f x])^{3/2} \sec[e+f x] \sqrt{a+b \sec[e+f x]}}{} \\
& \left(4 b c (b c-a d) \sqrt{\frac{(c+d) \cot\left[\frac{1}{2}(e+f x)\right]^2}{c-d}} \sqrt{\frac{(c+d) (b+a \cos[e+f x]) \csc\left[\frac{1}{2}(e+f x)\right]^2}{b c-a d}}\right. \\
& \left.\sqrt{-\frac{(a+b) (d+c \cos[e+f x]) \csc\left[\frac{1}{2}(e+f x)\right]^2}{b c-a d}} \csc[e+f x] \text{EllipticF}[\right.
\end{aligned}$$

$$\begin{aligned}
& \left. \left(\left(a+b \right) \left(c+d \right) \sqrt{b+a \cos[e+f x]} \sqrt{d+c \cos[e+f x]} \right) + 4 \left(b c - a d \right) \left(a c + b d \right) \right. \\
& \left. \left(\sqrt{\frac{(c+d) \cot[\frac{1}{2}(e+f x)]^2}{c-d}} \sqrt{\frac{(c+d) (b+a \cos[e+f x]) \csc[\frac{1}{2}(e+f x)]^2}{b c - a d}} \right. \right. \\
& \left. \left. \sqrt{-\frac{(a+b) (d+c \cos[e+f x]) \csc[\frac{1}{2}(e+f x)]^2}{b c - a d}} \csc[e+f x] \text{EllipticF} \right. \right. \\
& \left. \left. \left(\sqrt{-\frac{(a+b) (d+c \cos[e+f x]) \csc[\frac{1}{2}(e+f x)]^2}{b c - a d}} \right) , \frac{2 (b c - a d)}{(a+b) (c-d)} \right] \sin[\frac{1}{2}(e+f x)]^4 \right) / \\
& \left. \left(\left(a+b \right) \left(c+d \right) \sqrt{b+a \cos[e+f x]} \sqrt{d+c \cos[e+f x]} \right) - \right. \\
& \left. \left(\sqrt{\frac{(c+d) \cot[\frac{1}{2}(e+f x)]^2}{c-d}} \sqrt{\frac{(c+d) (b+a \cos[e+f x]) \csc[\frac{1}{2}(e+f x)]^2}{b c - a d}} \right. \right. \\
& \left. \left. \sqrt{-\frac{(a+b) (d+c \cos[e+f x]) \csc[\frac{1}{2}(e+f x)]^2}{b c - a d}} \csc[e+f x] \right. \right. \\
& \left. \left. \text{EllipticPi} \left[\frac{b c - a d}{(a+b) c}, \arcsin \left[\sqrt{-\frac{(a+b) (d+c \cos[e+f x]) \csc[\frac{1}{2}(e+f x)]^2}{b c - a d}} \right], \frac{2 (b c - a d)}{(a+b) (c-d)} \right] \right. \right. \\
& \left. \left. \left. \sin[\frac{1}{2}(e+f x)]^4 \right) / \left(\left(a+b \right) c \sqrt{b+a \cos[e+f x]} \sqrt{d+c \cos[e+f x]} \right) \right) + \\
& 2 a d \left(\sqrt{\frac{-a+b}{a+b}} (a+b) \cos[\frac{1}{2}(e+f x)] \sqrt{d+c \cos[e+f x]} \right)
\end{aligned}$$

$$\begin{aligned}
& \left. \text{EllipticE} \left[\text{ArcSin} \left[\frac{\sqrt{\frac{-a+b}{a+b}} \sin \left[\frac{1}{2} (e+f x) \right]}{\sqrt{\frac{b+a \cos [e+f x]}{a+b}}} \right], \frac{2 (b c - a d)}{(-a+b) (c+d)} \right] \right\} \\
& \left(a c \sqrt{\frac{(a+b) \cos \left[\frac{1}{2} (e+f x) \right]^2}{b+a \cos [e+f x]}} \sqrt{\frac{b+a \cos [e+f x]}{a+b}} \right. \\
& \left. \sqrt{\frac{(a+b) (d+c \cos [e+f x])}{(c+d) (b+a \cos [e+f x])}} \right) - \frac{1}{a c} 2 (b c - a d) \left(\begin{array}{l} (b c + (a+b) d) \\ \left(b c + (a+b) d \right) \end{array} \right. \\
& \left. \sqrt{\frac{(c+d) \cot \left[\frac{1}{2} (e+f x) \right]^2}{c-d}} \sqrt{\frac{(c+d) (b+a \cos [e+f x]) \csc \left[\frac{1}{2} (e+f x) \right]^2}{b c - a d}} \right. \\
& \left. \sqrt{-\frac{(a+b) (d+c \cos [e+f x]) \csc \left[\frac{1}{2} (e+f x) \right]^2}{b c - a d}} \csc [e+f x] \text{EllipticF} [\text{ArcSin} \right. \\
& \left. \left. \left. \left. \left. \sqrt{-\frac{(a+b) (d+c \cos [e+f x]) \csc \left[\frac{1}{2} (e+f x) \right]^2}{b c - a d}} \right], \frac{2 (b c - a d)}{(a+b) (c-d)} \right] \sin \left[\frac{1}{2} (e+f x) \right]^4 \right\} \right. \\
& \left((a+b) (c+d) \sqrt{b+a \cos [e+f x]} \sqrt{d+c \cos [e+f x]} \right) - \left(\begin{array}{l} (b c + a d) \\ (b c + a d) \end{array} \right. \\
& \left. \sqrt{\frac{(c+d) \cot \left[\frac{1}{2} (e+f x) \right]^2}{c-d}} \sqrt{\frac{(c+d) (b+a \cos [e+f x]) \csc \left[\frac{1}{2} (e+f x) \right]^2}{b c - a d}} \right. \\
& \left. \sqrt{-\frac{(a+b) (d+c \cos [e+f x]) \csc \left[\frac{1}{2} (e+f x) \right]^2}{b c - a d}} \csc [e+f x] \right. \\
& \left. \text{EllipticPi} \left[\frac{b c - a d}{(a+b) c}, \text{ArcSin} \left[\frac{\sqrt{-\frac{(a+b) (d+c \cos [e+f x]) \csc \left[\frac{1}{2} (e+f x) \right]^2}{b c - a d}}}{\sqrt{2}} \right] \right], \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \frac{2 (b c - a d)}{(a + b) (c - d)} \right] \sin \left[\frac{1}{2} (e + f x) \right]^4 \Bigg) \Bigg/ \left((a + b) c \sqrt{b + a \cos [e + f x]} \right. \\
& \left. \left. \sqrt{d + c \cos [e + f x]} \right) \right) + \frac{\sqrt{d + c \cos [e + f x]} \sin [e + f x]}{c \sqrt{b + a \cos [e + f x]}} \Bigg) \Bigg) + \\
& \frac{2 d (d + c \cos [e + f x]) \sqrt{a + b \sec [e + f x]} \tan [e + f x]}{(-c^2 + d^2) f (c + d \sec [e + f x])^{3/2}}
\end{aligned}$$

Problem 210: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{a + b \sec [e + f x]}}{(c + d \sec [e + f x])^{5/2}} dx$$

Optimal (type 4, 899 leaves, 7 steps):

$$\begin{aligned}
& \left(2 (a-b) \sqrt{a+b} d (6 b c^3 - 7 a c^2 d - 2 b c d^2 + 3 a d^3) \sqrt{-\frac{(b c - a d) (1 - \cos[e + f x])}{(a+b) (d + c \cos[e + f x])}} \right. \\
& \quad \left. \sqrt{-\frac{(b c - a d) (1 + \cos[e + f x])}{(a-b) (d + c \cos[e + f x])}} (d + c \cos[e + f x])^{3/2} \csc[e + f x] \right. \\
& \quad \left. \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{c+d} \sqrt{b+a \cos[e+f x]}}{\sqrt{a+b} \sqrt{d+c \cos[e+f x]}}\right], \frac{(a+b) (c-d)}{(a-b) (c+d)}\right] \sqrt{a+b \sec[e+f x]} \right) / \\
& \quad \left(3 c^2 (c-d)^2 (c+d)^{3/2} (b c - a d)^2 f \sqrt{b+a \cos[e+f x]} \sqrt{c+d \sec[e+f x]} \right) + \\
& \quad \left(2 \sqrt{a+b} (b c^2 (3 c^2 + 3 c d - 2 d^2) - a d (9 c^3 - 2 c^2 d - 6 c d^2 + 3 d^3)) \right. \\
& \quad \left. \sqrt{-\frac{(b c - a d) (1 - \cos[e + f x])}{(a+b) (d + c \cos[e + f x])}} \sqrt{-\frac{(b c - a d) (1 + \cos[e + f x])}{(a-b) (d + c \cos[e + f x])}} \right. \\
& \quad \left. (d + c \cos[e + f x])^{3/2} \csc[e + f x] \right. \\
& \quad \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{c+d} \sqrt{b+a \cos[e+f x]}}{\sqrt{a+b} \sqrt{d+c \cos[e+f x]}}\right], \frac{(a+b) (c-d)}{(a-b) (c+d)}\right] \sqrt{a+b \sec[e+f x]} \right) / \\
& \quad \left(3 c^3 (c-d)^2 (c+d)^{3/2} (b c - a d) f \sqrt{b+a \cos[e+f x]} \sqrt{c+d \sec[e+f x]} \right) - \\
& \quad \left(2 \sqrt{a+b} \sqrt{-\frac{(b c - a d) (1 - \cos[e + f x])}{(a+b) (d + c \cos[e + f x])}} \sqrt{-\frac{(b c - a d) (1 + \cos[e + f x])}{(a-b) (d + c \cos[e + f x])}} \right. \\
& \quad \left. (d + c \cos[e + f x])^{3/2} \csc[e + f x] \text{EllipticPi}\left[\frac{(a+b) c}{a (c+d)}, \right. \right. \\
& \quad \left. \left. \text{ArcSin}\left[\frac{\sqrt{c+d} \sqrt{b+a \cos[e+f x]}}{\sqrt{a+b} \sqrt{d+c \cos[e+f x]}}\right], \frac{(a+b) (c-d)}{(a-b) (c+d)}\right] \sqrt{a+b \sec[e+f x]} \right) / \\
& \quad \left(c^3 \sqrt{c+d} f \sqrt{b+a \cos[e+f x]} \sqrt{c+d \sec[e+f x]} \right) + \\
& \quad \frac{2 d^2 \sqrt{a+b \sec[e+f x]} \sin[e+f x]}{3 c (c^2 - d^2) f (d + c \cos[e + f x]) \sqrt{c+d \sec[e+f x]}}
\end{aligned}$$

Result (type 4, 1960 leaves):

$$\begin{aligned}
& \left((d + c \cos[e + f x])^3 \sec[e + f x]^2 \sqrt{a + b \sec[e + f x]} \left(\frac{2 d^2 \sin[e + f x]}{3 c (c^2 - d^2) (d + c \cos[e + f x])^2} - \right. \right. \\
& \quad \left. \left. (2 (6 b c^3 d \sin[e + f x] - 7 a c^2 d^2 \sin[e + f x] - 2 b c d^3 \sin[e + f x] + 3 a d^4 \sin[e + f x])) \right) \right) / \\
& \quad \left(3 c (b c - a d) (c^2 - d^2)^2 (d + c \cos[e + f x]) \right) \Bigg) \Bigg)
\end{aligned}$$

$$\begin{aligned}
& \left(f \left(c + d \sec [e + f x] \right)^{5/2} \right) + \left(\left(d + c \cos [e + f x] \right)^{5/2} \sec [e + f x]^2 \sqrt{a + b \sec [e + f x]} \right) \\
& \left(\left(4 (b c - a d) (3 b^2 c^4 - 3 a b c^3 d - a^2 c^2 d^2 + b^2 c^2 d^2 - a b c d^3 + a^2 d^4) \right. \right. \\
& \quad \sqrt{\frac{(c+d) \cot [\frac{1}{2} (e+f x)]^2}{c-d}} \sqrt{\frac{(c+d) (b+a \cos [e+f x]) \csc [\frac{1}{2} (e+f x)]^2}{b c - a d}} \\
& \quad \left. \sqrt{-\frac{(a+b) (d+c \cos [e+f x]) \csc [\frac{1}{2} (e+f x)]^2}{b c - a d}} \csc [e+f x] \text{EllipticF} \left[\right. \right. \\
& \quad \left. \left. \text{ArcSin} \left[\frac{\sqrt{-\frac{(a+b) (d+c \cos [e+f x]) \csc [\frac{1}{2} (e+f x)]^2}{b c - a d}}}{\sqrt{2}} \right], \frac{2 (b c - a d)}{(a+b) (c-d)} \right] \sin [\frac{1}{2} (e+f x)]^4 \right] \right) / \\
& \left((a+b) (c+d) \sqrt{b+a \cos [e+f x]} \sqrt{d+c \cos [e+f x]} \right) + \\
& 4 (b c - a d) (3 a b c^4 - 3 a^2 c^3 d + 6 b^2 c^3 d - 7 a b c^2 d^2 - a^2 c d^3 - 2 b^2 c d^3 + 4 a b d^4) \\
& \left(\left(\sqrt{\frac{(c+d) \cot [\frac{1}{2} (e+f x)]^2}{c-d}} \sqrt{\frac{(c+d) (b+a \cos [e+f x]) \csc [\frac{1}{2} (e+f x)]^2}{b c - a d}} \right. \right. \\
& \quad \left. \sqrt{-\frac{(a+b) (d+c \cos [e+f x]) \csc [\frac{1}{2} (e+f x)]^2}{b c - a d}} \csc [e+f x] \text{EllipticF} \left[\right. \right. \\
& \quad \left. \left. \text{ArcSin} \left[\frac{\sqrt{-\frac{(a+b) (d+c \cos [e+f x]) \csc [\frac{1}{2} (e+f x)]^2}{b c - a d}}}{\sqrt{2}} \right], \frac{2 (b c - a d)}{(a+b) (c-d)} \right] \sin [\frac{1}{2} (e+f x)]^4 \right] \right) / \\
& \left((a+b) (c+d) \sqrt{b+a \cos [e+f x]} \sqrt{d+c \cos [e+f x]} \right) - \\
& \left(\sqrt{\frac{(c+d) \cot [\frac{1}{2} (e+f x)]^2}{c-d}} \sqrt{\frac{(c+d) (b+a \cos [e+f x]) \csc [\frac{1}{2} (e+f x)]^2}{b c - a d}} \right)
\end{aligned}$$

$$\begin{aligned}
& \sqrt{-\frac{(a+b)(d+c \cos[e+f x]) \csc[\frac{1}{2}(e+f x)]^2}{b c - a d}} \csc[e+f x] \text{EllipticPi}\left[\frac{b c - a d}{(a+b)c}, \right. \\
& \left. \text{ArcSin}\left[\frac{\sqrt{-\frac{(a+b)(d+c \cos[e+f x]) \csc[\frac{1}{2}(e+f x)]^2}{b c - a d}}}{\sqrt{2}}\right], \frac{2(b c - a d)}{(a+b)(c-d)}\right] \sin[\frac{1}{2}(e+f x)]^4 \Bigg] / \\
& \left. \left((a+b)c \sqrt{b + a \cos[e+f x]} \sqrt{d + c \cos[e+f x]} \right) + 2(6 a b c^3 d - 7 a^2 c^2 d^2 - \right. \\
& \left. 2 a b c d^3 + 3 a^2 d^4) \left(\sqrt{\frac{-a+b}{a+b}} (a+b) \cos[\frac{1}{2}(e+f x)] \sqrt{d + c \cos[e+f x]} \right. \right. \\
& \left. \left. \text{EllipticE}[\text{ArcSin}\left[\frac{\sqrt{\frac{-a+b}{a+b}} \sin[\frac{1}{2}(e+f x)]}{\sqrt{\frac{b+a \cos[e+f x]}{a+b}}}\right], \frac{2(b c - a d)}{(-a+b)(c+d)}] \right) \Bigg] / \\
& \left. \left(a c \sqrt{\frac{(a+b) \cos[\frac{1}{2}(e+f x)]^2}{b + a \cos[e+f x]}} \sqrt{\frac{b + a \cos[e+f x]}{a+b}} \right. \right. \\
& \left. \left. \sqrt{\frac{(a+b)(d+c \cos[e+f x])}{(c+d)(b+a \cos[e+f x])}} \right) - \frac{1}{a c} 2(b c - a d) \left(\right. \right. \\
& \left. \left. (b c + (a+b)d) \sqrt{\frac{(c+d) \cot[\frac{1}{2}(e+f x)]^2}{c-d}} \sqrt{\frac{(c+d)(b+a \cos[e+f x]) \csc[\frac{1}{2}(e+f x)]^2}{b c - a d}} \right. \right. \\
& \left. \left. \sqrt{-\frac{(a+b)(d+c \cos[e+f x]) \csc[\frac{1}{2}(e+f x)]^2}{b c - a d}} \csc[e+f x] \right. \right. \\
& \left. \left. \text{EllipticF}[\text{ArcSin}\left[\frac{\sqrt{-\frac{(a+b)(d+c \cos[e+f x]) \csc[\frac{1}{2}(e+f x)]^2}{b c - a d}}}{\sqrt{2}}\right], \frac{2(b c - a d)}{(a+b)(c-d)}] \right)
\end{aligned}$$

$$\begin{aligned}
& \left. \frac{\sin \left[\frac{1}{2} (e + f x) \right]^4}{\left((a + b) (c + d) \sqrt{b + a \cos [e + f x]} \right)} \right\} \\
& - \left. \frac{\sqrt{d + c \cos [e + f x]}}{(b c + a d) \sqrt{\frac{(c + d) \cot \left[\frac{1}{2} (e + f x) \right]^2}{c - d}}} \right. \\
& \quad \left. \sqrt{\frac{(c + d) (b + a \cos [e + f x]) \csc \left[\frac{1}{2} (e + f x) \right]^2}{b c - a d}} \right. \\
& \quad \left. - \frac{\sqrt{-\frac{(a + b) (d + c \cos [e + f x]) \csc \left[\frac{1}{2} (e + f x) \right]^2}{b c - a d}} \csc [e + f x] \text{EllipticPi} \left[\frac{b c - a d}{(a + b) c}, \text{ArcSin} \left[\frac{\sqrt{-\frac{(a + b) (d + c \cos [e + f x]) \csc \left[\frac{1}{2} (e + f x) \right]^2}{b c - a d}}}{\sqrt{2}} \right], \frac{2 (b c - a d)}{(a + b) (c - d)} \right]} \right. \\
& \quad \left. \left. \left. \frac{\sqrt{d + c \cos [e + f x]} \sin [e + f x]}{c \sqrt{b + a \cos [e + f x]}} \right] \right] \right\} + \\
& \quad \left. \left. \left. \frac{(c + d)^2 (b c - a d) f \sqrt{b + a \cos [e + f x]}}{(c + d \sec [e + f x])^{5/2}} \right] \right] \right)
\end{aligned}$$

Problem 211: Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b \sec [e + f x])^{3/2}}{(c + d \sec [e + f x])^{3/2}} dx$$

Optimal (type 4, 744 leaves, 6 steps):

$$\begin{aligned}
& - \left(\left(2 (a-b) \sqrt{a+b} \sqrt{-\frac{(b c-a d) (1-\cos[e+f x])}{(a+b) (d+c \cos[e+f x])}} \right. \right. \\
& \quad \left. \left. \sqrt{-\frac{(b c-a d) (1+\cos[e+f x])}{(a-b) (d+c \cos[e+f x])}} (d+c \cos[e+f x])^{3/2} \csc[e+f x] \right. \right. \\
& \quad \left. \left. \text{EllipticE}[\text{ArcSin}\left[\frac{\sqrt{c+d} \sqrt{b+a \cos[e+f x]}}{\sqrt{a+b} \sqrt{d+c \cos[e+f x]}}, \frac{(a+b) (c-d)}{(a-b) (c+d)}\right] \sqrt{a+b \sec[e+f x]}] \right) / \\
& \quad \left. \left(c (c-d) \sqrt{c+d} f \sqrt{b+a \cos[e+f x]} \sqrt{c+d \sec[e+f x]} \right) \right) - \\
& \quad \left(2 \sqrt{a+b} (b c-a (2 c-d)) \sqrt{-\frac{(b c-a d) (1-\cos[e+f x])}{(a+b) (d+c \cos[e+f x])}} \right. \\
& \quad \left. \left. \sqrt{-\frac{(b c-a d) (1+\cos[e+f x])}{(a-b) (d+c \cos[e+f x])}} (d+c \cos[e+f x])^{3/2} \csc[e+f x] \right. \right. \\
& \quad \left. \left. \text{EllipticF}[\text{ArcSin}\left[\frac{\sqrt{c+d} \sqrt{b+a \cos[e+f x]}}{\sqrt{a+b} \sqrt{d+c \cos[e+f x]}}, \frac{(a+b) (c-d)}{(a-b) (c+d)}\right] \sqrt{a+b \sec[e+f x]}] \right) / \\
& \quad \left. \left(c^2 (c-d) \sqrt{c+d} f \sqrt{b+a \cos[e+f x]} \sqrt{c+d \sec[e+f x]} \right) \right. - \\
& \quad \left(2 a \sqrt{a+b} \sqrt{-\frac{(b c-a d) (1-\cos[e+f x])}{(a+b) (d+c \cos[e+f x])}} \sqrt{-\frac{(b c-a d) (1+\cos[e+f x])}{(a-b) (d+c \cos[e+f x])}} \right. \\
& \quad \left. \left. (d+c \cos[e+f x])^{3/2} \csc[e+f x] \text{EllipticPi}\left[\frac{(a+b) c}{a (c+d)}, \right. \right. \right. \\
& \quad \left. \left. \left. \text{ArcSin}\left[\frac{\sqrt{c+d} \sqrt{b+a \cos[e+f x]}}{\sqrt{a+b} \sqrt{d+c \cos[e+f x]}}, \frac{(a+b) (c-d)}{(a-b) (c+d)}\right] \sqrt{a+b \sec[e+f x]} \right) / \right. \\
& \quad \left. \left. \left(c^2 \sqrt{c+d} f \sqrt{b+a \cos[e+f x]} \sqrt{c+d \sec[e+f x]} \right) \right)
\end{aligned}$$

Result (type 4, 1720 leaves):

$$\begin{aligned}
& \left(2 (d+c \cos[e+f x]) (a+b \sec[e+f x])^{3/2} (-b c \sin[e+f x] + a d \sin[e+f x]) \right) / \\
& \quad \left((-c^2 + d^2) f (b+a \cos[e+f x]) (c+d \sec[e+f x])^{3/2} \right) + \\
& \quad \frac{1}{(c-d) (c+d) f (b+a \cos[e+f x])^{3/2} (c+d \sec[e+f x])^{3/2}}
\end{aligned}$$

$$\begin{aligned}
& \left(d + c \cos[e + f x] \right)^{3/2} \left(a + b \sec[e + f x] \right)^{3/2} \left(\sqrt{\frac{(c+d) \cot[\frac{1}{2}(e+f x)]^2}{c-d}} \sqrt{\frac{(c+d) (b+a \cos[e+f x]) \csc[\frac{1}{2}(e+f x)]^2}{b c - a d}} \right. \\
& \quad \left. \sqrt{-\frac{(a+b) (d+c \cos[e+f x]) \csc[\frac{1}{2}(e+f x)]^2}{b c - a d}} \csc[e+f x] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-\frac{(a+b) (d+c \cos[e+f x]) \csc[\frac{1}{2}(e+f x)]^2}{b c - a d}}}{\sqrt{2}} \right], \frac{2 (b c - a d)}{(a+b) (c-d)} \right] \sin[\frac{1}{2}(e+f x)]^4 \right) / \\
& \quad \left((a+b) (c+d) \sqrt{b+a \cos[e+f x]} \sqrt{d+c \cos[e+f x]} \right) + 4 (a^2 c - b^2 c) (b c - a d) \\
& \quad \left(\sqrt{\frac{(c+d) \cot[\frac{1}{2}(e+f x)]^2}{c-d}} \sqrt{\frac{(c+d) (b+a \cos[e+f x]) \csc[\frac{1}{2}(e+f x)]^2}{b c - a d}} \right. \\
& \quad \left. \sqrt{-\frac{(a+b) (d+c \cos[e+f x]) \csc[\frac{1}{2}(e+f x)]^2}{b c - a d}} \csc[e+f x] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-\frac{(a+b) (d+c \cos[e+f x]) \csc[\frac{1}{2}(e+f x)]^2}{b c - a d}}}{\sqrt{2}} \right], \frac{2 (b c - a d)}{(a+b) (c-d)} \right] \sin[\frac{1}{2}(e+f x)]^4 \right) / \\
& \quad \left((a+b) (c+d) \sqrt{b+a \cos[e+f x]} \sqrt{d+c \cos[e+f x]} \right) - \\
& \quad \left(\sqrt{\frac{(c+d) \cot[\frac{1}{2}(e+f x)]^2}{c-d}} \sqrt{\frac{(c+d) (b+a \cos[e+f x]) \csc[\frac{1}{2}(e+f x)]^2}{b c - a d}} \right. \\
& \quad \left. \sqrt{-\frac{(a+b) (d+c \cos[e+f x]) \csc[\frac{1}{2}(e+f x)]^2}{b c - a d}} \csc[e+f x] \right. \\
& \quad \left. \text{EllipticPi}\left[\frac{b c - a d}{(a+b) c}, \text{ArcSin}\left[\frac{\sqrt{-\frac{(a+b) (d+c \cos[e+f x]) \csc[\frac{1}{2}(e+f x)]^2}{b c - a d}}}{\sqrt{2}} \right], \frac{2 (b c - a d)}{(a+b) (c-d)} \right] \right)
\end{aligned}$$

$$\begin{aligned}
& \left. \left(\frac{\sin \left[\frac{1}{2} (e + f x) \right]^4}{\left((a + b) c \sqrt{b + a \cos [e + f x]} \sqrt{d + c \cos [e + f x]} \right)} \right) + \right. \\
& 2 (-a b c + a^2 d) \left(\sqrt{\frac{-a + b}{a + b}} (a + b) \cos \left[\frac{1}{2} (e + f x) \right] \sqrt{d + c \cos [e + f x]} \right. \\
& \left. \left. \text{EllipticE} \left[\text{ArcSin} \left[\frac{\sqrt{\frac{-a+b}{a+b}} \sin \left[\frac{1}{2} (e + f x) \right]}{\sqrt{\frac{b+a \cos [e+f x]}{a+b}}} \right], \frac{2 (b c - a d)}{(-a + b) (c + d)} \right] \right) / \right. \\
& \left. a c \sqrt{\frac{(a + b) \cos \left[\frac{1}{2} (e + f x) \right]^2}{b + a \cos [e + f x]}} \sqrt{b + a \cos [e + f x]} \sqrt{\frac{b + a \cos [e + f x]}{a + b}} \right. \\
& \left. \sqrt{\frac{(a + b) (d + c \cos [e + f x])}{(c + d) (b + a \cos [e + f x])}} \right) - \frac{1}{a c} 2 (b c - a d) \left(\left(b c + (a + b) d \right) \right. \\
& \left. \sqrt{\frac{(c + d) \cot \left[\frac{1}{2} (e + f x) \right]^2}{c - d}} \sqrt{\frac{(c + d) (b + a \cos [e + f x]) \csc \left[\frac{1}{2} (e + f x) \right]^2}{b c - a d}} \right. \\
& \left. \sqrt{-\frac{(a + b) (d + c \cos [e + f x]) \csc \left[\frac{1}{2} (e + f x) \right]^2}{b c - a d}} \csc [e + f x] \text{EllipticF} [\text{ArcSin} \left[\right. \right. \\
& \left. \left. \sqrt{-\frac{(a+b) (d+c \cos [e+f x]) \csc \left[\frac{1}{2} (e+f x) \right]^2}{b c-a d}} \right], \frac{2 (b c - a d)}{(a + b) (c - d)}] \sin \left[\frac{1}{2} (e + f x) \right]^4 \right) / \\
& \left((a + b) (c + d) \sqrt{b + a \cos [e + f x]} \sqrt{d + c \cos [e + f x]} \right) - \left(b c + a d \right) \\
& \left. \sqrt{\frac{(c + d) \cot \left[\frac{1}{2} (e + f x) \right]^2}{c - d}} \sqrt{\frac{(c + d) (b + a \cos [e + f x]) \csc \left[\frac{1}{2} (e + f x) \right]^2}{b c - a d}} \right)
\end{aligned}$$

$$\begin{aligned} & \sqrt{-\frac{(a+b)(d+c \cos[e+fx]) \csc^2[\frac{1}{2}(e+fx)]}{bc-ad}} \csc[e+fx] \\ & \text{EllipticPi}\left[\frac{bc-ad}{(a+b)c}, \text{ArcSin}\left[\frac{\sqrt{-\frac{(a+b)(d+c \cos[e+fx]) \csc^2[\frac{1}{2}(e+fx)]}{bc-ad}}}{\sqrt{2}}\right]\right], \\ & \left. \frac{2(bc-ad)}{(a+b)(c-d)} \sin^4\left[\frac{1}{2}(e+fx)\right] \right\} / \left((a+b)c \sqrt{b+a \cos[e+fx]} \right) \\ & \left. \sqrt{d+c \cos[e+fx]} \right) + \left. \frac{\sqrt{d+c \cos[e+fx]} \sin[e+fx]}{c \sqrt{b+a \cos[e+fx]}} \right) \end{aligned}$$

Problem 212: Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b \operatorname{Sec}[e + f x])^{3/2}}{(c + d \operatorname{Sec}[e + f x])^{5/2}} dx$$

Optimal (type 4, 919 leaves, 7 steps):

$$\begin{aligned}
& - \left(\left(2 (a-b) \sqrt{a+b} (3 b c^3 - 7 a c^2 d + b c d^2 + 3 a d^3) \sqrt{-\frac{(b c - a d) (1 - \cos[e+f x])}{(a+b) (d + c \cos[e+f x])}} \right. \right. \\
& \quad \left. \left. \sqrt{-\frac{(b c - a d) (1 + \cos[e+f x])}{(a-b) (d + c \cos[e+f x])}} (d + c \cos[e+f x])^{3/2} \csc[e+f x] \right. \right. \\
& \quad \left. \left. \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{c+d} \sqrt{b+a \cos[e+f x]}}{\sqrt{a+b} \sqrt{d+c \cos[e+f x]}}\right], \frac{(a+b) (c-d)}{(a-b) (c+d)}\right] \sqrt{a+b \sec[e+f x]} \right) \right) / \\
& \quad \left(3 c^2 (c-d)^2 (c+d)^{3/2} (b c - a d) f \sqrt{b+a \cos[e+f x]} \sqrt{c+d \sec[e+f x]} \right) - \\
& \quad \left(2 \sqrt{a+b} (b^2 c^3 (3 c+d) - 2 a b c^2 (3 c^2 + 2 c d - d^2) + a^2 d (9 c^3 - 2 c^2 d - 6 c d^2 + 3 d^3)) \right. \\
& \quad \left. \sqrt{-\frac{(b c - a d) (1 - \cos[e+f x])}{(a+b) (d + c \cos[e+f x])}} \sqrt{-\frac{(b c - a d) (1 + \cos[e+f x])}{(a-b) (d + c \cos[e+f x])}} \right. \\
& \quad \left. (d + c \cos[e+f x])^{3/2} \csc[e+f x] \right. \right. \\
& \quad \left. \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{c+d} \sqrt{b+a \cos[e+f x]}}{\sqrt{a+b} \sqrt{d+c \cos[e+f x]}}\right], \frac{(a+b) (c-d)}{(a-b) (c+d)}\right] \sqrt{a+b \sec[e+f x]} \right) \right) / \\
& \quad \left(3 c^3 (c-d)^2 (c+d)^{3/2} (b c - a d) f \sqrt{b+a \cos[e+f x]} \sqrt{c+d \sec[e+f x]} \right) - \\
& \quad \left(2 a \sqrt{a+b} \sqrt{-\frac{(b c - a d) (1 - \cos[e+f x])}{(a+b) (d + c \cos[e+f x])}} \sqrt{-\frac{(b c - a d) (1 + \cos[e+f x])}{(a-b) (d + c \cos[e+f x])}} \right. \\
& \quad \left. (d + c \cos[e+f x])^{3/2} \csc[e+f x] \text{EllipticPi}\left[\frac{(a+b) c}{a (c+d)}, \right. \right. \\
& \quad \left. \left. \text{ArcSin}\left[\frac{\sqrt{c+d} \sqrt{b+a \cos[e+f x]}}{\sqrt{a+b} \sqrt{d+c \cos[e+f x]}}\right], \frac{(a+b) (c-d)}{(a-b) (c+d)}\right] \sqrt{a+b \sec[e+f x]} \right) \right) / \\
& \quad \left(c^3 \sqrt{c+d} f \sqrt{b+a \cos[e+f x]} \sqrt{c+d \sec[e+f x]} \right) - \\
& \quad \frac{2 d (b c - a d) \sqrt{a+b \sec[e+f x]} \sin[e+f x]}{3 c (c^2 - d^2) f (d + c \cos[e+f x]) \sqrt{c+d \sec[e+f x]}}
\end{aligned}$$

Result (type 4, 1930 leaves):

$$\begin{aligned}
& \left((d + c \cos[e+f x])^3 \sec[e+f x] (a + b \sec[e+f x])^{3/2} \left(\frac{2 (-b c d \sin[e+f x] + a d^2 \sin[e+f x])}{3 c (c^2 - d^2) (d + c \cos[e+f x])^2} + \right. \right. \\
& \quad \left. \left. (2 (3 b c^3 \sin[e+f x] - 7 a c^2 d \sin[e+f x] + b c d^2 \sin[e+f x] + 3 a d^3 \sin[e+f x])) \right) \right. \\
& \quad \left. \left. \text{EllipticPi}\left[\frac{(a+b) c}{a (c+d)}, \text{ArcSin}\left[\frac{\sqrt{c+d} \sqrt{b+a \cos[e+f x]}}{\sqrt{a+b} \sqrt{d+c \cos[e+f x]}}\right], \frac{(a+b) (c-d)}{(a-b) (c+d)}\right] \sqrt{a+b \sec[e+f x]} \right) \right) / \\
& \quad \left(f (b + a \cos[e+f x]) (c + d \sec[e+f x])^{5/2} \right) +
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{3 c (c-d)^2 (c+d)^2 f (b+a \cos[e+f x])^{3/2} (c+d \sec[e+f x])^{5/2}} \\
& \quad \left(\frac{(d+c \cos[e+f x])^{5/2} \sec[e+f x] (a+b \sec[e+f x])^{3/2}}{4 (b c - a d) (3 a b c^3 + a^2 c^2 d - 4 b^2 c^2 d + a b c d^2 - a^2 d^3)} \right. \\
& \quad \sqrt{\frac{(c+d) \cot[\frac{1}{2} (e+f x)]^2}{c-d}} \sqrt{\frac{(c+d) (b+a \cos[e+f x]) \csc[\frac{1}{2} (e+f x)]^2}{b c - a d}} \\
& \quad \sqrt{-\frac{(a+b) (d+c \cos[e+f x]) \csc[\frac{1}{2} (e+f x)]^2}{b c - a d}} \csc[e+f x] \operatorname{EllipticF}\left[\\
& \quad \left. \operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b) (d+c \cos[e+f x]) \csc[\frac{1}{2} (e+f x)]^2}{b c - a d}}}{\sqrt{2}}\right], \frac{2 (b c - a d)}{(a+b) (c-d)}\right] \sin[\frac{1}{2} (e+f x)]^4 \right] / \\
& \quad \left((a+b) (c+d) \sqrt{b+a \cos[e+f x]} \sqrt{d+c \cos[e+f x]} \right) + \\
& \quad 4 (b c - a d) (3 a^2 c^3 - 3 b^2 c^3 + 4 a b c^2 d + a^2 c d^2 - b^2 c d^2 - 4 a b d^3) \\
& \quad \left(\sqrt{\frac{(c+d) \cot[\frac{1}{2} (e+f x)]^2}{c-d}} \sqrt{\frac{(c+d) (b+a \cos[e+f x]) \csc[\frac{1}{2} (e+f x)]^2}{b c - a d}} \right. \\
& \quad \sqrt{-\frac{(a+b) (d+c \cos[e+f x]) \csc[\frac{1}{2} (e+f x)]^2}{b c - a d}} \csc[e+f x] \operatorname{EllipticF}\left[\\
& \quad \left. \operatorname{ArcSin}\left[\frac{\sqrt{-\frac{(a+b) (d+c \cos[e+f x]) \csc[\frac{1}{2} (e+f x)]^2}{b c - a d}}}{\sqrt{2}}\right], \frac{2 (b c - a d)}{(a+b) (c-d)}\right] \sin[\frac{1}{2} (e+f x)]^4 \right] / \\
& \quad \left((a+b) (c+d) \sqrt{b+a \cos[e+f x]} \sqrt{d+c \cos[e+f x]} \right) - \\
& \quad \left(\sqrt{\frac{(c+d) \cot[\frac{1}{2} (e+f x)]^2}{c-d}} \sqrt{\frac{(c+d) (b+a \cos[e+f x]) \csc[\frac{1}{2} (e+f x)]^2}{b c - a d}} \right)
\end{aligned}$$

$$\begin{aligned}
& \sqrt{-\frac{(a+b)(d+c \cos[e+f x]) \csc[\frac{1}{2}(e+f x)]^2}{b c - a d}} \csc[e+f x] \\
& \text{EllipticPi}\left[\frac{b c - a d}{(a+b)c}, \text{ArcSin}\left[\sqrt{-\frac{(a+b)(d+c \cos[e+f x]) \csc[\frac{1}{2}(e+f x)]^2}{b c - a d}}\right], \frac{2(b c - a d)}{(a+b)(c-d)}\right] \\
& \left. \left. \sin\left[\frac{1}{2}(e+f x)\right]^4 \right/ \left((a+b)c \sqrt{b + a \cos[e+f x]} \sqrt{d + c \cos[e+f x]} \right) \right) + \\
& 2 \left(-3 a b c^3 + 7 a^2 c^2 d - a b c d^2 - 3 a^2 d^3 \right) \left(\left(\sqrt{\frac{-a+b}{a+b}} (a+b) \cos\left[\frac{1}{2}(e+f x)\right] \right. \right. \\
& \left. \left. \sqrt{d + c \cos[e+f x]} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{-a+b}{a+b}} \sin\left[\frac{1}{2}(e+f x)\right]}{\sqrt{\frac{b+a \cos[e+f x]}{a+b}}}\right], \frac{2(b c - a d)}{(-a+b)(c+d)}\right]\right) \right. \\
& \left. \left. a c \sqrt{\frac{(a+b) \cos\left[\frac{1}{2}(e+f x)\right]^2}{b + a \cos[e+f x]}} \sqrt{\frac{b + a \cos[e+f x]}{a + b}} \right. \right. \\
& \left. \left. \sqrt{\frac{(a+b)(d+c \cos[e+f x])}{(c+d)(b+a \cos[e+f x])}} - \frac{1}{a c} 2(b c - a d) \left(\left(b c + (a+b)d \right) \right. \right. \right. \\
& \left. \left. \left. \sqrt{\frac{(c+d) \cot\left[\frac{1}{2}(e+f x)\right]^2}{c-d}} \sqrt{\frac{(c+d)(b+a \cos[e+f x]) \csc\left[\frac{1}{2}(e+f x)\right]^2}{b c - a d}} \right. \right. \\
& \left. \left. \sqrt{-\frac{(a+b)(d+c \cos[e+f x]) \csc\left[\frac{1}{2}(e+f x)\right]^2}{b c - a d}} \csc[e+f x] \text{EllipticF}\left[\text{ArcSin}\left[\right. \right. \right. \right. \\
& \left. \left. \left. \left. \sqrt{-\frac{(a+b)(d+c \cos[e+f x]) \csc\left[\frac{1}{2}(e+f x)\right]^2}{b c - a d}}\right], \frac{2(b c - a d)}{(a+b)(c-d)}\right] \sin\left[\frac{1}{2}(e+f x)\right]^4 \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left((a+b) (c+d) \sqrt{b+a \cos[e+f x]} \sqrt{d+c \cos[e+f x]} \right) - \left(b c + a d \right) \\
& \sqrt{\frac{(c+d) \cot[\frac{1}{2} (e+f x)]^2}{c-d}} \sqrt{\frac{(c+d) (b+a \cos[e+f x]) \csc[\frac{1}{2} (e+f x)]^2}{b c - a d}} \\
& \sqrt{-\frac{(a+b) (d+c \cos[e+f x]) \csc[\frac{1}{2} (e+f x)]^2}{b c - a d}} \csc[e+f x] \\
& \text{EllipticPi}\left[\frac{b c - a d}{(a+b) c}, \text{ArcSin}\left[\frac{\sqrt{-\frac{(a+b) (d+c \cos[e+f x]) \csc[\frac{1}{2} (e+f x)]^2}{b c - a d}}}{\sqrt{2}}\right]\right], \\
& \left. \frac{2 (b c - a d)}{(a+b) (c-d)} \right] \sin[\frac{1}{2} (e+f x)]^4 \Bigg/ \left((a+b) c \sqrt{b+a \cos[e+f x]} \right. \\
& \left. \sqrt{d+c \cos[e+f x]} \right) + \frac{\sqrt{d+c \cos[e+f x]} \sin[e+f x]}{c \sqrt{b+a \cos[e+f x]}} \Bigg)
\end{aligned}$$

Problem 213: Result more than twice size of optimal antiderivative.

$$\int \frac{(a+b \sec[e+f x])^{3/2}}{(c+d \sec[e+f x])^{7/2}} dx$$

Optimal (type 4, 1122 leaves, 8 steps):

$$\begin{aligned}
& \left(2 (a - b) \sqrt{a + b} \right. \\
& \quad \left(2 a b c d (35 c^4 - 8 c^2 d^2 + 5 d^4) - a^2 d^2 (58 c^4 - 41 c^2 d^2 + 15 d^4) - b^2 (15 c^6 + 19 c^4 d^2 - 2 c^2 d^4) \right) \\
& \quad \sqrt{-\frac{(b c - a d) (1 - \cos[e + f x])}{(a + b) (d + c \cos[e + f x])}} \sqrt{-\frac{(b c - a d) (1 + \cos[e + f x])}{(a - b) (d + c \cos[e + f x])}} \\
& \quad (d + c \cos[e + f x])^{3/2} \csc[e + f x] \\
& \quad \left. \text{EllipticE}[\text{ArcSin}\left[\frac{\sqrt{c + d} \sqrt{b + a \cos[e + f x]}}{\sqrt{a + b} \sqrt{d + c \cos[e + f x]}}\right], \frac{(a + b) (c - d)}{(a - b) (c + d)}] \sqrt{a + b \sec[e + f x]} \right) / \\
& \quad \left(15 c^3 (c - d)^3 (c + d)^{5/2} (b c - a d)^2 f \sqrt{b + a \cos[e + f x]} \sqrt{c + d \sec[e + f x]} \right) - \\
& \quad \left(2 \sqrt{a + b} (b^2 c^3 (15 c^3 + 10 c^2 d + 9 c d^2 - 2 d^3) - 2 a b c^2 (15 c^4 + 20 c^3 d - 4 c^2 d^2 - 4 c d^3 + 5 d^4)) + \right. \\
& \quad a^2 d (60 c^5 - 2 c^4 d - 66 c^3 d^2 + 25 c^2 d^3 + 30 c d^4 - 15 d^5)) \sqrt{-\frac{(b c - a d) (1 - \cos[e + f x])}{(a + b) (d + c \cos[e + f x])}} \\
& \quad \left. \sqrt{-\frac{(b c - a d) (1 + \cos[e + f x])}{(a - b) (d + c \cos[e + f x])}} (d + c \cos[e + f x])^{3/2} \csc[e + f x] \right. \\
& \quad \left. \text{EllipticF}[\text{ArcSin}\left[\frac{\sqrt{c + d} \sqrt{b + a \cos[e + f x]}}{\sqrt{a + b} \sqrt{d + c \cos[e + f x]}}\right], \frac{(a + b) (c - d)}{(a - b) (c + d)}] \sqrt{a + b \sec[e + f x]} \right) / \\
& \quad \left(15 c^4 (c - d)^3 (c + d)^{5/2} (b c - a d) f \sqrt{b + a \cos[e + f x]} \sqrt{c + d \sec[e + f x]} \right) - \\
& \quad \left(2 a \sqrt{a + b} \sqrt{-\frac{(b c - a d) (1 - \cos[e + f x])}{(a + b) (d + c \cos[e + f x])}} \sqrt{-\frac{(b c - a d) (1 + \cos[e + f x])}{(a - b) (d + c \cos[e + f x])}} \right. \\
& \quad (d + c \cos[e + f x])^{3/2} \csc[e + f x] \text{EllipticPi}\left[\frac{(a + b) c}{a (c + d)}, \right. \\
& \quad \left. \left. \text{ArcSin}\left[\frac{\sqrt{c + d} \sqrt{b + a \cos[e + f x]}}{\sqrt{a + b} \sqrt{d + c \cos[e + f x]}}\right], \frac{(a + b) (c - d)}{(a - b) (c + d)}] \sqrt{a + b \sec[e + f x]} \right) / \\
& \quad \left(c^4 \sqrt{c + d} f \sqrt{b + a \cos[e + f x]} \sqrt{c + d \sec[e + f x]} \right) + \\
& \quad \frac{2 d^2 (b + a \cos[e + f x]) \sqrt{a + b \sec[e + f x]} \sin[e + f x]}{5 c (c^2 - d^2) f (d + c \cos[e + f x])^2 \sqrt{c + d \sec[e + f x]}} - \\
& \quad \left(2 d (10 b c^3 - 13 a c^2 d - 2 b c d^2 + 5 a d^3) \sqrt{a + b \sec[e + f x]} \sin[e + f x] \right) / \\
& \quad \left(15 c^2 (c^2 - d^2)^2 f (d + c \cos[e + f x]) \sqrt{c + d \sec[e + f x]} \right)
\end{aligned}$$

Result (type 4, 2355 leaves):

$$\begin{aligned}
& \frac{1}{f(b + a \cos[e + fx]) (c + d \sec[e + fx])^{7/2}} (d + c \cos[e + fx])^4 \\
& \sec[e + fx]^2 (a + b \sec[e + fx])^{3/2} \left(-\frac{2 (-b c d^2 \sin[e + fx] + a d^3 \sin[e + fx])}{5 c^2 (c^2 - d^2) (d + c \cos[e + fx])^3} - \right. \\
& (4 (5 b c^3 d \sin[e + fx] - 8 a c^2 d^2 \sin[e + fx] - b c d^3 \sin[e + fx] + 4 a d^4 \sin[e + fx])) / \\
& (15 c^2 (c^2 - d^2)^2 (d + c \cos[e + fx])^2) + \\
& (2 (15 b^2 c^6 \sin[e + fx] - 70 a b c^5 d \sin[e + fx] + 58 a^2 c^4 d^2 \sin[e + fx] + \\
& 19 b^2 c^4 d^2 \sin[e + fx] + 16 a b c^3 d^3 \sin[e + fx] - 41 a^2 c^2 d^4 \sin[e + fx] - \\
& 2 b^2 c^2 d^4 \sin[e + fx] - 10 a b c d^5 \sin[e + fx] + 15 a^2 d^6 \sin[e + fx])) / \\
& \left. (15 c^2 (b c - a d) (c^2 - d^2)^3 (d + c \cos[e + fx])) \right) + \\
& \left((d + c \cos[e + fx])^{7/2} \sec[e + fx]^2 (a + b \sec[e + fx])^{3/2} \right. \\
& \left(\frac{1}{(a + b) (c + d) \sqrt{b + a \cos[e + fx]} \sqrt{d + c \cos[e + fx]}} \right. \\
& 4 (b c - a d) (-15 a b^2 c^6 + 5 a^2 b c^5 d + 25 b^3 c^5 d + 13 a^3 c^4 d^2 - 38 a b^2 c^4 d^2 + \\
& 25 a^2 b c^3 d^3 + 7 b^3 c^3 d^3 - 18 a^3 c^2 d^4 - 11 a b^2 c^2 d^4 + 2 a^2 b c d^5 + 5 a^3 d^6) \\
& \sqrt{\frac{(c + d) \cot[\frac{1}{2} (e + fx)]^2}{c - d}} \sqrt{\frac{(c + d) (b + a \cos[e + fx]) \csc[\frac{1}{2} (e + fx)]^2}{b c - a d}} \\
& - \frac{(a + b) (d + c \cos[e + fx]) \csc[\frac{1}{2} (e + fx)]^2}{b c - a d} \csc[e + fx] \text{EllipticF}[\\
& \text{ArcSin}\left[\sqrt{-\frac{(a + b) (d + c \cos[e + fx]) \csc[\frac{1}{2} (e + fx)]^2}{b c - a d}}\right], \frac{2 (b c - a d)}{(a + b) (c - d)}] \sin[\frac{1}{2} (e + fx)]^4 + \\
& 4 (b c - a d) (-15 a^2 b c^6 + 15 b^3 c^6 + 15 a^3 c^5 d - 55 a b^2 c^5 d + 33 a^2 b c^4 d^2 + 19 b^3 c^4 d^2 + \\
& 13 a^3 c^3 d^3 + 35 a b^2 c^3 d^3 - 70 a^2 b c^2 d^4 - 2 b^3 c^2 d^4 + 4 a^3 c d^5 - 12 a b^2 c d^5 + 20 a^2 b d^6) \\
& \left. \left(\sqrt{\frac{(c + d) \cot[\frac{1}{2} (e + fx)]^2}{c - d}} \sqrt{\frac{(c + d) (b + a \cos[e + fx]) \csc[\frac{1}{2} (e + fx)]^2}{b c - a d}} \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \sqrt{-\frac{(a+b)(d+c \cos[e+f x]) \csc[\frac{1}{2}(e+f x)]^2}{b c - a d}} \csc[e+f x] \text{EllipticF}\left[\\
& \quad \text{ArcSin}\left[\frac{\sqrt{-\frac{(a+b)(d+c \cos[e+f x]) \csc[\frac{1}{2}(e+f x)]^2}{b c - a d}}}{\sqrt{2}}\right], \frac{2(b c - a d)}{(a+b)(c-d)}\right] \sin[\frac{1}{2}(e+f x)]^4\right] \\
& \left((a+b)(c+d) \sqrt{b + a \cos[e+f x]} \sqrt{d + c \cos[e+f x]} \right) - \\
& \left(\sqrt{\frac{(c+d) \cot[\frac{1}{2}(e+f x)]^2}{c-d}} \sqrt{\frac{(c+d)(b+a \cos[e+f x]) \csc[\frac{1}{2}(e+f x)]^2}{b c - a d}} \right. \\
& \quad \sqrt{-\frac{(a+b)(d+c \cos[e+f x]) \csc[\frac{1}{2}(e+f x)]^2}{b c - a d}} \csc[e+f x] \\
& \quad \text{EllipticPi}\left[\frac{b c - a d}{(a+b)c}, \text{ArcSin}\left[\frac{\sqrt{-\frac{(a+b)(d+c \cos[e+f x]) \csc[\frac{1}{2}(e+f x)]^2}{b c - a d}}}{\sqrt{2}}\right], \frac{2(b c - a d)}{(a+b)(c-d)}\right] \\
& \quad \left. \sin[\frac{1}{2}(e+f x)]^4\right) \\
& 2(15 a b^2 c^6 - 70 a^2 b c^5 d + 58 a^3 c^4 d^2 + 19 a b^2 c^4 d^2 + 16 a^2 b c^3 d^3 - 41 a^3 c^2 d^4 - 2 a b^2 c^2 d^4 - \\
& 10 a^2 b c d^5 + 15 a^3 d^6) \left(\sqrt{\frac{-a+b}{a+b}} (a+b) \cos[\frac{1}{2}(e+f x)] \sqrt{d + c \cos[e+f x]} \right. \\
& \quad \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{-a+b}{a+b}} \sin[\frac{1}{2}(e+f x)]}{\sqrt{\frac{b+a \cos[e+f x]}{a+b}}}\right], \frac{2(b c - a d)}{(-a+b)(c+d)}\right] \\
& \quad \left. a c \sqrt{\frac{(a+b) \cos[\frac{1}{2}(e+f x)]^2}{b + a \cos[e+f x]}} \sqrt{b + a \cos[e+f x]} \sqrt{\frac{b + a \cos[e+f x]}{a + b}}\right)
\end{aligned}$$

$$\begin{aligned}
& \sqrt{\frac{(a+b)(d+c \cos[e+f x])}{(c+d)(b+a \cos[e+f x])}} - \frac{1}{a c} 2(b c - a d) \left| \begin{array}{l} (b c + (a+b)d) \\ (b c + (a+b)d) \end{array} \right. \\
& \sqrt{\frac{(c+d) \cot[\frac{1}{2}(e+f x)]^2}{c-d}} \sqrt{\frac{(c+d)(b+a \cos[e+f x]) \csc[\frac{1}{2}(e+f x)]^2}{b c - a d}} \\
& \sqrt{-\frac{(a+b)(d+c \cos[e+f x]) \csc[\frac{1}{2}(e+f x)]^2}{b c - a d}} \csc[e+f x] \\
& \text{EllipticF}[\text{ArcSin}\left[\sqrt{-\frac{(a+b)(d+c \cos[e+f x]) \csc[\frac{1}{2}(e+f x)]^2}{b c - a d}}\right], \frac{2(b c - a d)}{(a+b)(c-d)}] \\
& \sin\left[\frac{1}{2}(e+f x)\right]^4 \left| \begin{array}{l} ((a+b)(c+d) \sqrt{b+a \cos[e+f x]} \\ (b c + a d) \sqrt{\frac{(c+d) \cot[\frac{1}{2}(e+f x)]^2}{c-d}} \end{array} \right. \\
& \sqrt{d+c \cos[e+f x]} \left| \begin{array}{l} ((a+b)(c+d) \csc[\frac{1}{2}(e+f x)]^2 \\ (b c + a d) \sqrt{\frac{(c+d) \cot[\frac{1}{2}(e+f x)]^2}{c-d}} \end{array} \right. \\
& \sqrt{\frac{(c+d)(b+a \cos[e+f x]) \csc[\frac{1}{2}(e+f x)]^2}{b c - a d}} \\
& \sqrt{-\frac{(a+b)(d+c \cos[e+f x]) \csc[\frac{1}{2}(e+f x)]^2}{b c - a d}} \csc[e+f x] \text{EllipticPi}\left[\frac{b c - a d}{(a+b)c}, \text{ArcSin}\left[\sqrt{-\frac{(a+b)(d+c \cos[e+f x]) \csc[\frac{1}{2}(e+f x)]^2}{b c - a d}}\right], \frac{2(b c - a d)}{(a+b)(c-d)}\right] \\
& \sin\left[\frac{1}{2}(e+f x)\right]^4 \left| \begin{array}{l} ((a+b)c \sqrt{b+a \cos[e+f x]} \sqrt{d+c \cos[e+f x]}) \end{array} \right. \left. \right| +
\end{aligned}$$

$$\left. \left(\frac{\sqrt{d+c \cos[e+f x]} \sin[e+f x]}{c \sqrt{b+a \cos[e+f x]}} \right) \right\} / \left(15 c^2 (c-d)^3 (c+d)^3 (-b c + a d) f (b + a \cos[e+f x])^{3/2} (c + d \sec[e+f x])^{7/2} \right)$$

Problem 214: Result more than twice size of optimal antiderivative.

$$\int \frac{(a+b \sec[e+f x])^{5/2}}{(c+d \sec[e+f x])^{5/2}} dx$$

Optimal (type 4, 891 leaves, 7 steps):

$$\begin{aligned}
& - \left(\left(2 (a-b) \sqrt{a+b} (7 a c^2 - 4 b c d - 3 a d^2) \sqrt{-\frac{(b c - a d) (1 - \cos[e + f x])}{(a+b) (d + c \cos[e + f x])}} \right. \right. \\
& \quad \left. \left. \sqrt{-\frac{(b c - a d) (1 + \cos[e + f x])}{(a-b) (d + c \cos[e + f x])}} (d + c \cos[e + f x])^{3/2} \csc[e + f x] \right. \right. \\
& \quad \left. \left. \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{c+d} \sqrt{b+a \cos[e+f x]}}{\sqrt{a+b} \sqrt{d+c \cos[e+f x]}}\right], \frac{(a+b) (c-d)}{(a-b) (c+d)}\right] \sqrt{a+b \sec[e+f x]} \right) \right) / \\
& \quad \left(3 c^2 (c-d)^2 (c+d)^{3/2} f \sqrt{b+a \cos[e+f x]} \sqrt{c+d \sec[e+f x]} \right) + \\
& \quad \left(2 \sqrt{a+b} (b^2 c^2 (c+3 d) - a b c (7 c^2 + 4 c d - 3 d^2) + a^2 (9 c^3 - 2 c^2 d - 6 c d^2 + 3 d^3)) \right. \\
& \quad \left. \sqrt{-\frac{(b c - a d) (1 - \cos[e + f x])}{(a+b) (d + c \cos[e + f x])}} \sqrt{-\frac{(b c - a d) (1 + \cos[e + f x])}{(a-b) (d + c \cos[e + f x])}} \right. \\
& \quad \left. (d + c \cos[e + f x])^{3/2} \csc[e + f x] \right. \right. \\
& \quad \left. \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{c+d} \sqrt{b+a \cos[e+f x]}}{\sqrt{a+b} \sqrt{d+c \cos[e+f x]}}\right], \frac{(a+b) (c-d)}{(a-b) (c+d)}\right] \sqrt{a+b \sec[e+f x]} \right) \right) / \\
& \quad \left(3 c^3 (c-d)^2 (c+d)^{3/2} f \sqrt{b+a \cos[e+f x]} \sqrt{c+d \sec[e+f x]} \right) - \\
& \quad \left(2 a^2 \sqrt{a+b} \sqrt{-\frac{(b c - a d) (1 - \cos[e + f x])}{(a+b) (d + c \cos[e + f x])}} \sqrt{-\frac{(b c - a d) (1 + \cos[e + f x])}{(a-b) (d + c \cos[e + f x])}} \right. \\
& \quad \left. (d + c \cos[e + f x])^{3/2} \csc[e + f x] \text{EllipticPi}\left[\frac{(a+b) c}{a (c+d)}, \right. \right. \\
& \quad \left. \left. \text{ArcSin}\left[\frac{\sqrt{c+d} \sqrt{b+a \cos[e+f x]}}{\sqrt{a+b} \sqrt{d+c \cos[e+f x]}}\right], \frac{(a+b) (c-d)}{(a-b) (c+d)}\right] \sqrt{a+b \sec[e+f x]} \right) \right) / \\
& \quad \left(c^3 \sqrt{c+d} f \sqrt{b+a \cos[e+f x]} \sqrt{c+d \sec[e+f x]} \right) + \\
& \quad \frac{2 (b c - a d)^2 \sqrt{a+b \sec[e+f x]} \sin[e+f x]}{3 c (c^2 - d^2) f (d + c \cos[e + f x]) \sqrt{c+d \sec[e+f x]}}
\end{aligned}$$

Result (type 4, 1996 leaves) :

$$\begin{aligned}
& \left((d + c \cos[e + f x])^3 (a + b \sec[e + f x])^{5/2} \right. \\
& \quad \left(\frac{2 (b^2 c^2 \sin[e + f x] - 2 a b c d \sin[e + f x] + a^2 d^2 \sin[e + f x])}{3 c (c^2 - d^2) (d + c \cos[e + f x])^2} + \right. \\
& \quad \left. \left(2 (7 a b c^3 \sin[e + f x] - 7 a^2 c^2 d \sin[e + f x] - 4 b^2 c^2 d \sin[e + f x] + \right. \right. \\
& \quad \left. \left. 2 a^3 c d^2 \sin[e + f x] - 3 a b c d^2 \sin[e + f x]) \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left(f(b + a \cos[e + fx])^2 (c + d \sec[e + fx])^{5/2} \right) + \\
& \frac{1}{3c(c-d)^2(c+d)^2 f(b + a \cos[e + fx])^{5/2} (c + d \sec[e + fx])^{5/2}} \\
& \left(d + c \cos[e + fx] \right)^{5/2} (a + b \sec[e + fx])^{5/2} \\
& \left(\left(4(bc - ad)(2a^2bc^3 + b^3c^3 + a^3c^2d - 8ab^2c^2d + 2a^2bcd^2 + 3b^3cd^2 - a^3d^3) \right. \right. \\
& \sqrt{\frac{(c+d)\cot[\frac{1}{2}(e+fx)]^2}{c-d}} \sqrt{\frac{(c+d)(b+a \cos[e+fx])\csc[\frac{1}{2}(e+fx)]^2}{bc-ad}} \\
& \sqrt{-\frac{(a+b)(d+c \cos[e+fx])\csc[\frac{1}{2}(e+fx)]^2}{bc-ad}} \csc[e+fx] \operatorname{EllipticF} \left. \left[\right. \right. \\
& \left. \left. \operatorname{ArcSin} \left[\sqrt{-\frac{(a+b)(d+c \cos[e+fx])\csc[\frac{1}{2}(e+fx)]^2}{bc-ad}} \right], \frac{2(bc-ad)}{(a+b)(c-d)} \right] \sin[\frac{1}{2}(e+fx)]^4 \right) \\
& \left((a+b)(c+d)\sqrt{b+a \cos[e+fx]}\sqrt{d+c \cos[e+fx]} \right) + \\
& 4(bc-ad)(3a^3c^3 - 7ab^2c^3 + 4b^3c^2d + a^3cd^2 + 3ab^2cd^2 - 4a^2bd^3) \\
& \left(\sqrt{\frac{(c+d)\cot[\frac{1}{2}(e+fx)]^2}{c-d}} \sqrt{\frac{(c+d)(b+a \cos[e+fx])\csc[\frac{1}{2}(e+fx)]^2}{bc-ad}} \right. \\
& \sqrt{-\frac{(a+b)(d+c \cos[e+fx])\csc[\frac{1}{2}(e+fx)]^2}{bc-ad}} \csc[e+fx] \operatorname{EllipticF} \left. \left[\right. \right. \\
& \left. \left. \operatorname{ArcSin} \left[\sqrt{-\frac{(a+b)(d+c \cos[e+fx])\csc[\frac{1}{2}(e+fx)]^2}{bc-ad}} \right], \frac{2(bc-ad)}{(a+b)(c-d)} \right] \sin[\frac{1}{2}(e+fx)]^4 \right) \\
& \left((a+b)(c+d)\sqrt{b+a \cos[e+fx]}\sqrt{d+c \cos[e+fx]} \right) -
\end{aligned}$$

$$\begin{aligned}
& \left(\sqrt{\frac{(c+d) \cot[\frac{1}{2}(e+f x)]^2}{c-d}} \sqrt{\frac{(c+d)(b+a \cos[e+f x]) \csc[\frac{1}{2}(e+f x)]^2}{b c - a d}} \right. \\
& \quad \left. \sqrt{-\frac{(a+b)(d+c \cos[e+f x]) \csc[\frac{1}{2}(e+f x)]^2}{b c - a d}} \csc[e+f x] \right. \\
& \quad \left. \text{EllipticPi}\left[\frac{b c - a d}{(a+b)c}, \text{ArcSin}\left[\frac{\sqrt{-\frac{(a+b)(d+c \cos[e+f x]) \csc[\frac{1}{2}(e+f x)]^2}{b c - a d}}}{\sqrt{2}}\right], \frac{2(b c - a d)}{(a+b)(c-d)}\right]\right. \\
& \quad \left. \left. \sin\left[\frac{1}{2}(e+f x)\right]^4\right) \right/ \left((a+b)c \sqrt{b+a \cos[e+f x]} \sqrt{d+c \cos[e+f x]} \right) + \\
& 2(-7 a^2 b c^3 + 7 a^3 c^2 d + 4 a b^2 c^2 d - a^2 b c d^2 - 3 a^3 d^3) \left(\sqrt{\frac{-a+b}{a+b}} (a+b) \cos\left[\frac{1}{2}(e+f x)\right] \right. \\
& \quad \left. \sqrt{d+c \cos[e+f x]} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{-a+b}{a+b}} \sin\left[\frac{1}{2}(e+f x)\right]}{\sqrt{\frac{b+a \cos[e+f x]}{a+b}}}\right], \frac{2(b c - a d)}{(-a+b)(c+d)}\right]\right) \right/ \\
& \left(a c \sqrt{\frac{(a+b) \cos[\frac{1}{2}(e+f x)]^2}{b+a \cos[e+f x]}} \sqrt{\frac{b+a \cos[e+f x]}{a+b}} \right. \\
& \quad \left. \sqrt{\frac{(a+b)(d+c \cos[e+f x])}{(c+d)(b+a \cos[e+f x])}} - \frac{1}{a c} 2(b c - a d) \right) \left(\begin{array}{l} (b c + (a+b) d) \\ \end{array} \right. \\
& \quad \left. \sqrt{\frac{(c+d) \cot[\frac{1}{2}(e+f x)]^2}{c-d}} \sqrt{\frac{(c+d)(b+a \cos[e+f x]) \csc[\frac{1}{2}(e+f x)]^2}{b c - a d}} \right. \\
& \quad \left. \sqrt{-\frac{(a+b)(d+c \cos[e+f x]) \csc[\frac{1}{2}(e+f x)]^2}{b c - a d}} \csc[e+f x] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-\frac{(a+b)(d+c \cos[e+f x]) \csc[\frac{1}{2}(e+f x)]^2}{b c - a d}}}{\sqrt{2}}\right], \frac{2(b c - a d)}{(a+b)(c-d)}\right]\right)
\end{aligned}$$

$$\begin{aligned}
& \left. \frac{\sqrt{-\frac{(a+b)(d+c \cos[e+f x]) \csc[\frac{1}{2}(e+f x)]^2}{b c - a d}}}{\sqrt{2}}, \frac{2(b c - a d)}{(a + b)(c - d)} \sin[\frac{1}{2}(e + f x)]^4 \right\} \\
& \left((a + b)(c + d) \sqrt{b + a \cos[e + f x]} \sqrt{d + c \cos[e + f x]} \right) - \left. \begin{array}{l} (b c + a d) \\ \sqrt{\frac{(c + d) \cot[\frac{1}{2}(e + f x)]^2}{c - d}} \sqrt{\frac{(c + d)(b + a \cos[e + f x]) \csc[\frac{1}{2}(e + f x)]^2}{b c - a d}} \\ \sqrt{-\frac{(a + b)(d + c \cos[e + f x]) \csc[\frac{1}{2}(e + f x)]^2}{b c - a d}} \csc[e + f x] \end{array} \right. \\
& \text{EllipticPi}\left[\frac{b c - a d}{(a + b)c}, \text{ArcSin}\left[\frac{\sqrt{-\frac{(a+b)(d+c \cos[e+f x]) \csc[\frac{1}{2}(e+f x)]^2}{b c - a d}}}{\sqrt{2}}\right]\right], \\
& \left. \frac{2(b c - a d)}{(a + b)(c - d)} \sin[\frac{1}{2}(e + f x)]^4 \right\} / \left((a + b)c \sqrt{b + a \cos[e + f x]} \right. \\
& \left. \left. \sqrt{d + c \cos[e + f x]}\right) \right) + \left. \frac{\sqrt{d + c \cos[e + f x]} \sin[e + f x]}{c \sqrt{b + a \cos[e + f x]}} \right\}
\end{aligned}$$

Problem 215: Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b \sec[e + f x])^{5/2}}{(c + d \sec[e + f x])^{7/2}} dx$$

Optimal (type 4, 1150 leaves, 8 steps):

$$\begin{aligned}
& \left(2 (a-b) \sqrt{a+b} (b^2 c^2 d (29 c^2 + 3 d^2) - a b c (35 c^4 + 34 c^2 d^2 - 5 d^4) + a^2 (58 c^4 d - 41 c^2 d^3 + 15 d^5)) \right. \\
& \quad \left. \sqrt{-\frac{(b c - a d) (1 - \cos[e + f x])}{(a+b) (d + c \cos[e + f x])}} \sqrt{-\frac{(b c - a d) (1 + \cos[e + f x])}{(a-b) (d + c \cos[e + f x])}} \right. \\
& \quad \left. (d + c \cos[e + f x])^{3/2} \csc[e + f x] \right. \\
& \quad \left. \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{c+d} \sqrt{b+a \cos[e+f x]}}{\sqrt{a+b} \sqrt{d+c \cos[e+f x]}}\right], \frac{(a+b) (c-d)}{(a-b) (c+d)}\right] \sqrt{a+b \sec[e+f x]} \right) / \\
& \quad \left(15 c^3 (c-d)^3 (c+d)^{5/2} (b c - a d) f \sqrt{b+a \cos[e+f x]} \sqrt{c+d \sec[e+f x]} \right) + \\
& \quad \left(2 \sqrt{a+b} (b^3 c^4 (5 c^2 + 24 c d + 3 d^2) - a b^2 c^3 (35 c^3 + 42 c^2 d + 21 c d^2 - 2 d^3) + \right. \\
& \quad \left. a^2 b c^2 (45 c^4 + 48 c^3 d + c^2 d^2 - 8 c d^3 + 10 d^4) - \right. \\
& \quad \left. a^3 d (60 c^5 - 2 c^4 d - 66 c^3 d^2 + 25 c^2 d^3 + 30 c d^4 - 15 d^5) \right) \sqrt{-\frac{(b c - a d) (1 - \cos[e + f x])}{(a+b) (d + c \cos[e + f x])}} \\
& \quad \left. (d + c \cos[e + f x])^{3/2} \csc[e + f x] \right. \\
& \quad \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{c+d} \sqrt{b+a \cos[e+f x]}}{\sqrt{a+b} \sqrt{d+c \cos[e+f x]}}\right], \frac{(a+b) (c-d)}{(a-b) (c+d)}\right] \sqrt{a+b \sec[e+f x]} \right) / \\
& \quad \left(15 c^4 (c-d)^3 (c+d)^{5/2} (b c - a d) f \sqrt{b+a \cos[e+f x]} \sqrt{c+d \sec[e+f x]} \right) - \\
& \quad \left(2 a^2 \sqrt{a+b} \sqrt{-\frac{(b c - a d) (1 - \cos[e + f x])}{(a+b) (d + c \cos[e + f x])}} \sqrt{-\frac{(b c - a d) (1 + \cos[e + f x])}{(a-b) (d + c \cos[e + f x])}} \right. \\
& \quad \left. (d + c \cos[e + f x])^{3/2} \csc[e + f x] \right. \\
& \quad \left. \text{EllipticPi}\left[\frac{(a+b) c}{a (c+d)}, \text{ArcSin}\left[\frac{\sqrt{c+d} \sqrt{b+a \cos[e+f x]}}{\sqrt{a+b} \sqrt{d+c \cos[e+f x]}}\right], \frac{(a+b) (c-d)}{(a-b) (c+d)}\right] \sqrt{a+b \sec[e+f x]} \right) / \\
& \quad \left(c^4 \sqrt{c+d} f \sqrt{b+a \cos[e+f x]} \sqrt{c+d \sec[e+f x]} \right) - \\
& \quad \frac{2 d (b c - a d) (b + a \cos[e + f x]) \sqrt{a+b \sec[e+f x]} \sin[e+f x]}{5 c (c^2 - d^2) f (d + c \cos[e + f x])^2 \sqrt{c+d \sec[e+f x]}} + \\
& \quad \left(2 (b c - a d) (5 b c^3 - 13 a c^2 d + 3 b c d^2 + 5 a d^3) \sqrt{a+b \sec[e+f x]} \sin[e+f x] \right) / \\
& \quad \left(15 c^2 (c^2 - d^2)^2 f (d + c \cos[e + f x]) \sqrt{c+d \sec[e+f x]} \right)
\end{aligned}$$

Result (type 4, 2314 leaves):

$$\begin{aligned}
& \frac{1}{f (b + a \cos[e + f x])^2 (c + d \sec[e + f x])^{7/2}} (d + c \cos[e + f x])^4 \sec[e + f x] (a + b \sec[e + f x])^{5/2} \\
& \left(- \left((2 (b^2 c^2 d \sin[e + f x] - 2 a b c d^2 \sin[e + f x] + a^2 d^3 \sin[e + f x])) \right) / \right. \\
& \quad \left(5 c^2 (c^2 - d^2) (d + c \cos[e + f x])^3 \right) + \\
& \quad \left(2 (5 b^2 c^4 \sin[e + f x] - 21 a b c^3 d \sin[e + f x] + 16 a^2 c^2 d^2 \sin[e + f x] + \right. \\
& \quad \left. \left. 3 b^2 c^2 d^2 \sin[e + f x] + 5 a b c d^3 \sin[e + f x] - 8 a^2 d^4 \sin[e + f x]) \right) / \right. \\
& \quad \left(15 c^2 (c^2 - d^2)^2 (d + c \cos[e + f x])^2 \right) + \\
& \quad \left(2 (35 a b c^5 \sin[e + f x] - 58 a^2 c^4 d \sin[e + f x] - 29 b^2 c^4 d \sin[e + f x] + \right. \\
& \quad \left. 34 a b c^3 d^2 \sin[e + f x] + 41 a^2 c^2 d^3 \sin[e + f x] - 3 b^2 c^2 d^3 \sin[e + f x] - \right. \\
& \quad \left. 5 a b c d^4 \sin[e + f x] - 15 a^2 d^5 \sin[e + f x]) \right) / \left(15 c^2 (c^2 - d^2)^3 (d + c \cos[e + f x]) \right) + \\
& \frac{1}{15 c^2 (c - d)^3 (c + d)^3 f (b + a \cos[e + f x])^{5/2} (c + d \sec[e + f x])^{7/2}} \\
& \quad (d + c \cos[e + f x])^{7/2} \\
& \quad \sec[e + f x] \\
& \quad (a + b \sec[e + f x])^{5/2} \\
& \left(\frac{1}{(a + b) (c + d) \sqrt{b + a \cos[e + f x]} \sqrt{d + c \cos[e + f x]}} \right. \\
& \quad \left. \sqrt{\frac{(c + d) \cot[\frac{1}{2} (e + f x)]^2}{c - d}} \sqrt{\frac{(c + d) (b + a \cos[e + f x]) \csc[\frac{1}{2} (e + f x)]^2}{b c - a d}} \right. \\
& \quad \left. - \frac{(a + b) (d + c \cos[e + f x]) \csc[\frac{1}{2} (e + f x)]^2}{b c - a d} \csc[e + f x] \text{EllipticF}[\right. \\
& \quad \left. \text{ArcSin}\left[\sqrt{-\frac{(a+b) (d+c \cos[e+f x]) \csc[\frac{1}{2} (e+f x)]^2}{b c - a d}}\right], \frac{2 (b c - a d)}{(a + b) (c - d)} \right] \sin[\frac{1}{2} (e + f x)]^4 + \\
& \quad 4 (b c - a d) (15 a^3 c^5 - 35 a b^2 c^5 + 23 a^2 b c^4 d + 29 b^3 c^4 d + 13 a^3 c^3 d^2 - 5 a b^2 c^3 d^2 - \\
& \quad 27 b^3 c^3 d^2 - 18 a^3 c^2 d^3 - 16 a b^2 c^2 d^3 + 7 a^2 b c d^4 + 5 a^3 d^5) \\
& \quad \sqrt{\frac{(c + d) \cot[\frac{1}{2} (e + f x)]^2}{c - d}} \sqrt{\frac{(c + d) (b + a \cos[e + f x]) \csc[\frac{1}{2} (e + f x)]^2}{b c - a d}} \\
& \quad - \frac{(a + b) (d + c \cos[e + f x]) \csc[\frac{1}{2} (e + f x)]^2}{b c - a d} \csc[e + f x] \text{EllipticF}[\\
& \quad \text{ArcSin}\left[\sqrt{-\frac{(a+b) (d+c \cos[e+f x]) \csc[\frac{1}{2} (e+f x)]^2}{b c - a d}}\right], \frac{2 (b c - a d)}{(a + b) (c - d)} \right] \sin[\frac{1}{2} (e + f x)]^4 + \\
& \quad 4 (b c - a d) (15 a^3 c^5 - 35 a b^2 c^5 + 23 a^2 b c^4 d + 29 b^3 c^4 d + 13 a^3 c^3 d^2 - 5 a b^2 c^3 d^2 - \\
& \quad 75 a^2 b c^2 d^3 + 3 b^3 c^2 d^3 + 4 a^3 c d^4 + 8 a b^2 c d^4 + 20 a^2 b d^5) \\
& \left(\sqrt{\frac{(c + d) \cot[\frac{1}{2} (e + f x)]^2}{c - d}} \sqrt{\frac{(c + d) (b + a \cos[e + f x]) \csc[\frac{1}{2} (e + f x)]^2}{b c - a d}} \right. \\
& \quad \left. - \frac{(a + b) (d + c \cos[e + f x]) \csc[\frac{1}{2} (e + f x)]^2}{b c - a d} \csc[e + f x] \text{EllipticF}[\right.
\end{aligned}$$

$$\begin{aligned}
& \left. \left(\text{ArcSin} \left[\frac{\sqrt{-\frac{(a+b)(d+c \cos[e+f x]) \csc[\frac{1}{2}(e+f x)]^2}{b c - a d}}}{\sqrt{2}} \right], \frac{2(b c - a d)}{(a+b)(c-d)} \right] \sin \left[\frac{1}{2}(e+f x) \right]^4 \right) / \\
& \left((a+b)(c+d) \sqrt{b + a \cos[e+f x]} \sqrt{d + c \cos[e+f x]} \right) - \\
& \left(\sqrt{\frac{(c+d) \cot[\frac{1}{2}(e+f x)]^2}{c-d}} \sqrt{\frac{(c+d)(b+a \cos[e+f x]) \csc[\frac{1}{2}(e+f x)]^2}{b c - a d}} \right. \\
& \left. \sqrt{-\frac{(a+b)(d+c \cos[e+f x]) \csc[\frac{1}{2}(e+f x)]^2}{b c - a d}} \csc[e+f x] \right. \\
& \text{EllipticPi} \left[\frac{b c - a d}{(a+b)c}, \text{ArcSin} \left[\frac{\sqrt{-\frac{(a+b)(d+c \cos[e+f x]) \csc[\frac{1}{2}(e+f x)]^2}{b c - a d}}}{\sqrt{2}} \right], \frac{2(b c - a d)}{(a+b)(c-d)} \right] \\
& \left. \sin \left[\frac{1}{2}(e+f x) \right]^4 \right) / \left((a+b)c \sqrt{b + a \cos[e+f x]} \sqrt{d + c \cos[e+f x]} \right) \Bigg) + \\
& 2(-35 a^2 b c^5 + 58 a^3 c^4 d + 29 a b^2 c^4 d - 34 a^2 b c^3 d^2 - 41 a^3 c^2 d^3 + 3 a b^2 c^2 d^3 + \\
& 5 a^2 b c d^4 + 15 a^3 d^5) \left(\sqrt{\frac{-a+b}{a+b}} (a+b) \cos \left[\frac{1}{2}(e+f x) \right] \sqrt{d + c \cos[e+f x]} \right. \\
& \text{EllipticE} \left[\text{ArcSin} \left[\frac{\sqrt{\frac{-a+b}{a+b}} \sin \left[\frac{1}{2}(e+f x) \right]}{\sqrt{\frac{b+a \cos[e+f x]}{a+b}}} \right], \frac{2(b c - a d)}{(-a+b)(c+d)} \right] \Bigg) / \\
& \left(a c \sqrt{\frac{(a+b) \cos[\frac{1}{2}(e+f x)]^2}{b + a \cos[e+f x]}} \sqrt{\frac{b + a \cos[e+f x]}{a+b}} \right. \\
& \left. \sqrt{\frac{(a+b)(d+c \cos[e+f x])}{(c+d)(b+a \cos[e+f x])}} \right) - \frac{1}{a c} 2(b c - a d) \left(\begin{array}{l} (b c + (a+b)d) \end{array} \right)
\end{aligned}$$

$$\begin{aligned}
& \sqrt{\frac{(c+d) \cot[\frac{1}{2}(e+f x)]^2}{c-d}} \sqrt{\frac{(c+d)(b+a \cos[e+f x]) \csc[\frac{1}{2}(e+f x)]^2}{b c-a d}} \\
& \sqrt{-\frac{(a+b)(d+c \cos[e+f x]) \csc[\frac{1}{2}(e+f x)]^2}{b c-a d}} \csc[e+f x] \text{EllipticF}[\text{ArcSin}\left[\\
& \frac{\sqrt{-\frac{(a+b)(d+c \cos[e+f x]) \csc[\frac{1}{2}(e+f x)]^2}{b c-a d}}}{\sqrt{2}}, \frac{2(b c-a d)}{(a+b)(c-d)}\right] \sin[\frac{1}{2}(e+f x)]^4\right] / \\
& \left((a+b)(c+d) \sqrt{b+a \cos[e+f x]} \sqrt{d+c \cos[e+f x]} \right) - \left(b c+a d \right) \\
& \sqrt{\frac{(c+d) \cot[\frac{1}{2}(e+f x)]^2}{c-d}} \sqrt{\frac{(c+d)(b+a \cos[e+f x]) \csc[\frac{1}{2}(e+f x)]^2}{b c-a d}} \\
& \sqrt{-\frac{(a+b)(d+c \cos[e+f x]) \csc[\frac{1}{2}(e+f x)]^2}{b c-a d}} \csc[e+f x] \\
& \text{EllipticPi}\left[\frac{b c-a d}{(a+b)c}, \text{ArcSin}\left[\frac{\sqrt{-\frac{(a+b)(d+c \cos[e+f x]) \csc[\frac{1}{2}(e+f x)]^2}{b c-a d}}}{\sqrt{2}}\right], \right. \\
& \left. \frac{2(b c-a d)}{(a+b)(c-d)}\right] \sin[\frac{1}{2}(e+f x)]^4\right] / \left((a+b)c \sqrt{b+a \cos[e+f x]}\right. \\
& \left. \sqrt{d+c \cos[e+f x]}\right) + \left. \frac{\sqrt{d+c \cos[e+f x]} \sin[e+f x]}{c \sqrt{b+a \cos[e+f x]}}\right)
\end{aligned}$$

Problem 216: Result more than twice size of optimal antiderivative.

$$\int \frac{(a+b \sec[e+f x])^{5/2}}{(c+d \sec[e+f x])^{9/2}} dx$$

Optimal (type 4, 1428 leaves, 9 steps):

$$\begin{aligned}
& \left(2 (a-b) \sqrt{a+b} (2 b^3 c^3 d (133 c^4 + 62 c^2 d^2 - 3 d^4) + 2 a^2 b c d (406 c^6 + 73 c^4 d^2 + 132 c^2 d^4 - 35 d^6) - \right. \\
& \quad a b^2 c^2 (245 c^6 + 852 c^4 d^2 + 41 c^2 d^4 + 14 d^6) - a^3 (582 c^6 d^2 - 485 c^4 d^4 + 392 c^2 d^6 - 105 d^8)) \\
& \quad \sqrt{-\frac{(b c - a d) (1 - \cos[e + f x])}{(a+b) (d + c \cos[e + f x])}} \sqrt{-\frac{(b c - a d) (1 + \cos[e + f x])}{(a-b) (d + c \cos[e + f x])}} \\
& \quad (d + c \cos[e + f x])^{3/2} \csc[e + f x] \\
& \quad \text{EllipticE}[\text{ArcSin}\left[\frac{\sqrt{c+d} \sqrt{b+a \cos[e+f x]}}{\sqrt{a+b} \sqrt{d+c \cos[e+f x]}}\right], \frac{(a+b) (c-d)}{(a-b) (c+d)}] \sqrt{a+b \sec[e+f x]} \Big) / \\
& \quad (105 c^4 (c-d)^4 (c+d)^{7/2} (b c - a d)^2 f \sqrt{b+a \cos[e+f x]} \sqrt{c+d \sec[e+f x]}) + \\
& \quad \left(2 \sqrt{a+b} (b^3 c^4 (35 c^4 + 231 c^3 d + 67 c^2 d^2 + 57 c d^3 - 6 d^4) - \right. \\
& \quad a b^2 c^3 (245 c^5 + 413 c^4 d + 439 c^3 d^2 + 53 c^2 d^3 - 12 c d^4 + 14 d^5) + \\
& \quad a^2 b c^2 (315 c^6 + 497 c^5 d + 219 c^4 d^2 - 73 c^3 d^3 + 208 c^2 d^4 + 56 c d^5 - 70 d^6) - \\
& \quad a^3 d (525 c^7 + 57 c^6 d - 699 c^5 d^2 + 214 c^4 d^3 + 672 c^3 d^4 - 280 c^2 d^5 - 210 c d^6 + 105 d^7)) \\
& \quad \sqrt{-\frac{(b c - a d) (1 - \cos[e + f x])}{(a+b) (d + c \cos[e + f x])}} \sqrt{-\frac{(b c - a d) (1 + \cos[e + f x])}{(a-b) (d + c \cos[e + f x])}} \\
& \quad (d + c \cos[e + f x])^{3/2} \csc[e + f x] \\
& \quad \text{EllipticF}[\text{ArcSin}\left[\frac{\sqrt{c+d} \sqrt{b+a \cos[e+f x]}}{\sqrt{a+b} \sqrt{d+c \cos[e+f x]}}\right], \frac{(a+b) (c-d)}{(a-b) (c+d)}] \sqrt{a+b \sec[e+f x]} \Big) / \\
& \quad (105 c^5 (c-d)^4 (c+d)^{7/2} (b c - a d) f \sqrt{b+a \cos[e+f x]} \sqrt{c+d \sec[e+f x]}) - \\
& \quad \left(2 a^2 \sqrt{a+b} \sqrt{-\frac{(b c - a d) (1 - \cos[e + f x])}{(a+b) (d + c \cos[e + f x])}} \sqrt{-\frac{(b c - a d) (1 + \cos[e + f x])}{(a-b) (d + c \cos[e + f x])}} \right. \\
& \quad (d + c \cos[e + f x])^{3/2} \csc[e + f x] \text{EllipticPi}\left[\frac{(a+b) c}{a (c+d)}, \right. \\
& \quad \text{ArcSin}\left[\frac{\sqrt{c+d} \sqrt{b+a \cos[e+f x]}}{\sqrt{a+b} \sqrt{d+c \cos[e+f x]}}\right], \frac{(a+b) (c-d)}{(a-b) (c+d)}] \sqrt{a+b \sec[e+f x]} \Big) / \\
& \quad (c^5 \sqrt{c+d} f \sqrt{b+a \cos[e+f x]} \sqrt{c+d \sec[e+f x]}) + \\
& \quad \frac{2 d^2 (b + a \cos[e + f x])^2 \sqrt{a + b \sec[e + f x]} \sin[e + f x]}{7 c (c^2 - d^2) f (d + c \cos[e + f x])^3 \sqrt{c + d \sec[e + f x]}} - \\
& \quad \left(2 d (14 b c^3 - 19 a c^2 d - 2 b c d^2 + 7 a d^3) (b + a \cos[e + f x]) \sqrt{a + b \sec[e + f x]} \sin[e + f x] \right) / \\
& \quad \left(35 c^2 (c^2 - d^2)^2 f (d + c \cos[e + f x])^2 \sqrt{c + d \sec[e + f x]} \right) - \\
& \quad \left(2 (2 a b c d (91 c^4 - 2 c^2 d^2 + 7 d^4) - a^2 d^2 (162 c^4 - 101 c^2 d^2 + 35 d^4) - b^2 (35 c^6 + 67 c^4 d^2 - 6 c^2 d^4)) \right)
\end{aligned}$$

$$\frac{\sqrt{a+b \sec[e+f x]} \sin[e+f x]}{(105 c^3 (c^2-d^2)^3 f (d+c \cos[e+f x]) \sqrt{c+d \sec[e+f x]})} /$$

Result (type 4, 2949 leaves):

$$\begin{aligned} & \frac{1}{f (b+a \cos[e+f x])^2 (c+d \sec[e+f x])^{9/2}} (d+c \cos[e+f x])^5 \sec[e+f x]^2 \\ & \left((a+b \sec[e+f x])^{5/2} \left((2 (b^2 c^2 d^2 \sin[e+f x] - 2 a b c d^3 \sin[e+f x] + a^2 d^4 \sin[e+f x])) / \right. \right. \\ & \left. \left. (7 c^3 (c^2-d^2) (d+c \cos[e+f x])^4) + \right. \right. \\ & \left. \left. (2 (-14 b^2 c^4 d \sin[e+f x] + 43 a b c^3 d^2 \sin[e+f x] - 29 a^2 c^2 d^3 \sin[e+f x] + \right. \right. \\ & \left. \left. 2 b^2 c^2 d^3 \sin[e+f x] - 19 a b c d^4 \sin[e+f x] + 17 a^2 d^5 \sin[e+f x])) / \right. \right. \\ & \left. \left. (35 c^3 (c^2-d^2)^2 (d+c \cos[e+f x])^3) + (2 (35 b^2 c^6 \sin[e+f x] - 224 a b c^5 d \sin[e+f x] + \right. \right. \\ & \left. \left. 234 a^2 c^4 d^2 \sin[e+f x] + 67 b^2 c^4 d^2 \sin[e+f x] + 52 a b c^3 d^3 \sin[e+f x] - 209 a^2 c^2 d^4 \right. \right. \\ & \left. \left. \sin[e+f x] - 6 b^2 c^2 d^4 \sin[e+f x] - 20 a b c d^5 \sin[e+f x] + 71 a^2 d^6 \sin[e+f x])) / \right. \right. \\ & \left. \left. (105 c^3 (c^2-d^2)^3 (d+c \cos[e+f x])^2) + \frac{1}{105 c^3 (b c-a d) (c^2-d^2)^4 (d+c \cos[e+f x])} \right. \right. \\ & \left. \left. 2 (245 a b^2 c^8 \sin[e+f x] - 812 a^2 b c^7 d \sin[e+f x] - 266 b^3 c^7 d \sin[e+f x] + \right. \right. \\ & \left. \left. 582 a^3 c^6 d^2 \sin[e+f x] + 852 a b^2 c^6 d^2 \sin[e+f x] - 146 a^2 b c^5 d^3 \sin[e+f x] - \right. \right. \\ & \left. \left. 124 b^3 c^5 d^3 \sin[e+f x] - 485 a^3 c^4 d^4 \sin[e+f x] + 41 a b^2 c^4 d^4 \sin[e+f x] - \right. \right. \\ & \left. \left. 264 a^2 b c^3 d^5 \sin[e+f x] + 6 b^3 c^3 d^5 \sin[e+f x] + 392 a^3 c^2 d^6 \sin[e+f x] + \right. \right. \\ & \left. \left. 14 a b^2 c^2 d^6 \sin[e+f x] + 70 a^2 b c d^7 \sin[e+f x] - 105 a^3 d^8 \sin[e+f x]) \right) + \right. \\ & \left. \left((d+c \cos[e+f x])^{9/2} \sec[e+f x]^2 (a+b \sec[e+f x])^{5/2} \right. \right. \\ & \left. \left. \left(\frac{1}{(a+b) (c+d) \sqrt{b+a \cos[e+f x]} \sqrt{d+c \cos[e+f x]}} \right. \right. \right. \\ & \left. \left. \left. 4 (b c-a d) (-70 a^2 b^2 c^8 - 35 b^4 c^8 - 77 a^3 b c^7 d + 427 a b^3 c^7 d + 162 a^4 c^6 d^2 - 522 a^2 b^2 c^6 d^2 - \right. \right. \\ & \left. \left. 298 b^4 c^6 d^2 + 348 a^3 b c^5 d^3 + 666 a b^3 c^5 d^3 - 263 a^4 c^4 d^4 - 586 a^2 b^2 c^4 d^4 - 51 b^4 c^4 d^4 + \right. \right. \\ & \left. \left. 127 a^3 b c^3 d^5 + 59 a b^3 c^3 d^5 + 136 a^4 c^2 d^6 + 26 a^2 b^2 c^2 d^6 - 14 a^3 b c d^7 - 35 a^4 d^8) \right. \right. \\ & \left. \left. \sqrt{\frac{(c+d) \cot[\frac{1}{2} (e+f x)]^2}{c-d}} \sqrt{\frac{(c+d) (b+a \cos[e+f x]) \csc[\frac{1}{2} (e+f x)]^2}{b c-a d}} \right. \right. \\ & \left. \left. \sqrt{-\frac{(a+b) (d+c \cos[e+f x]) \csc[\frac{1}{2} (e+f x)]^2}{b c-a d}} \csc[e+f x] \text{EllipticF}[\right. \right. \right. \end{aligned}$$

$$\begin{aligned}
& \text{ArcSin} \left[\frac{\sqrt{-\frac{(a+b)(d+c \cos[e+f x]) \csc[\frac{1}{2}(e+f x)]^2}{b c - a d}}}{\sqrt{2}}, \frac{2(b c - a d)}{(a+b)(c-d)} \right] \sin \left[\frac{1}{2}(e+f x) \right]^4 + \\
& 4(b c - a d) (-105 a^3 b c^8 + 245 a b^3 c^8 + 105 a^4 c^7 d - 567 a^2 b^2 c^7 d - 266 b^4 c^7 d + \\
& 190 a^3 b c^6 d^2 + 586 a b^3 c^6 d^2 + 162 a^4 c^5 d^3 + 706 a^2 b^2 c^5 d^3 - 124 b^4 c^5 d^3 - \\
& 1261 a^3 b c^4 d^4 - 83 a b^3 c^4 d^4 + 145 a^4 c^3 d^5 - 223 a^2 b^2 c^3 d^5 + 6 b^4 c^3 d^5 + \\
& 548 a^3 b c^2 d^6 + 20 a b^3 c^2 d^6 - 28 a^4 c d^7 + 84 a^2 b^2 c d^7 - 140 a^3 b d^8) \\
& \left(\left(\sqrt{\frac{(c+d) \cot[\frac{1}{2}(e+f x)]^2}{c-d}} \sqrt{\frac{(c+d)(b+a \cos[e+f x]) \csc[\frac{1}{2}(e+f x)]^2}{b c - a d}} \right. \right. \\
& \left. \left. \sqrt{-\frac{(a+b)(d+c \cos[e+f x]) \csc[\frac{1}{2}(e+f x)]^2}{b c - a d}} \csc[e+f x] \text{EllipticF} \right. \right. \\
& \left. \left. \text{ArcSin} \left[\frac{\sqrt{-\frac{(a+b)(d+c \cos[e+f x]) \csc[\frac{1}{2}(e+f x)]^2}{b c - a d}}}{\sqrt{2}}, \frac{2(b c - a d)}{(a+b)(c-d)} \right] \sin \left[\frac{1}{2}(e+f x) \right]^4 \right) \right) \\
& \left((a+b)(c+d) \sqrt{b+a \cos[e+f x]} \sqrt{d+c \cos[e+f x]} \right) - \\
& \left(\sqrt{\frac{(c+d) \cot[\frac{1}{2}(e+f x)]^2}{c-d}} \sqrt{\frac{(c+d)(b+a \cos[e+f x]) \csc[\frac{1}{2}(e+f x)]^2}{b c - a d}} \right. \\
& \left. \sqrt{-\frac{(a+b)(d+c \cos[e+f x]) \csc[\frac{1}{2}(e+f x)]^2}{b c - a d}} \csc[e+f x] \right. \\
& \left. \text{EllipticPi} \left[\frac{b c - a d}{(a+b)c}, \text{ArcSin} \left[\frac{\sqrt{-\frac{(a+b)(d+c \cos[e+f x]) \csc[\frac{1}{2}(e+f x)]^2}{b c - a d}}}{\sqrt{2}}, \frac{2(b c - a d)}{(a+b)(c-d)} \right], \right. \right. \\
& \left. \left. \sin \left[\frac{1}{2}(e+f x) \right]^4 \right) \right) / \left((a+b)c \sqrt{b+a \cos[e+f x]} \sqrt{d+c \cos[e+f x]} \right) + \\
& 2(245 a^2 b^2 c^8 - 812 a^3 b c^7 d - 266 a b^3 c^7 d + 582 a^4 c^6 d^2 + 852 a^2 b^2 c^6 d^2 - \\
& 146 a^3 b c^5 d^3 - 124 a b^3 c^5 d^3 - 485 a^4 c^4 d^4 + 41 a^2 b^2 c^4 d^4 - 264 a^3 b c^3 d^5 + \\
& 6 a b^3 c^3 d^5 + 392 a^4 c^2 d^6 + 14 a^2 b^2 c^2 d^6 + 70 a^3 b c d^7 - 105 a^4 d^8)
\end{aligned}$$

$$\begin{aligned}
& \left(\sqrt{\frac{-a+b}{a+b}} (a+b) \cos \left[\frac{1}{2} (e+f x) \right] \sqrt{d+c \cos [e+f x]} \right. \\
& \left. \text{EllipticE} \left[\text{ArcSin} \left[\frac{\sqrt{\frac{-a+b}{a+b}} \sin \left[\frac{1}{2} (e+f x) \right]}{\sqrt{\frac{b+a \cos [e+f x]}{a+b}}} \right], \frac{2 (b c - a d)}{(-a+b) (c+d)} \right] \right) / \\
& \left(a c \sqrt{\frac{(a+b) \cos \left[\frac{1}{2} (e+f x) \right]^2}{b+a \cos [e+f x]}} \sqrt{\frac{b+a \cos [e+f x]}{a+b}} \right. \\
& \left. \sqrt{\frac{(a+b) (d+c \cos [e+f x])}{(c+d) (b+a \cos [e+f x])}} \right) - \frac{1}{a c} 2 (b c - a d) \left(\begin{array}{l} (b c + (a+b) d) \\
\sqrt{\frac{(c+d) \cot \left[\frac{1}{2} (e+f x) \right]^2}{c-d}} \sqrt{\frac{(c+d) (b+a \cos [e+f x]) \csc \left[\frac{1}{2} (e+f x) \right]^2}{b c - a d}} \\
\sqrt{-\frac{(a+b) (d+c \cos [e+f x]) \csc \left[\frac{1}{2} (e+f x) \right]^2}{b c - a d}} \csc [e+f x] \\
\text{EllipticF} \left[\text{ArcSin} \left[\sqrt{-\frac{(a+b) (d+c \cos [e+f x]) \csc \left[\frac{1}{2} (e+f x) \right]^2}{b c - a d}} \right], \frac{2 (b c - a d)}{(a+b) (c-d)} \right] \\
\sin \left[\frac{1}{2} (e+f x) \right]^4 \right) / \left((a+b) (c+d) \sqrt{b+a \cos [e+f x]} \right. \\
& \left. \sqrt{d+c \cos [e+f x]} \right) - \left(b c + a d \right) \sqrt{\frac{(c+d) \cot \left[\frac{1}{2} (e+f x) \right]^2}{c-d}} \\
& \sqrt{\frac{(c+d) (b+a \cos [e+f x]) \csc \left[\frac{1}{2} (e+f x) \right]^2}{b c - a d}}
\end{aligned}$$

Problem 217: Unable to integrate problem.

$$\int \frac{(c + d \operatorname{Sec}[e + f x])^{3/2}}{\sqrt{a + b \operatorname{Sec}[e + f x]}} dx$$

Optimal (type 4, 652 leaves, ? steps):

$$\begin{aligned}
& - \left(\left(2 c (c+d) \operatorname{Cot}[e+f x] \operatorname{EllipticPi} \left[\frac{a (c+d)}{(a+b) c}, \operatorname{ArcSin} \left[\sqrt{\frac{(a+b) (c+d \sec[e+f x])}{(c+d) (a+b \sec[e+f x])}} \right], \right. \right. \right. \\
& \left. \left. \left. \frac{(a-b) (c+d)}{(a+b) (c-d)} \right] \sqrt{\frac{(b c-a d) (1+\sec[e+f x])}{(c-d) (a+b \sec[e+f x])}} (a+b \sec[e+f x])^{3/2} \right. \right. \\
& \left. \left. \left. \sqrt{\frac{(a+b) (b c-a d) (-1+\sec[e+f x]) (c+d \sec[e+f x])}{(c+d)^2 (a+b \sec[e+f x])^2}} \right] \right) / \\
& \left(a (a+b) f \sqrt{c+d \sec[e+f x]} \right) + \left(2 d (c+d) \operatorname{Cot}[e+f x] \right. \\
& \left. \operatorname{EllipticPi} \left[\frac{b (c+d)}{(a+b) d}, \operatorname{ArcSin} \left[\sqrt{\frac{(a+b) (c+d \sec[e+f x])}{(c+d) (a+b \sec[e+f x])}} \right], \frac{(a-b) (c+d)}{(a+b) (c-d)} \right] \right. \\
& \left. \sqrt{\frac{(b c-a d) (1+\sec[e+f x])}{(c-d) (a+b \sec[e+f x])}} (a+b \sec[e+f x])^{3/2} \right. \\
& \left. \left. \sqrt{-\frac{(a+b) (-b c+a d) (-1+\sec[e+f x]) (c+d \sec[e+f x])}{(c+d)^2 (a+b \sec[e+f x])^2}} \right] \right) / \\
& \left(b (a+b) f \sqrt{c+d \sec[e+f x]} \right) + \\
& \left(2 (b c-a d) \operatorname{Cot}[e+f x] \operatorname{EllipticF}[\operatorname{ArcSin} \left[\sqrt{\frac{(a+b) (c+d \sec[e+f x])}{(c+d) (a+b \sec[e+f x])}} \right], \frac{(a-b) (c+d)}{(a+b) (c-d)} \right. \\
& \left. \sqrt{\frac{(b c-a d) (-1+\sec[e+f x])}{(c+d) (a+b \sec[e+f x])}} \sqrt{\frac{(b c-a d) (1+\sec[e+f x])}{(c-d) (a+b \sec[e+f x])}} \right. \\
& \left. \sqrt{a+b \sec[e+f x]} \sqrt{c+d \sec[e+f x]} \right) / \left(a b f \sqrt{\frac{(a+b) (c+d \sec[e+f x])}{(c+d) (a+b \sec[e+f x])}} \right)
\end{aligned}$$

Result (type 8, 31 leaves):

$$\int \frac{(c+d \sec[e+f x])^{3/2}}{\sqrt{a+b \sec[e+f x]}} dx$$

Problem 218: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{c+d \sec[e+f x]}}{\sqrt{a+b \sec[e+f x]}} dx$$

Optimal (type 4, 198 leaves, 1 step):

$$\begin{aligned}
& -\frac{1}{a \sqrt{c+d} f} \\
& 2 \sqrt{a+b} \operatorname{Cot}[e+f x] \operatorname{EllipticPi}\left[\frac{(a+b) c}{a (c+d)}, \operatorname{ArcSin}\left[\frac{\sqrt{c+d} \sqrt{a+b} \operatorname{Sec}[e+f x]}{\sqrt{a+b} \sqrt{c+d} \operatorname{Sec}[e+f x]}\right], \frac{(a+b) (c-d)}{(a-b) (c+d)}\right] \\
& \sqrt{\frac{(b c-a d) (1-\operatorname{Sec}[e+f x])}{(a+b) (c+d \operatorname{Sec}[e+f x])}} \sqrt{-\frac{(b c-a d) (1+\operatorname{Sec}[e+f x])}{(a-b) (c+d \operatorname{Sec}[e+f x])}} (c+d \operatorname{Sec}[e+f x])
\end{aligned}$$

Result (type 4, 554 leaves):

$$\begin{aligned}
& \frac{1}{f \sqrt{d+c \cos[e+f x]} \sqrt{a+b \sec[e+f x]}} 4 (-b c + a d) \sqrt{b+a \cos[e+f x]} \sqrt{c+d \sec[e+f x]} \\
& \left(\sqrt{\frac{(a+b) \cot[\frac{1}{2} (e+f x)]^2}{a-b}} \sqrt{-\frac{(c+d) (b+a \cos[e+f x]) \csc[\frac{1}{2} (e+f x)]^2}{-b c + a d}} \right. \\
& \sqrt{\frac{(a+b) (d+c \cos[e+f x]) \csc[\frac{1}{2} (e+f x)]^2}{-b c + a d}} \csc[e+f x] \\
& \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{-\frac{(c+d) (b+a \cos[e+f x]) \csc[\frac{1}{2} (e+f x)]^2}{-b c + a d}}}{\sqrt{2}}\right], \frac{2 (-b c + a d)}{(a-b) (c+d)}\right] \\
& \left. \sin\left[\frac{1}{2} (e+f x)\right]^4 \right) / \left((a+b) \sqrt{b+a \cos[e+f x]} \sqrt{d+c \cos[e+f x]} \right) - \\
& \left(c \sqrt{\frac{(a+b) \cot[\frac{1}{2} (e+f x)]^2}{a-b}} \sqrt{-\frac{(c+d) (b+a \cos[e+f x]) \csc[\frac{1}{2} (e+f x)]^2}{-b c + a d}} \right. \\
& \sqrt{\frac{(a+b) (d+c \cos[e+f x]) \csc[\frac{1}{2} (e+f x)]^2}{-b c + a d}} \csc[e+f x] \\
& \text{EllipticPi}\left[\frac{-b c + a d}{a (c+d)}, \text{ArcSin}\left[\frac{\sqrt{-\frac{(c+d) (b+a \cos[e+f x]) \csc[\frac{1}{2} (e+f x)]^2}{-b c + a d}}}{\sqrt{2}}\right], \frac{2 (-b c + a d)}{(a-b) (c+d)}\right] \\
& \left. \sin\left[\frac{1}{2} (e+f x)\right]^4 \right) / \left(a (c+d) \sqrt{b+a \cos[e+f x]} \sqrt{d+c \cos[e+f x]} \right)
\end{aligned}$$

Problem 219: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{a+b \sec[e+f x]} \sqrt{c+d \sec[e+f x]}} dx$$

Optimal (type 4, 398 leaves, 3 steps):

$$\begin{aligned}
& -\frac{1}{a \sqrt{a+b} c f} 2 \sqrt{c+d} \cot[e+f x] \\
& \text{EllipticPi}\left[\frac{a(c+d)}{(a+b)c}, \text{ArcSin}\left[\frac{\sqrt{a+b} \sqrt{c+d} \sec[e+f x]}{\sqrt{c+d} \sqrt{a+b} \sec[e+f x]}\right], \frac{(a-b)(c+d)}{(a+b)(c-d)}\right] \\
& \sqrt{-\frac{(b c-a d) (1-\sec[e+f x])}{(c+d) (a+b \sec[e+f x])}} \sqrt{\frac{(b c-a d) (1+\sec[e+f x])}{(c-d) (a+b \sec[e+f x])}} (a+b \sec[e+f x]) - \\
& \left(2 b \sqrt{a+b} \cot[e+f x] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{c+d} \sqrt{a+b} \sec[e+f x]}{\sqrt{a+b} \sqrt{c+d} \sec[e+f x]}\right], \frac{(a+b)(c-d)}{(a-b)(c+d)}\right]\right. \\
& \left.\sqrt{\frac{(b c-a d) (1-\sec[e+f x])}{(a+b) (c+d \sec[e+f x])}} \sqrt{-\frac{(b c-a d) (1+\sec[e+f x])}{(a-b) (c+d \sec[e+f x])}}\right. \\
& \left.(c+d \sec[e+f x])\right) / \left(a \sqrt{c+d} (b c-a d) f\right)
\end{aligned}$$

Result (type 4, 249 leaves) :

$$\begin{aligned}
& \left(4 i \cos\left[\frac{1}{2} (e+f x)\right]^2 \sqrt{\frac{b+a \cos[e+f x]}{(a+b) (1+\cos[e+f x])}} \sqrt{\frac{d+c \cos[e+f x]}{(c+d) (1+\cos[e+f x])}}\right. \\
& \left(\text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \tan\left[\frac{1}{2} (e+f x)\right]\right], \frac{(a+b)(c-d)}{(a-b)(c+d)}\right] - \right. \\
& \left.2 \text{EllipticPi}\left[-\frac{a+b}{a-b}, i \text{ArcSinh}\left[\sqrt{\frac{-a+b}{a+b}} \tan\left[\frac{1}{2} (e+f x)\right]\right], \frac{(a+b)(c-d)}{(a-b)(c+d)}\right]\right) \\
& \left.\sec[e+f x]\right) / \left(\sqrt{\frac{-a+b}{a+b}} f \sqrt{a+b \sec[e+f x]} \sqrt{c+d \sec[e+f x]}\right)
\end{aligned}$$

Problem 220: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{a+b \sec[e+f x]} (c+d \sec[e+f x])^{3/2}} dx$$

Optimal (type 4, 622 leaves, 6 steps) :

$$\begin{aligned}
& - \left(\left(2 (a-b) \sqrt{a+b} d^2 \operatorname{Cot}[e+f x] \operatorname{EllipticE} \left[\right. \right. \right. \\
& \quad \left. \left. \left. \operatorname{ArcSin} \left[\frac{\sqrt{c+d} \sqrt{a+b} \operatorname{Sec}[e+f x]}{\sqrt{a+b} \sqrt{c+d} \operatorname{Sec}[e+f x]} \right], \frac{(a+b)(c-d)}{(a-b)(c+d)} \right] \sqrt{\frac{(b c - a d) (1 - \operatorname{Sec}[e+f x])}{(a+b) (c+d \operatorname{Sec}[e+f x])}} \right. \right. \\
& \quad \left. \left. \left. \sqrt{-\frac{(b c - a d) (1 + \operatorname{Sec}[e+f x])}{(a-b) (c+d \operatorname{Sec}[e+f x])}} (c+d \operatorname{Sec}[e+f x]) \right) \right) \Big/ \left(c (c-d) \sqrt{c+d} (b c - a d)^2 f \right) \right) - \\
& \left(2 \sqrt{a+b} (2 c - d) d \operatorname{Cot}[e+f x] \operatorname{EllipticF} \left[\operatorname{ArcSin} \left[\frac{\sqrt{c+d} \sqrt{a+b} \operatorname{Sec}[e+f x]}{\sqrt{a+b} \sqrt{c+d} \operatorname{Sec}[e+f x]} \right], \frac{(a+b)(c-d)}{(a-b)(c+d)} \right] \right. \\
& \quad \left. \sqrt{\frac{(b c - a d) (1 - \operatorname{Sec}[e+f x])}{(a+b) (c+d \operatorname{Sec}[e+f x])}} \sqrt{-\frac{(b c - a d) (1 + \operatorname{Sec}[e+f x])}{(a-b) (c+d \operatorname{Sec}[e+f x])}} (c+d \operatorname{Sec}[e+f x]) \right) \Big/ \\
& \quad \left(c^2 (c-d) \sqrt{c+d} (b c - a d) f \right) - \frac{1}{a c^2 \sqrt{c+d} f} 2 \sqrt{a+b} \operatorname{Cot}[e+f x] \\
& \operatorname{EllipticPi} \left[\frac{(a+b)c}{a(c+d)}, \operatorname{ArcSin} \left[\frac{\sqrt{c+d} \sqrt{a+b} \operatorname{Sec}[e+f x]}{\sqrt{a+b} \sqrt{c+d} \operatorname{Sec}[e+f x]} \right], \frac{(a+b)(c-d)}{(a-b)(c+d)} \right] \\
& \quad \sqrt{\frac{(b c - a d) (1 - \operatorname{Sec}[e+f x])}{(a+b) (c+d \operatorname{Sec}[e+f x])}} \\
& \quad \sqrt{-\frac{(b c - a d) (1 + \operatorname{Sec}[e+f x])}{(a-b) (c+d \operatorname{Sec}[e+f x])}} (c+d \operatorname{Sec}[e+f x])
\end{aligned}$$

Result (type 4, 1731 leaves):

$$\begin{aligned}
& \frac{1}{(c-d) (c+d) (b c - a d) f \sqrt{a+b} \operatorname{Sec}[e+f x] (c+d \operatorname{Sec}[e+f x])^{3/2}} \\
& \quad \sqrt{b+a \operatorname{Cos}[e+f x]} (d+c \operatorname{Cos}[e+f x])^{3/2} \operatorname{Sec}[e+f x]^2 \left(- \left(\left(\left(\right. \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. \left. \left. 4 b c d (b c - a d) \right) \right) \right) \right) \\
& \quad \sqrt{\frac{(c+d) \operatorname{Cot}[\frac{1}{2} (e+f x)]^2}{c-d}} \sqrt{\frac{(c+d) (b+a \operatorname{Cos}[e+f x]) \operatorname{Csc}[\frac{1}{2} (e+f x)]^2}{b c - a d}} \\
& \quad \sqrt{-\frac{(a+b) (d+c \operatorname{Cos}[e+f x]) \operatorname{Csc}[\frac{1}{2} (e+f x)]^2}{b c - a d}} \operatorname{Csc}[e+f x] \operatorname{EllipticF} [
\end{aligned}$$

$$\begin{aligned}
& \left. \left(\text{ArcSin} \left[\frac{\sqrt{-\frac{(a+b)(d+c \cos(e+f x)) \csc(\frac{1}{2}(e+f x))^2}{b c - a d}}}{\sqrt{2}} \right], \frac{2(b c - a d)}{(a+b)(c-d)} \right) \sin \left[\frac{1}{2}(e+f x) \right]^4 \right) / \\
& \left. \left((a+b)(c+d) \sqrt{b + a \cos(e+f x)} \sqrt{d + c \cos(e+f x)} \right) \right) + 4(b c - a d)(b c^2 - a c d - \\
& 2 b d^2) \left(\left(\sqrt{\frac{(c+d) \cot(\frac{1}{2}(e+f x))^2}{c-d}} \sqrt{\frac{(c+d)(b+a \cos(e+f x)) \csc(\frac{1}{2}(e+f x))^2}{b c - a d}} \right. \right. \\
& \left. \left. \sqrt{-\frac{(a+b)(d+c \cos(e+f x)) \csc(\frac{1}{2}(e+f x))^2}{b c - a d}} \csc(e+f x) \text{EllipticF} \left[\right. \right. \right. \\
& \left. \left. \left. \text{ArcSin} \left[\frac{\sqrt{-\frac{(a+b)(d+c \cos(e+f x)) \csc(\frac{1}{2}(e+f x))^2}{b c - a d}}}{\sqrt{2}} \right], \frac{2(b c - a d)}{(a+b)(c-d)} \right] \sin \left[\frac{1}{2}(e+f x) \right]^4 \right) \right) / \\
& \left. \left((a+b)(c+d) \sqrt{b + a \cos(e+f x)} \sqrt{d + c \cos(e+f x)} \right) - \right. \\
& \left(\sqrt{\frac{(c+d) \cot(\frac{1}{2}(e+f x))^2}{c-d}} \sqrt{\frac{(c+d)(b+a \cos(e+f x)) \csc(\frac{1}{2}(e+f x))^2}{b c - a d}} \right. \\
& \left. \left. \sqrt{-\frac{(a+b)(d+c \cos(e+f x)) \csc(\frac{1}{2}(e+f x))^2}{b c - a d}} \csc(e+f x) \right. \right. \\
& \left. \left. \text{EllipticPi} \left[\frac{b c - a d}{(a+b)c}, \text{ArcSin} \left[\frac{\sqrt{-\frac{(a+b)(d+c \cos(e+f x)) \csc(\frac{1}{2}(e+f x))^2}{b c - a d}}}{\sqrt{2}} \right], \frac{2(b c - a d)}{(a+b)(c-d)} \right] \right. \right. \\
& \left. \left. \sin \left[\frac{1}{2}(e+f x) \right]^4 \right) \right) / \left((a+b)c \sqrt{b + a \cos(e+f x)} \sqrt{d + c \cos(e+f x)} \right) \right) -
\end{aligned}$$

$$\begin{aligned}
& 2 a d^2 \left(\sqrt{\frac{-a+b}{a+b}} (a+b) \cos \left[\frac{1}{2} (e+f x) \right] \sqrt{d+c \cos [e+f x]} \right. \\
& \left. \text{EllipticE} \left[\text{ArcSin} \left[\frac{\sqrt{\frac{-a+b}{a+b}} \sin \left[\frac{1}{2} (e+f x) \right]}{\sqrt{\frac{b+a \cos [e+f x]}{a+b}}} \right], \frac{2 (b c - a d)}{(-a+b) (c+d)} \right] \right) / \\
& \left(a c \sqrt{\frac{(a+b) \cos \left[\frac{1}{2} (e+f x) \right]^2}{b+a \cos [e+f x]}} \sqrt{\frac{b+a \cos [e+f x]}{a+b}} \right. \\
& \left. \sqrt{\frac{(a+b) (d+c \cos [e+f x])}{(c+d) (b+a \cos [e+f x])}} \right) - \frac{1}{a c} 2 (b c - a d) \left(\begin{array}{l} (b c + (a+b) d) \\ \left(b c + (a+b) d \right) \end{array} \right. \\
& \left. \sqrt{\frac{(c+d) \cot \left[\frac{1}{2} (e+f x) \right]^2}{c-d}} \sqrt{\frac{(c+d) (b+a \cos [e+f x]) \csc \left[\frac{1}{2} (e+f x) \right]^2}{b c - a d}} \right. \\
& \left. \sqrt{-\frac{(a+b) (d+c \cos [e+f x]) \csc \left[\frac{1}{2} (e+f x) \right]^2}{b c - a d}} \csc [e+f x] \text{EllipticF} \left[\text{ArcSin} \left[\frac{\sqrt{-\frac{(a+b) (d+c \cos [e+f x]) \csc \left[\frac{1}{2} (e+f x) \right]^2}{b c - a d}}}{\sqrt{2}} \right], \frac{2 (b c - a d)}{(a+b) (c-d)} \right] \sin \left[\frac{1}{2} (e+f x) \right]^4 \right) / \\
& \left((a+b) (c+d) \sqrt{b+a \cos [e+f x]} \sqrt{d+c \cos [e+f x]} \right) - \left(\begin{array}{l} (b c + a d) \\ (b c + a d) \end{array} \right. \\
& \left. \sqrt{\frac{(c+d) \cot \left[\frac{1}{2} (e+f x) \right]^2}{c-d}} \sqrt{\frac{(c+d) (b+a \cos [e+f x]) \csc \left[\frac{1}{2} (e+f x) \right]^2}{b c - a d}} \right. \\
& \left. \sqrt{-\frac{(a+b) (d+c \cos [e+f x]) \csc \left[\frac{1}{2} (e+f x) \right]^2}{b c - a d}} \csc [e+f x] \right)
\end{aligned}$$

$$\begin{aligned}
& \text{EllipticPi}\left[\frac{b c - a d}{(a + b) c}, \text{ArcSin}\left[\frac{\sqrt{-\frac{(a+b)(d+c \cos[e+f x]) \csc\left[\frac{1}{2}(e+f x)\right]^2}{b c - a d}}}{\sqrt{2}}\right], \right. \\
& \left. \frac{2(b c - a d)}{(a + b)(c - d)} \right] \sin\left[\frac{1}{2}(e + f x)\right]^4 \Bigg/ \left((a + b) c \sqrt{b + a \cos[e + f x]} \right. \\
& \left. \sqrt{d + c \cos[e + f x]}\right) \Bigg) + \frac{\sqrt{d + c \cos[e + f x]} \sin[e + f x]}{c \sqrt{b + a \cos[e + f x]}} \Bigg) + \\
& \frac{2 d^2 (b + a \cos[e + f x]) (d + c \cos[e + f x]) \sec[e + f x] \tan[e + f x]}{(-b c + a d) (-c^2 + d^2) f \sqrt{a + b \sec[e + f x]} (c + d \sec[e + f x])^{3/2}}
\end{aligned}$$

Problem 222: Attempted integration timed out after 120 seconds.

$$\int \frac{(a + b \sec[e + f x])^{1/3}}{(c + d \sec[e + f x])^{4/3}} dx$$

Optimal (type 8, 32 leaves, 0 steps):

$$\text{Int}\left[\frac{(a + b \sec[e + f x])^{1/3}}{(c + d \sec[e + f x])^{4/3}}, x\right]$$

Result (type 1, 1 leaves):

???

Problem 223: Attempted integration timed out after 120 seconds.

$$\int \frac{(a + b \sec[e + f x])^{1/3}}{(c + d \sec[e + f x])^{7/3}} dx$$

Optimal (type 8, 32 leaves, 0 steps):

$$\text{Int}\left[\frac{(a + b \sec[e + f x])^{1/3}}{(c + d \sec[e + f x])^{7/3}}, x\right]$$

Result (type 1, 1 leaves):

???

Problem 225: Attempted integration timed out after 120 seconds.

$$\int \frac{(a+b \sec[e+f x])^{2/3}}{(c+d \sec[e+f x])^{5/3}} dx$$

Optimal (type 8, 32 leaves, 0 steps):

$$\text{Int} \left[\frac{(a+b \sec[e+f x])^{2/3}}{(c+d \sec[e+f x])^{5/3}}, x \right]$$

Result (type 1, 1 leaves):

???

Problem 226: Attempted integration timed out after 120 seconds.

$$\int \frac{(a+b \sec[e+f x])^{2/3}}{(c+d \sec[e+f x])^{8/3}} dx$$

Optimal (type 8, 32 leaves, 0 steps):

$$\text{Int} \left[\frac{(a+b \sec[e+f x])^{2/3}}{(c+d \sec[e+f x])^{8/3}}, x \right]$$

Result (type 1, 1 leaves):

???

Problem 227: Attempted integration timed out after 120 seconds.

$$\int \frac{(a+b \sec[e+f x])^{4/3}}{(c+d \sec[e+f x])^{4/3}} dx$$

Optimal (type 8, 89 leaves, 1 step):

$$\frac{(d+c \cos[e+f x])^{4/3} (a+b \sec[e+f x])^{4/3} \text{Int} \left[\frac{(b+a \cos[e+f x])^{4/3}}{(d+c \cos[e+f x])^{4/3}}, x \right]}{(b+a \cos[e+f x])^{4/3} (c+d \sec[e+f x])^{4/3}}$$

Result (type 1, 1 leaves):

???

Problem 228: Attempted integration timed out after 120 seconds.

$$\int \frac{(a+b \sec[e+f x])^{4/3}}{(c+d \sec[e+f x])^{7/3}} dx$$

Optimal (type 8, 32 leaves, 0 steps):

$$\text{Int} \left[\frac{(a+b \sec[e+f x])^{4/3}}{(c+d \sec[e+f x])^{7/3}}, x \right]$$

Result (type 1, 1 leaves) :

???

Problem 229: Attempted integration timed out after 120 seconds.

$$\int \frac{(a+b \sec[e+f x])^{4/3}}{(c+d \sec[e+f x])^{10/3}} dx$$

Optimal (type 8, 32 leaves, 0 steps) :

$$\text{Int} \left[\frac{(a+b \sec[e+f x])^{4/3}}{(c+d \sec[e+f x])^{10/3}}, x \right]$$

Result (type 1, 1 leaves) :

???

Problem 230: Result more than twice size of optimal antiderivative.

$$\int (c (d \sec[e+f x])^p)^n (a + a \sec[e+f x])^m dx$$

Optimal (type 6, 106 leaves, 4 steps) :

$$-\left(\left(\text{AppellF1} \left[n p, \frac{1}{2}, \frac{1}{2} - m, 1 + n p, \sec[e+f x], -\sec[e+f x] \right] (c (d \sec[e+f x])^p)^n (1 + \sec[e+f x])^{-\frac{1}{2}-m} (a + a \sec[e+f x])^m \tan[e+f x] \right) / (f n p \sqrt{1 - \sec[e+f x]}) \right)$$

Result (type 6, 2425 leaves) :

$$\begin{aligned} & \left(3 \times 2^{1+m} \text{AppellF1} \left[\frac{1}{2}, m+n p, 1-n p, \frac{3}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] \right. \\ & \quad \left(\sec \left[\frac{1}{2} (e+f x) \right]^2 \right)^{-1+n p} \left(\cos \left[\frac{1}{2} (e+f x) \right]^2 \sec[e+f x] \right)^{m+n p} \\ & \quad \left. (c (d \sec[e+f x])^p)^n (a (1 + \sec[e+f x]))^m \tan \left[\frac{1}{2} (e+f x) \right] \right) / \\ & \left(f \left(3 \text{AppellF1} \left[\frac{1}{2}, m+n p, 1-n p, \frac{3}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] + \right. \right. \\ & \quad 2 \left((-1+n p) \text{AppellF1} \left[\frac{3}{2}, m+n p, 2-n p, \frac{5}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] + \right. \\ & \quad \left. \left. (m+n p) \text{AppellF1} \left[\frac{3}{2}, 1+m+n p, 1-n p, \frac{5}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] \right) \right. \\ & \quad \left. \tan \left[\frac{1}{2} (e+f x) \right]^2 \right) \left(\left(3 \times 2^m \text{AppellF1} \left[\frac{1}{2}, m+n p, 1-n p, \frac{3}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, \right. \right. \right. \\ & \quad \left. \left. \left. -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] \left(\sec \left[\frac{1}{2} (e+f x) \right]^2 \right)^{n p} \left(\cos \left[\frac{1}{2} (e+f x) \right]^2 \sec[e+f x] \right)^{m+n p} \right) \right) / \end{aligned}$$

$$\begin{aligned}
& \left(3 \operatorname{AppellF1} \left[\frac{1}{2}, m+n p, 1-n p, \frac{3}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] + \right. \\
& \quad 2 \left((-1+n p) \operatorname{AppellF1} \left[\frac{3}{2}, m+n p, 2-n p, \frac{5}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] + \right. \\
& \quad \left. \left. (\mathfrak{m}+n p) \operatorname{AppellF1} \left[\frac{3}{2}, 1+m+n p, 1-n p, \frac{5}{2}, \right. \right. \right. \\
& \quad \left. \left. \left. \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] \right) \tan \left[\frac{1}{2} (e+f x) \right]^2 \right] + \\
& \left(3 \times 2^{1+m} (-1+n p) \operatorname{AppellF1} \left[\frac{1}{2}, m+n p, 1-n p, \frac{3}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] \right. \\
& \quad \left(\sec \left[\frac{1}{2} (e+f x) \right]^2 \right)^{-1+n p} \left(\cos \left[\frac{1}{2} (e+f x) \right]^2 \sec [e+f x] \right)^{m+n p} \tan \left[\frac{1}{2} (e+f x) \right]^2 \Bigg) / \\
& \left(3 \operatorname{AppellF1} \left[\frac{1}{2}, m+n p, 1-n p, \frac{3}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] + \right. \\
& \quad 2 \left((-1+n p) \operatorname{AppellF1} \left[\frac{3}{2}, m+n p, 2-n p, \frac{5}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] + \right. \\
& \quad \left. \left. (\mathfrak{m}+n p) \operatorname{AppellF1} \left[\frac{3}{2}, 1+m+n p, 1-n p, \frac{5}{2}, \right. \right. \right. \\
& \quad \left. \left. \left. \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] \right) \tan \left[\frac{1}{2} (e+f x) \right]^2 \right] + \\
& \left(3 \times 2^{1+m} \left(\sec \left[\frac{1}{2} (e+f x) \right]^2 \right)^{-1+n p} \left(\cos \left[\frac{1}{2} (e+f x) \right]^2 \sec [e+f x] \right)^{m+n p} \tan \left[\frac{1}{2} (e+f x) \right] \right. \\
& \quad \left(-\frac{1}{3} (1-n p) \operatorname{AppellF1} \left[\frac{3}{2}, m+n p, 2-n p, \frac{5}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] \right. \\
& \quad \left. \sec \left[\frac{1}{2} (e+f x) \right]^2 \tan \left[\frac{1}{2} (e+f x) \right] + \frac{1}{3} (\mathfrak{m}+n p) \operatorname{AppellF1} \left[\frac{3}{2}, 1+m+n p, 1-n p, \frac{5}{2}, \right. \right. \\
& \quad \left. \left. \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] \sec \left[\frac{1}{2} (e+f x) \right]^2 \tan \left[\frac{1}{2} (e+f x) \right] \right) \Bigg) / \\
& \left(3 \operatorname{AppellF1} \left[\frac{1}{2}, m+n p, 1-n p, \frac{3}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] + \right. \\
& \quad 2 \left((-1+n p) \operatorname{AppellF1} \left[\frac{3}{2}, m+n p, 2-n p, \frac{5}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] + \right. \\
& \quad \left. \left. (\mathfrak{m}+n p) \operatorname{AppellF1} \left[\frac{3}{2}, 1+m+n p, 1-n p, \frac{5}{2}, \right. \right. \right. \\
& \quad \left. \left. \left. \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] \right) \tan \left[\frac{1}{2} (e+f x) \right]^2 \right] - \\
& \left(3 \times 2^{1+m} \operatorname{AppellF1} \left[\frac{1}{2}, m+n p, 1-n p, \frac{3}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] \right. \\
& \quad \left(\sec \left[\frac{1}{2} (e+f x) \right]^2 \right)^{-1+n p} \left(\cos \left[\frac{1}{2} (e+f x) \right]^2 \sec [e+f x] \right)^{m+n p} \tan \left[\frac{1}{2} (e+f x) \right] \Bigg) \\
& \left(2 \left((-1+n p) \operatorname{AppellF1} \left[\frac{3}{2}, m+n p, 2-n p, \frac{5}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] + \right. \right. \\
& \quad \left. \left. (\mathfrak{m}+n p) \operatorname{AppellF1} \left[\frac{3}{2}, 1+m+n p, 1-n p, \frac{5}{2}, \tan \left[\frac{1}{2} (e+f x) \right]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\tan \left[\frac{1}{2} (e+f x) \right]^2 \right] \right) \sec \left[\frac{1}{2} (e+f x) \right]^2 \tan \left[\frac{1}{2} (e+f x) \right] + \right.
\end{aligned}$$

$$\begin{aligned}
& 3 \left(-\frac{1}{3} (1-np) \operatorname{AppellF1} \left[\frac{3}{2}, m+np, 2-np, \frac{5}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] \right. \\
& \quad \left. \sec \left[\frac{1}{2} (e+fx) \right]^2 \tan \left[\frac{1}{2} (e+fx) \right] + \frac{1}{3} (m+np) \operatorname{AppellF1} \left[\frac{3}{2}, 1+m+np, 1-np, \right. \right. \\
& \quad \left. \left. \frac{5}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] \sec \left[\frac{1}{2} (e+fx) \right]^2 \tan \left[\frac{1}{2} (e+fx) \right] \right) + \\
& 2 \tan \left[\frac{1}{2} (e+fx) \right]^2 \left((-1+np) \left(-\frac{3}{5} (2-np) \operatorname{AppellF1} \left[\frac{5}{2}, m+np, 3-np, \frac{7}{2}, \right. \right. \right. \\
& \quad \left. \left. \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] \sec \left[\frac{1}{2} (e+fx) \right]^2 \tan \left[\frac{1}{2} (e+fx) \right] \right. \\
& \quad \left. \left. \left. + \frac{3}{5} (m+np) \operatorname{AppellF1} \left[\frac{5}{2}, 1+m+np, 2-np, \frac{7}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] \sec \left[\frac{1}{2} (e+fx) \right]^2 \tan \left[\frac{1}{2} (e+fx) \right] \right. \right. \\
& \quad \left. \left. \left. + (m+np) \left(-\frac{3}{5} (1-np) \operatorname{AppellF1} \left[\frac{5}{2}, 1+m+np, 2-np, \frac{7}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] \sec \left[\frac{1}{2} (e+fx) \right]^2 \tan \left[\frac{1}{2} (e+fx) \right] \right. \right. \right. \\
& \quad \left. \left. \left. + \frac{3}{5} (1+m+np) \operatorname{AppellF1} \left[\frac{5}{2}, 2+m+np, 1-np, \frac{7}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] \sec \left[\frac{1}{2} (e+fx) \right]^2 \tan \left[\frac{1}{2} (e+fx) \right] \right) \right) \right) \Bigg) / \\
& \left(3 \operatorname{AppellF1} \left[\frac{1}{2}, m+np, 1-np, \frac{3}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] + \right. \\
& \quad 2 \left((-1+np) \operatorname{AppellF1} \left[\frac{3}{2}, m+np, 2-np, \frac{5}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] + \right. \\
& \quad (m+np) \operatorname{AppellF1} \left[\frac{3}{2}, 1+m+np, 1-np, \frac{5}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, \right. \\
& \quad \left. \left. -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] \tan \left[\frac{1}{2} (e+fx) \right]^2 \right)^2 + \\
& 3 \times 2^{1+m} (m+np) \operatorname{AppellF1} \left[\frac{1}{2}, m+np, 1-np, \frac{3}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] \\
& \quad \left(\sec \left[\frac{1}{2} (e+fx) \right]^2 \right)^{-1+n p} \left(\cos \left[\frac{1}{2} (e+fx) \right]^2 \sec [e+fx] \right)^{-1+m+n p} \\
& \quad \tan \left[\frac{1}{2} (e+fx) \right] \left(-\cos \left[\frac{1}{2} (e+fx) \right] \sec [e+fx] \sin \left[\frac{1}{2} (e+fx) \right] + \right. \\
& \quad \left. \cos \left[\frac{1}{2} (e+fx) \right]^2 \sec [e+fx] \tan [e+fx] \right) \Bigg) / \\
& \left(3 \operatorname{AppellF1} \left[\frac{1}{2}, m+np, 1-np, \frac{3}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] + \right. \\
& \quad 2 \left((-1+np) \operatorname{AppellF1} \left[\frac{3}{2}, m+np, 2-np, \frac{5}{2}, \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] + \right. \\
& \quad (m+np) \operatorname{AppellF1} \left[\frac{3}{2}, 1+m+np, 1-np, \frac{5}{2}, \right. \\
& \quad \left. \left. \tan \left[\frac{1}{2} (e+fx) \right]^2, -\tan \left[\frac{1}{2} (e+fx) \right]^2 \right] \tan \left[\frac{1}{2} (e+fx) \right]^2 \right) \Bigg)
\end{aligned}$$

Problem 231: Unable to integrate problem.

$$\int (c (d \sec(e + f x))^p)^n (a + a \sec(e + f x))^3 dx$$

Optimal (type 5, 275 leaves, 8 steps):

$$\begin{aligned} & \left(a^3 (7 + 4 n p) \text{Hypergeometric2F1}\left[\frac{1}{2}, -\frac{n p}{2}, \frac{1}{2} (2 - n p), \cos[e + f x]^2\right] \right. \\ & \quad \left. (c (d \sec[e + f x])^p)^n \sin[e + f x] \right) / \left(f n p (2 + n p) \sqrt{\sin[e + f x]^2} \right) - \\ & \left(a^3 (1 + 4 n p) \cos[e + f x] \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{2} (1 - n p), \frac{1}{2} (3 - n p), \cos[e + f x]^2\right] \right. \\ & \quad \left. (c (d \sec[e + f x])^p)^n \sin[e + f x] \right) / \left(f (1 - n^2 p^2) \sqrt{\sin[e + f x]^2} \right) + \\ & \frac{a^3 (5 + 2 n p) (c (d \sec[e + f x])^p)^n \tan[e + f x]}{f (1 + n p) (2 + n p)} + \\ & \frac{(c (d \sec[e + f x])^p)^n (a^3 + a^3 \sec[e + f x]) \tan[e + f x]}{f (2 + n p)} \end{aligned}$$

Result (type 8, 29 leaves):

$$\int (c (d \sec(e + f x))^p)^n (a + a \sec(e + f x))^2 dx$$

Problem 232: Unable to integrate problem.

$$\int (c (d \sec(e + f x))^p)^n (a + a \sec(e + f x))^2 dx$$

Optimal (type 5, 205 leaves, 7 steps):

$$\begin{aligned} & \left(2 a^2 \text{Hypergeometric2F1}\left[\frac{1}{2}, -\frac{n p}{2}, \frac{1}{2} (2 - n p), \cos[e + f x]^2\right] (c (d \sec[e + f x])^p)^n \sin[e + f x] \right) / \\ & \quad \left(f n p \sqrt{\sin[e + f x]^2} \right) - \\ & \left(a^2 (1 + 2 n p) \cos[e + f x] \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{2} (1 - n p), \frac{1}{2} (3 - n p), \cos[e + f x]^2\right] \right. \\ & \quad \left. (c (d \sec[e + f x])^p)^n \sin[e + f x] \right) / \\ & \left(f (1 - n^2 p^2) \sqrt{\sin[e + f x]^2} \right) + \frac{a^2 (c (d \sec[e + f x])^p)^n \tan[e + f x]}{f (1 + n p)} \end{aligned}$$

Result (type 8, 29 leaves):

$$\int (c (d \sec(e + f x))^p)^n (a + a \sec(e + f x))^2 dx$$

Problem 233: Result unnecessarily involves higher level functions and more

than twice size of optimal antiderivative.

$$\int (c (d \sec(e + f x))^p)^n (a + a \sec(e + f x)) dx$$

Optimal (type 5, 156 leaves, 6 steps):

$$\begin{aligned} & \left(a \text{Hypergeometric2F1}\left[\frac{1}{2}, -\frac{n p}{2}, \frac{1}{2} (2-n p), \cos[e+f x]^2\right] (c (d \sec(e + f x))^p)^n \sin[e+f x] \right) / \\ & \quad \left(f n p \sqrt{\sin[e+f x]^2} \right) - \\ & \left(a \cos[e+f x] \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{2} (1-n p), \frac{1}{2} (3-n p), \cos[e+f x]^2\right] \right. \\ & \quad \left. (c (d \sec(e + f x))^p)^n \sin[e+f x] \right) / \left(f (1-n p) \sqrt{\sin[e+f x]^2} \right) \end{aligned}$$

Result (type 6, 4295 leaves):

$$\begin{aligned} & - \left(\left(a \sec[e+f x]^{n p} (c (d \sec[e+f x])^p)^n (1 + \sec[e+f x]) \tan\left[\frac{1}{2} (e+f x)\right] \right. \right. \\ & \quad \left(3 \text{AppellF1}\left[\frac{1}{2}, n p, 1-n p, \frac{3}{2}, \tan\left[\frac{1}{2} (e+f x)\right]^2, -\tan\left[\frac{1}{2} (e+f x)\right]^2\right] \cos[e+f x] \right) / \\ & \quad \left(3 \text{AppellF1}\left[\frac{1}{2}, n p, 1-n p, \frac{3}{2}, \tan\left[\frac{1}{2} (e+f x)\right]^2, -\tan\left[\frac{1}{2} (e+f x)\right]^2\right] + \right. \\ & \quad 2 \left((-1+n p) \text{AppellF1}\left[\frac{3}{2}, n p, 2-n p, \frac{5}{2}, \tan\left[\frac{1}{2} (e+f x)\right]^2, -\tan\left[\frac{1}{2} (e+f x)\right]^2\right] + \right. \\ & \quad n p \text{AppellF1}\left[\frac{3}{2}, 1+n p, 1-n p, \frac{5}{2}, \tan\left[\frac{1}{2} (e+f x)\right]^2, \right. \\ & \quad \left. \left. -\tan\left[\frac{1}{2} (e+f x)\right]^2\right] \right) \tan\left[\frac{1}{2} (e+f x)\right]^2 + \\ & \quad \text{AppellF1}\left[\frac{1}{2}, 1+n p, -n p, \frac{3}{2}, \tan\left[\frac{1}{2} (e+f x)\right]^2, -\tan\left[\frac{1}{2} (e+f x)\right]^2\right] / \\ & \quad \left(\text{AppellF1}\left[\frac{1}{2}, 1+n p, -n p, \frac{3}{2}, \tan\left[\frac{1}{2} (e+f x)\right]^2, -\tan\left[\frac{1}{2} (e+f x)\right]^2\right] + \right. \\ & \quad \frac{2}{3} \left(n p \text{AppellF1}\left[\frac{3}{2}, 1+n p, 1-n p, \frac{5}{2}, \tan\left[\frac{1}{2} (e+f x)\right]^2, -\tan\left[\frac{1}{2} (e+f x)\right]^2\right] + \right. \\ & \quad (1+n p) \text{AppellF1}\left[\frac{3}{2}, 2+n p, -n p, \frac{5}{2}, \tan\left[\frac{1}{2} (e+f x)\right]^2, -\tan\left[\frac{1}{2} (e+f x)\right]^2\right] \Big) \\ & \quad \left. \tan\left[\frac{1}{2} (e+f x)\right]^2 \right) \Big) / \left(f \left(-1 + \tan\left[\frac{1}{2} (e+f x)\right]^2 \right) \right. \\ & \quad \left(\frac{1}{(-1 + \tan\left[\frac{1}{2} (e+f x)\right]^2)^2} \sec\left[\frac{1}{2} (e+f x)\right]^2 \sec[e+f x]^{n p} \tan\left[\frac{1}{2} (e+f x)\right]^2 \right. \\ & \quad \left(\left(3 \text{AppellF1}\left[\frac{1}{2}, n p, 1-n p, \frac{3}{2}, \tan\left[\frac{1}{2} (e+f x)\right]^2, -\tan\left[\frac{1}{2} (e+f x)\right]^2\right] \cos[e+f x] \right) / \right. \\ & \quad \left(3 \text{AppellF1}\left[\frac{1}{2}, n p, 1-n p, \frac{3}{2}, \tan\left[\frac{1}{2} (e+f x)\right]^2, -\tan\left[\frac{1}{2} (e+f x)\right]^2\right] + \right. \end{aligned}$$

$$\begin{aligned}
& 2 \left((-1 + np) \operatorname{AppellF1} \left[\frac{3}{2}, np, 2 - np, \frac{5}{2}, \tan \left[\frac{1}{2} (e + fx) \right]^2, \right. \right. \\
& \quad \left. \left. - \tan \left[\frac{1}{2} (e + fx) \right]^2 \right] + np \operatorname{AppellF1} \left[\frac{3}{2}, 1 + np, 1 - np, \frac{5}{2}, \right. \right. \\
& \quad \left. \left. \tan \left[\frac{1}{2} (e + fx) \right]^2, - \tan \left[\frac{1}{2} (e + fx) \right]^2 \right] \right) \tan \left[\frac{1}{2} (e + fx) \right]^2 + \\
& \operatorname{AppellF1} \left[\frac{1}{2}, 1 + np, -np, \frac{3}{2}, \tan \left[\frac{1}{2} (e + fx) \right]^2, - \tan \left[\frac{1}{2} (e + fx) \right]^2 \right] / \\
& \left(\operatorname{AppellF1} \left[\frac{1}{2}, 1 + np, -np, \frac{3}{2}, \tan \left[\frac{1}{2} (e + fx) \right]^2, - \tan \left[\frac{1}{2} (e + fx) \right]^2 \right] + \right. \\
& \quad \left. \frac{2}{3} \left(np \operatorname{AppellF1} \left[\frac{3}{2}, 1 + np, 1 - np, \frac{5}{2}, \tan \left[\frac{1}{2} (e + fx) \right]^2, - \tan \left[\frac{1}{2} (e + fx) \right]^2 \right] + \right. \right. \\
& \quad \left. \left. (1 + np) \operatorname{AppellF1} \left[\frac{3}{2}, 2 + np, -np, \frac{5}{2}, \tan \left[\frac{1}{2} (e + fx) \right]^2, \right. \right. \\
& \quad \left. \left. - \tan \left[\frac{1}{2} (e + fx) \right]^2 \right] \right) \tan \left[\frac{1}{2} (e + fx) \right]^2 \right) - \\
& \frac{1}{2 \left(-1 + \tan \left[\frac{1}{2} (e + fx) \right]^2 \right)} \sec \left[\frac{1}{2} (e + fx) \right]^2 \sec [e + fx]^{np} \\
& \left(\left(3 \operatorname{AppellF1} \left[\frac{1}{2}, np, 1 - np, \frac{3}{2}, \tan \left[\frac{1}{2} (e + fx) \right]^2, - \tan \left[\frac{1}{2} (e + fx) \right]^2 \right] \cos [e + fx] \right) / \right. \\
& \quad \left. \left(3 \operatorname{AppellF1} \left[\frac{1}{2}, np, 1 - np, \frac{3}{2}, \tan \left[\frac{1}{2} (e + fx) \right]^2, - \tan \left[\frac{1}{2} (e + fx) \right]^2 \right] + \right. \right. \\
& \quad \left. \left. 2 \left((-1 + np) \operatorname{AppellF1} \left[\frac{3}{2}, np, 2 - np, \frac{5}{2}, \tan \left[\frac{1}{2} (e + fx) \right]^2, \right. \right. \right. \\
& \quad \left. \left. \left. - \tan \left[\frac{1}{2} (e + fx) \right]^2 \right] + np \operatorname{AppellF1} \left[\frac{3}{2}, 1 + np, 1 - np, \frac{5}{2}, \right. \right. \\
& \quad \left. \left. \tan \left[\frac{1}{2} (e + fx) \right]^2, - \tan \left[\frac{1}{2} (e + fx) \right]^2 \right] \right) \tan \left[\frac{1}{2} (e + fx) \right]^2 \right) + \\
& \operatorname{AppellF1} \left[\frac{1}{2}, 1 + np, -np, \frac{3}{2}, \tan \left[\frac{1}{2} (e + fx) \right]^2, - \tan \left[\frac{1}{2} (e + fx) \right]^2 \right] / \\
& \left(\operatorname{AppellF1} \left[\frac{1}{2}, 1 + np, -np, \frac{3}{2}, \tan \left[\frac{1}{2} (e + fx) \right]^2, - \tan \left[\frac{1}{2} (e + fx) \right]^2 \right] + \right. \\
& \quad \left. \frac{2}{3} \left(np \operatorname{AppellF1} \left[\frac{3}{2}, 1 + np, 1 - np, \frac{5}{2}, \tan \left[\frac{1}{2} (e + fx) \right]^2, - \tan \left[\frac{1}{2} (e + fx) \right]^2 \right] + \right. \right. \\
& \quad \left. \left. (1 + np) \operatorname{AppellF1} \left[\frac{3}{2}, 2 + np, -np, \frac{5}{2}, \tan \left[\frac{1}{2} (e + fx) \right]^2, \right. \right. \\
& \quad \left. \left. - \tan \left[\frac{1}{2} (e + fx) \right]^2 \right] \right) \tan \left[\frac{1}{2} (e + fx) \right]^2 \right) - \\
& \frac{1}{-1 + \tan \left[\frac{1}{2} (e + fx) \right]^2} np \sec [e + fx]^{1+np} \sin [e + fx] \tan \left[\frac{1}{2} (e + fx) \right] \\
& \left(\left(3 \operatorname{AppellF1} \left[\frac{1}{2}, np, 1 - np, \frac{3}{2}, \tan \left[\frac{1}{2} (e + fx) \right]^2, - \tan \left[\frac{1}{2} (e + fx) \right]^2 \right] \cos [e + fx] \right) / \right. \\
& \quad \left. \left(3 \operatorname{AppellF1} \left[\frac{1}{2}, np, 1 - np, \frac{3}{2}, \tan \left[\frac{1}{2} (e + fx) \right]^2, - \tan \left[\frac{1}{2} (e + fx) \right]^2 \right] + \right. \right.
\end{aligned}$$

$$\begin{aligned}
& 2 \left((-1 + np) \operatorname{AppellF1} \left[\frac{3}{2}, np, 2 - np, \frac{5}{2}, \tan \left[\frac{1}{2} (e + fx) \right]^2, \right. \right. \\
& \quad \left. \left. - \tan \left[\frac{1}{2} (e + fx) \right]^2 \right] + np \operatorname{AppellF1} \left[\frac{3}{2}, 1 + np, 1 - np, \frac{5}{2}, \right. \right. \\
& \quad \left. \left. \tan \left[\frac{1}{2} (e + fx) \right]^2, - \tan \left[\frac{1}{2} (e + fx) \right]^2 \right] \right) \tan \left[\frac{1}{2} (e + fx) \right]^2 + \\
& \operatorname{AppellF1} \left[\frac{1}{2}, 1 + np, -np, \frac{3}{2}, \tan \left[\frac{1}{2} (e + fx) \right]^2, - \tan \left[\frac{1}{2} (e + fx) \right]^2 \right] / \\
& \left(\operatorname{AppellF1} \left[\frac{1}{2}, 1 + np, -np, \frac{3}{2}, \tan \left[\frac{1}{2} (e + fx) \right]^2, - \tan \left[\frac{1}{2} (e + fx) \right]^2 \right] + \right. \\
& \quad \left. \frac{2}{3} \left(np \operatorname{AppellF1} \left[\frac{3}{2}, 1 + np, 1 - np, \frac{5}{2}, \tan \left[\frac{1}{2} (e + fx) \right]^2, - \tan \left[\frac{1}{2} (e + fx) \right]^2 \right] + \right. \right. \\
& \quad \left. \left. (1 + np) \operatorname{AppellF1} \left[\frac{3}{2}, 2 + np, -np, \frac{5}{2}, \tan \left[\frac{1}{2} (e + fx) \right]^2, \right. \right. \\
& \quad \left. \left. - \tan \left[\frac{1}{2} (e + fx) \right]^2 \right] \right) \tan \left[\frac{1}{2} (e + fx) \right]^2 \right) - \\
& \frac{1}{-1 + \tan \left[\frac{1}{2} (e + fx) \right]^2} \sec [e + fx]^{np} \tan \left[\frac{1}{2} (e + fx) \right] \\
& - \left(\left(3 \operatorname{AppellF1} \left[\frac{1}{2}, np, 1 - np, \frac{3}{2}, \tan \left[\frac{1}{2} (e + fx) \right]^2, - \tan \left[\frac{1}{2} (e + fx) \right]^2 \right] \right. \right. \\
& \quad \left. \left. \sin [e + fx] \right) / \left(3 \operatorname{AppellF1} \left[\frac{1}{2}, np, 1 - np, \frac{3}{2}, \tan \left[\frac{1}{2} (e + fx) \right]^2, \right. \right. \\
& \quad \left. \left. - \tan \left[\frac{1}{2} (e + fx) \right]^2 \right] + 2 \left((-1 + np) \operatorname{AppellF1} \left[\frac{3}{2}, np, 2 - np, \frac{5}{2}, \right. \right. \\
& \quad \left. \left. \tan \left[\frac{1}{2} (e + fx) \right]^2, - \tan \left[\frac{1}{2} (e + fx) \right]^2 \right] + np \operatorname{AppellF1} \left[\frac{3}{2}, 1 + np, 1 - np, \right. \right. \\
& \quad \left. \left. \frac{5}{2}, \tan \left[\frac{1}{2} (e + fx) \right]^2, - \tan \left[\frac{1}{2} (e + fx) \right]^2 \right] \right) \tan \left[\frac{1}{2} (e + fx) \right]^2 \right) + \\
& \left(3 \cos [e + fx] \left(-\frac{1}{3} (1 - np) \operatorname{AppellF1} \left[\frac{3}{2}, np, 2 - np, \frac{5}{2}, \tan \left[\frac{1}{2} (e + fx) \right]^2, \right. \right. \right. \\
& \quad \left. \left. - \tan \left[\frac{1}{2} (e + fx) \right]^2 \right] \sec \left[\frac{1}{2} (e + fx) \right]^2 \tan \left[\frac{1}{2} (e + fx) \right] + \right. \\
& \quad \left. \left. \frac{1}{3} np \operatorname{AppellF1} \left[\frac{3}{2}, 1 + np, 1 - np, \frac{5}{2}, \tan \left[\frac{1}{2} (e + fx) \right]^2, \right. \right. \\
& \quad \left. \left. - \tan \left[\frac{1}{2} (e + fx) \right]^2 \right] \sec \left[\frac{1}{2} (e + fx) \right]^2 \tan \left[\frac{1}{2} (e + fx) \right] \right) / \right. \\
& \left(3 \operatorname{AppellF1} \left[\frac{1}{2}, np, 1 - np, \frac{3}{2}, \tan \left[\frac{1}{2} (e + fx) \right]^2, - \tan \left[\frac{1}{2} (e + fx) \right]^2 \right] + \right. \\
& \quad \left. 2 \left((-1 + np) \operatorname{AppellF1} \left[\frac{3}{2}, np, 2 - np, \frac{5}{2}, \tan \left[\frac{1}{2} (e + fx) \right]^2, \right. \right. \right. \\
& \quad \left. \left. - \tan \left[\frac{1}{2} (e + fx) \right]^2 \right] + np \operatorname{AppellF1} \left[\frac{3}{2}, 1 + np, 1 - np, \frac{5}{2}, \right. \right. \\
& \quad \left. \left. \tan \left[\frac{1}{2} (e + fx) \right]^2, - \tan \left[\frac{1}{2} (e + fx) \right]^2 \right] \right) \tan \left[\frac{1}{2} (e + fx) \right]^2 \right) + \\
& \left(\frac{1}{3} np \operatorname{AppellF1} \left[\frac{3}{2}, 1 + np, 1 - np, \frac{5}{2}, \tan \left[\frac{1}{2} (e + fx) \right]^2, - \tan \left[\frac{1}{2} (e + fx) \right]^2 \right] \right)
\end{aligned}$$

$$\begin{aligned}
& \operatorname{Sec}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]+\frac{1}{3}(1+n p) \operatorname{AppellF1}\left[\frac{3}{2}, 2+n p, -n p, \frac{5}{2},\right. \\
& \left.\operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]\Bigg)/ \\
& \left(\operatorname{AppellF1}\left[\frac{1}{2}, 1+n p, -n p, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2\right]+ \right. \\
& \left.\frac{2}{3}\left(n p \operatorname{AppellF1}\left[\frac{3}{2}, 1+n p, 1-n p, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2\right]+\right.\right. \\
& \left.\left.(1+n p) \operatorname{AppellF1}\left[\frac{3}{2}, 2+n p, -n p, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2,\right.\right. \\
& \left.\left.-\operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2\right]\right) \operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2\Big)- \\
& \left(3 \operatorname{AppellF1}\left[\frac{1}{2}, n p, 1-n p, \frac{3}{2}, \operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2\right]\right. \\
& \left.\cos [\mathbf{e}+\mathbf{f} x]\left(2\left((-1+n p) \operatorname{AppellF1}\left[\frac{3}{2}, n p, 2-n p, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2,\right.\right.\right.\right. \\
& \left.\left.\left.\left.-\operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2\right]+n p \operatorname{AppellF1}\left[\frac{3}{2}, 1+n p, 1-n p, \frac{5}{2},\right.\right.\right.\right. \\
& \left.\left.\left.\left.\operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2\right]\right) \operatorname{Sec}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2\right. \\
& \left.\operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]+3\left(-\frac{1}{3}(1-n p) \operatorname{AppellF1}\left[\frac{3}{2}, n p, 2-n p, \frac{5}{2},\right.\right.\right.\right. \\
& \left.\left.\left.\left.\operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2\right]\right) \operatorname{Sec}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2\right. \\
& \left.\operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]+\frac{1}{3} n p \operatorname{AppellF1}\left[\frac{3}{2}, 1+n p, 1-n p, \frac{5}{2}, \operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2,\right.\right.\right. \\
& \left.\left.\left.-\operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2\right]\right) \operatorname{Sec}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]\right)+ \\
& 2 \operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2\left((-1+n p)\left(-\frac{3}{5}(2-n p) \operatorname{AppellF1}\left[\frac{5}{2}, n p, 3-n p,\right.\right.\right.\right. \\
& \left.\left.\left.\left.\frac{7}{2}, \operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2\right]\right) \operatorname{Sec}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]\right. \\
& \left.\left.\left.\left.\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]+\frac{3}{5} n p \operatorname{AppellF1}\left[\frac{5}{2}, 1+n p, 2-n p, \frac{7}{2}, \operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2,\right.\right.\right. \\
& \left.\left.\left.-\operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2\right]\right) \operatorname{Sec}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]\right)+ \\
& n p\left(-\frac{3}{5}(1-n p) \operatorname{AppellF1}\left[\frac{5}{2}, 1+n p, 2-n p, \frac{7}{2}, \operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2,\right.\right.\right. \\
& \left.\left.\left.-\operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2\right]\right) \operatorname{Sec}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]+ \\
& \left.\left.\left.\frac{3}{5}(1+n p) \operatorname{AppellF1}\left[\frac{5}{2}, 2+n p, 1-n p, \frac{7}{2}, \operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2,\right.\right.\right. \\
& \left.\left.\left.-\operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2\right]\right) \operatorname{Sec}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]^2 \operatorname{Tan}\left[\frac{1}{2}(\mathbf{e}+\mathbf{f} x)\right]\right)\right)\Big)
\end{aligned}$$

$$\begin{aligned}
& 2 \left(\left(-1 + np \right) \text{AppellF1} \left[\frac{3}{2}, np, 2 - np, \frac{5}{2}, \tan \left[\frac{1}{2} (e + fx) \right]^2, \right. \right. \\
& \quad \left. \left. - \tan \left[\frac{1}{2} (e + fx) \right]^2 \right] + np \text{AppellF1} \left[\frac{3}{2}, 1 + np, 1 - np, \frac{5}{2}, \right. \right. \\
& \quad \left. \left. \tan \left[\frac{1}{2} (e + fx) \right]^2, - \tan \left[\frac{1}{2} (e + fx) \right]^2 \right] \right) \tan \left[\frac{1}{2} (e + fx) \right]^2 - \\
& \left(\text{AppellF1} \left[\frac{1}{2}, 1 + np, -np, \frac{3}{2}, \tan \left[\frac{1}{2} (e + fx) \right]^2, - \tan \left[\frac{1}{2} (e + fx) \right]^2 \right] \right. \\
& \quad \left. \left(\frac{1}{3} np \text{AppellF1} \left[\frac{3}{2}, 1 + np, 1 - np, \frac{5}{2}, \tan \left[\frac{1}{2} (e + fx) \right]^2, - \tan \left[\frac{1}{2} (e + fx) \right]^2 \right] \right. \right. \\
& \quad \left. \left. \sec \left[\frac{1}{2} (e + fx) \right]^2 \tan \left[\frac{1}{2} (e + fx) \right] + \frac{1}{3} (1 + np) \text{AppellF1} \left[\frac{3}{2}, 2 + np, -np, \frac{5}{2}, \right. \right. \\
& \quad \left. \left. \tan \left[\frac{1}{2} (e + fx) \right]^2, - \tan \left[\frac{1}{2} (e + fx) \right]^2 \right] \sec \left[\frac{1}{2} (e + fx) \right]^2 \tan \left[\frac{1}{2} (e + fx) \right] + \right. \\
& \quad \left. \frac{2}{3} \left(np \text{AppellF1} \left[\frac{3}{2}, 1 + np, 1 - np, \frac{5}{2}, \tan \left[\frac{1}{2} (e + fx) \right]^2, - \tan \left[\frac{1}{2} (e + fx) \right]^2 \right] + \right. \right. \\
& \quad \left. \left. (1 + np) \text{AppellF1} \left[\frac{3}{2}, 2 + np, -np, \frac{5}{2}, \tan \left[\frac{1}{2} (e + fx) \right]^2, - \tan \left[\frac{1}{2} (e + fx) \right]^2 \right] \right) \right. \\
& \quad \left. \sec \left[\frac{1}{2} (e + fx) \right]^2 \tan \left[\frac{1}{2} (e + fx) \right] + \frac{2}{3} \tan \left[\frac{1}{2} (e + fx) \right]^2 \right. \\
& \quad \left(np \left(-\frac{3}{5} (1 - np) \text{AppellF1} \left[\frac{5}{2}, 1 + np, 2 - np, \frac{7}{2}, \tan \left[\frac{1}{2} (e + fx) \right]^2, \right. \right. \right. \\
& \quad \left. \left. \left. - \tan \left[\frac{1}{2} (e + fx) \right]^2 \right] \sec \left[\frac{1}{2} (e + fx) \right]^2 \tan \left[\frac{1}{2} (e + fx) \right] + \right. \right. \\
& \quad \left. \left. \frac{3}{5} (1 + np) \text{AppellF1} \left[\frac{5}{2}, 2 + np, 1 - np, \frac{7}{2}, \tan \left[\frac{1}{2} (e + fx) \right]^2, \right. \right. \\
& \quad \left. \left. - \tan \left[\frac{1}{2} (e + fx) \right]^2 \right] \sec \left[\frac{1}{2} (e + fx) \right]^2 \tan \left[\frac{1}{2} (e + fx) \right] \right) + \right. \\
& \quad \left. (1 + np) \left(\frac{3}{5} np \text{AppellF1} \left[\frac{5}{2}, 2 + np, 1 - np, \frac{7}{2}, \tan \left[\frac{1}{2} (e + fx) \right]^2, \right. \right. \right. \\
& \quad \left. \left. \left. - \tan \left[\frac{1}{2} (e + fx) \right]^2 \right] \sec \left[\frac{1}{2} (e + fx) \right]^2 \tan \left[\frac{1}{2} (e + fx) \right] + \right. \right. \\
& \quad \left. \left. \frac{3}{5} (2 + np) \text{AppellF1} \left[\frac{5}{2}, 3 + np, -np, \frac{7}{2}, \tan \left[\frac{1}{2} (e + fx) \right]^2, \right. \right. \\
& \quad \left. \left. - \tan \left[\frac{1}{2} (e + fx) \right]^2 \right] \sec \left[\frac{1}{2} (e + fx) \right]^2 \tan \left[\frac{1}{2} (e + fx) \right] \right) \right) \right) \right) / \\
& \left(\text{AppellF1} \left[\frac{1}{2}, 1 + np, -np, \frac{3}{2}, \tan \left[\frac{1}{2} (e + fx) \right]^2, - \tan \left[\frac{1}{2} (e + fx) \right]^2 \right] + \right. \\
& \quad \left. \frac{2}{3} \left(np \text{AppellF1} \left[\frac{3}{2}, 1 + np, 1 - np, \frac{5}{2}, \tan \left[\frac{1}{2} (e + fx) \right]^2, - \tan \left[\frac{1}{2} (e + fx) \right]^2 \right] + \right. \right. \\
& \quad \left. \left. (1 + np) \text{AppellF1} \left[\frac{3}{2}, 2 + np, -np, \frac{5}{2}, \tan \left[\frac{1}{2} (e + fx) \right]^2, \right. \right. \\
& \quad \left. \left. - \tan \left[\frac{1}{2} (e + fx) \right]^2 \right] \tan \left[\frac{1}{2} (e + fx) \right]^2 \right)^2 \right) \right) \right)
\end{aligned}$$

Problem 234: Unable to integrate problem.

$$\int \frac{(c (d \sec [e + f x])^p)^n}{a + a \sec [e + f x]} dx$$

Optimal (type 5, 208 leaves, 7 steps):

$$\begin{aligned} & \frac{(c (d \sec [e + f x])^p)^n \sin [e + f x]}{f (a + a \sec [e + f x])} - \\ & \left(\cos [e + f x] \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{2} (1 - n p), \frac{1}{2} (3 - n p), \cos [e + f x]^2\right] \right. \\ & \quad \left. (c (d \sec [e + f x])^p)^n \sin [e + f x]\right) / \left(a f \sqrt{\sin [e + f x]^2}\right) + \\ & \left((1 - n p) \cos [e + f x]^2 \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{2} (2 - n p), \frac{1}{2} (4 - n p), \cos [e + f x]^2\right] \right. \\ & \quad \left. (c (d \sec [e + f x])^p)^n \sin [e + f x]\right) / \left(a f (2 - n p) \sqrt{\sin [e + f x]^2}\right) \end{aligned}$$

Result (type 8, 29 leaves):

$$\int \frac{(c (d \sec [e + f x])^p)^n}{a + a \sec [e + f x]} dx$$

Problem 235: Unable to integrate problem.

$$\int \frac{(c (d \sec [e + f x])^p)^n}{(a + a \sec [e + f x])^2} dx$$

Optimal (type 5, 248 leaves, 8 steps):

$$\begin{aligned} & \left(2 (2 - n p) \text{Hypergeometric2F1}\left[\frac{1}{2}, -\frac{n p}{2}, \frac{1}{2} (2 - n p), \cos [e + f x]^2\right] \right. \\ & \quad \left. (c (d \sec [e + f x])^p)^n \sin [e + f x]\right) / \left(3 a^2 f \sqrt{\sin [e + f x]^2}\right) - \\ & \left((3 - 2 n p) \cos [e + f x] \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{2} (1 - n p), \frac{1}{2} (3 - n p), \cos [e + f x]^2\right] \right. \\ & \quad \left. (c (d \sec [e + f x])^p)^n \sin [e + f x]\right) / \left(3 a^2 f \sqrt{\sin [e + f x]^2}\right) - \\ & \frac{2 (2 - n p) (c (d \sec [e + f x])^p)^n \tan [e + f x]}{3 a^2 f (1 + \sec [e + f x])} - \frac{(c (d \sec [e + f x])^p)^n \tan [e + f x]}{3 f (a + a \sec [e + f x])^2} \end{aligned}$$

Result (type 8, 29 leaves):

$$\int \frac{(c (d \sec [e + f x])^p)^n}{(a + a \sec [e + f x])^2} dx$$

Problem 240: Result more than twice size of optimal antiderivative.

$$\int \frac{(c (d \sec [e + f x])^p)^n}{a + b \sec [e + f x]} dx$$

Optimal (type 6, 206 leaves, 7 steps):

$$\begin{aligned} & -\frac{1}{(a^2 - b^2) f} b \text{AppellF1}\left[\frac{1}{2}, \frac{n p}{2}, 1, \frac{3}{2}, \sin[e + f x]^2, \frac{a^2 \sin[e + f x]^2}{a^2 - b^2}\right] \\ & (\cos[e + f x]^2)^{\frac{n p}{2}} (c (d \sec [e + f x])^p)^n \sin[e + f x] + \frac{1}{(a^2 - b^2) f} \\ & a \text{AppellF1}\left[\frac{1}{2}, \frac{1}{2} (-1 + n p), 1, \frac{3}{2}, \sin[e + f x]^2, \frac{a^2 \sin[e + f x]^2}{a^2 - b^2}\right] \cos[e + f x] \\ & (\cos[e + f x]^2)^{\frac{1}{2} (-1 + n p)} (c (d \sec [e + f x])^p)^n \sin[e + f x] \end{aligned}$$

Result (type 6, 5411 leaves):

$$\begin{aligned} & \left((c (d \sec [e + f x])^p)^n \tan[e + f x] \left(-b \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{2} - \frac{n p}{2}, \frac{3}{2}, -\tan[e + f x]^2\right] + \right. \right. \\ & a \text{Hypergeometric2F1}\left[\frac{1}{2}, 1 - \frac{n p}{2}, \frac{3}{2}, -\tan[e + f x]^2\right] + \left(3 a b^2 (a^2 - b^2) \right. \\ & \left. \text{AppellF1}\left[\frac{1}{2}, -\frac{n p}{2}, 1, \frac{3}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2}\right] (1 + \tan[e + f x]^2)^{\frac{n p}{2}} \right) / \\ & \left(\left(3 (a^2 - b^2) \text{AppellF1}\left[\frac{1}{2}, -\frac{n p}{2}, 1, \frac{3}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2}\right] + \right. \right. \\ & \left. \left(2 b^2 \text{AppellF1}\left[\frac{3}{2}, -\frac{n p}{2}, 2, \frac{5}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2}\right] + (a^2 - b^2) n p \right. \\ & \left. \text{AppellF1}\left[\frac{3}{2}, 1 - \frac{n p}{2}, 1, \frac{5}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2}\right]\right) \tan[e + f x]^2 \\ & \left. \left(a^2 - b^2 (1 + \tan[e + f x]^2) \right) \right) + \left(3 b^3 (a^2 - b^2) \text{AppellF1}\left[\frac{1}{2}, -\frac{1}{2} - \frac{n p}{2}, 1, \frac{3}{2}, \right. \right. \\ & \left. \left. -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2}\right] (1 + \tan[e + f x]^2)^{\frac{1}{2} (1+n p)} \right) / \\ & \left(\left(3 (a^2 - b^2) \text{AppellF1}\left[\frac{1}{2}, -\frac{1}{2} - \frac{n p}{2}, 1, \frac{3}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2}\right] + \right. \right. \\ & \left. \left(2 b^2 \text{AppellF1}\left[\frac{3}{2}, -\frac{1}{2} - \frac{n p}{2}, 2, \frac{5}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2}\right] + \right. \right. \\ & \left. \left. (a^2 - b^2) (1 + n p) \text{AppellF1}\left[\frac{3}{2}, \frac{1}{2} - \frac{n p}{2}, 1, \frac{5}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2}\right]\right) \right. \\ & \left. \left. \tan[e + f x]^2 \right) \left(-a^2 + b^2 (1 + \tan[e + f x]^2) \right) \right) \right) / \left(a^2 f (a + b \sec [e + f x]) \right. \\ & \left. \left(\frac{1}{a^2} \sec [e + f x]^2 \left(-b \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{2} - \frac{n p}{2}, \frac{3}{2}, -\tan[e + f x]^2\right] + \right. \right. \right. \right. \end{aligned}$$

$$\begin{aligned}
& a \text{Hypergeometric2F1}\left[\frac{1}{2}, 1 - \frac{n p}{2}, \frac{3}{2}, -\tan[e + f x]^2\right] + \left(3 a b^2 (a^2 - b^2)\right. \\
& \quad \left. \text{AppellF1}\left[\frac{1}{2}, -\frac{n p}{2}, 1, \frac{3}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2}\right] (1 + \tan[e + f x]^2)^{\frac{n p}{2}}\right) / \\
& \quad \left(\left(3 (a^2 - b^2) \text{AppellF1}\left[\frac{1}{2}, -\frac{n p}{2}, 1, \frac{3}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2}\right] + \right. \right. \\
& \quad \left. \left. \left(2 b^2 \text{AppellF1}\left[\frac{3}{2}, -\frac{n p}{2}, 2, \frac{5}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2}\right] + (a^2 - b^2) n p\right.\right. \\
& \quad \left.\left. \text{AppellF1}\left[\frac{3}{2}, 1 - \frac{n p}{2}, 1, \frac{5}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2}\right]\right) \tan[e + f x]^2 \right) \\
& \quad \left(a^2 - b^2 (1 + \tan[e + f x]^2)\right)) + \left(3 b^3 (a^2 - b^2) \text{AppellF1}\left[\frac{1}{2}, -\frac{1}{2} - \frac{n p}{2}, 1,\right.\right. \\
& \quad \left.\left. \frac{3}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2}\right] (1 + \tan[e + f x]^2)^{\frac{1}{2}(1+n p)}\right) / \\
& \quad \left(\left(3 (a^2 - b^2) \text{AppellF1}\left[\frac{1}{2}, -\frac{1}{2} - \frac{n p}{2}, 1, \frac{3}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2}\right] + \right. \right. \\
& \quad \left. \left. \left(2 b^2 \text{AppellF1}\left[\frac{3}{2}, -\frac{1}{2} - \frac{n p}{2}, 2, \frac{5}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2}\right] + \right.\right. \\
& \quad \left.\left. (a^2 - b^2) (1 + n p) \text{AppellF1}\left[\frac{3}{2}, \frac{1}{2} - \frac{n p}{2}, 1, \frac{5}{2}, -\tan[e + f x]^2,\right.\right. \\
& \quad \left.\left. \frac{b^2 \tan[e + f x]^2}{a^2 - b^2}\right] \tan[e + f x]^2\right) (-a^2 + b^2 (1 + \tan[e + f x]^2)) \right)) + \\
& \frac{1}{a^2} \tan[e + f x] \left(\left(6 a b^4 (a^2 - b^2) \text{AppellF1}\left[\frac{1}{2}, -\frac{n p}{2}, 1, \frac{3}{2}, -\tan[e + f x]^2,\right.\right. \right. \\
& \quad \left.\left. \frac{b^2 \tan[e + f x]^2}{a^2 - b^2}\right] \sec[e + f x]^2 \tan[e + f x] (1 + \tan[e + f x]^2)^{\frac{n p}{2}}\right) / \\
& \quad \left(\left(3 (a^2 - b^2) \text{AppellF1}\left[\frac{1}{2}, -\frac{n p}{2}, 1, \frac{3}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2}\right] + \right. \right. \\
& \quad \left. \left. \left(2 b^2 \text{AppellF1}\left[\frac{3}{2}, -\frac{n p}{2}, 2, \frac{5}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2}\right] + (a^2 - b^2) n p\right.\right. \\
& \quad \left.\left. \text{AppellF1}\left[\frac{3}{2}, 1 - \frac{n p}{2}, 1, \frac{5}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2}\right]\right) \tan[e + f x]^2 \right) \\
& \quad \left(a^2 - b^2 (1 + \tan[e + f x]^2)\right)^2) + \left(3 a b^2 (a^2 - b^2) \left(\frac{1}{3 (a^2 - b^2)} 2 b^2 \text{AppellF1}\left[\frac{3}{2}, -\frac{n p}{2}, 2, \frac{5}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2}\right] \sec[e + f x]^2 \tan[e + f x] + \right.\right. \\
& \quad \left.\left. \frac{1}{3} n p \text{AppellF1}\left[\frac{3}{2}, 1 - \frac{n p}{2}, 1, \frac{5}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2}\right]\right. \right. \\
& \quad \left.\left. \sec[e + f x]^2 \tan[e + f x]\right) (1 + \tan[e + f x]^2)^{\frac{n p}{2}}\right) / \\
& \quad \left(\left(3 (a^2 - b^2) \text{AppellF1}\left[\frac{1}{2}, -\frac{n p}{2}, 1, \frac{3}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2}\right] + \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left(2 b^2 \text{AppellF1} \left[\frac{3}{2}, -\frac{n p}{2}, 2, \frac{5}{2}, -\tan[e+f x]^2, \frac{b^2 \tan[e+f x]^2}{a^2-b^2} \right] + \right. \\
& \quad \left. (a^2-b^2) n p \text{AppellF1} \left[\frac{3}{2}, 1-\frac{n p}{2}, 1, \frac{5}{2}, -\tan[e+f x]^2, \frac{b^2 \tan[e+f x]^2}{a^2-b^2} \right] \right) \\
& \quad \left. \tan[e+f x]^2 \right) \left(a^2-b^2 (1+\tan[e+f x]^2) \right) + \\
& \left(3 a b^2 (a^2-b^2) n p \text{AppellF1} \left[\frac{1}{2}, -\frac{n p}{2}, 1, \frac{3}{2}, -\tan[e+f x]^2, \frac{b^2 \tan[e+f x]^2}{a^2-b^2} \right] \right. \\
& \quad \left. \sec[e+f x]^2 \tan[e+f x] (1+\tan[e+f x]^2)^{-1+\frac{n p}{2}} \right) / \\
& \left(\left(3 (a^2-b^2) \text{AppellF1} \left[\frac{1}{2}, -\frac{n p}{2}, 1, \frac{3}{2}, -\tan[e+f x]^2, \frac{b^2 \tan[e+f x]^2}{a^2-b^2} \right] + \right. \right. \\
& \quad \left. \left. 2 b^2 \text{AppellF1} \left[\frac{3}{2}, -\frac{n p}{2}, 2, \frac{5}{2}, -\tan[e+f x]^2, \frac{b^2 \tan[e+f x]^2}{a^2-b^2} \right] + \right. \\
& \quad \left. \left. (a^2-b^2) n p \text{AppellF1} \left[\frac{3}{2}, 1-\frac{n p}{2}, 1, \frac{5}{2}, -\tan[e+f x]^2, \frac{b^2 \tan[e+f x]^2}{a^2-b^2} \right] \right) \\
& \quad \left. \tan[e+f x]^2 \right) \left(a^2-b^2 (1+\tan[e+f x]^2) \right) - \\
& \left(6 b^5 (a^2-b^2) \text{AppellF1} \left[\frac{1}{2}, -\frac{1}{2}-\frac{n p}{2}, 1, \frac{3}{2}, -\tan[e+f x]^2, \frac{b^2 \tan[e+f x]^2}{a^2-b^2} \right] \right. \\
& \quad \left. \sec[e+f x]^2 \tan[e+f x] (1+\tan[e+f x]^2)^{\frac{1}{2}(1+n p)} \right) / \\
& \left(\left(3 (a^2-b^2) \text{AppellF1} \left[\frac{1}{2}, -\frac{1}{2}-\frac{n p}{2}, 1, \frac{3}{2}, -\tan[e+f x]^2, \frac{b^2 \tan[e+f x]^2}{a^2-b^2} \right] + \right. \right. \\
& \quad \left. \left. 2 b^2 \text{AppellF1} \left[\frac{3}{2}, -\frac{1}{2}-\frac{n p}{2}, 2, \frac{5}{2}, -\tan[e+f x]^2, \frac{b^2 \tan[e+f x]^2}{a^2-b^2} \right] + \right. \\
& \quad \left. \left. (a^2-b^2) (1+n p) \text{AppellF1} \left[\frac{3}{2}, \frac{1}{2}-\frac{n p}{2}, 1, \frac{5}{2}, -\tan[e+f x]^2, \right. \right. \\
& \quad \left. \left. \frac{b^2 \tan[e+f x]^2}{a^2-b^2} \right] \right) \tan[e+f x]^2 \left(-a^2+b^2 (1+\tan[e+f x]^2) \right)^2 + \\
& \left(3 b^3 (a^2-b^2) \left(\frac{1}{3 (a^2-b^2)} 2 b^2 \text{AppellF1} \left[\frac{3}{2}, -\frac{1}{2}-\frac{n p}{2}, 2, \frac{5}{2}, -\tan[e+f x]^2, \right. \right. \right. \\
& \quad \left. \left. \frac{b^2 \tan[e+f x]^2}{a^2-b^2} \right] \sec[e+f x]^2 \tan[e+f x] - \\
& \quad \left. \left. \left. 2 \left(-\frac{1}{2}-\frac{n p}{2} \right) \text{AppellF1} \left[\frac{3}{2}, \frac{1}{2}-\frac{n p}{2}, 1, \frac{5}{2}, -\tan[e+f x]^2, \right. \right. \right. \\
& \quad \left. \left. \left. \frac{b^2 \tan[e+f x]^2}{a^2-b^2} \right] \sec[e+f x]^2 \tan[e+f x] \right) \left(1+\tan[e+f x]^2 \right)^{\frac{1}{2}(1+n p)} \right) / \\
& \left(\left(3 (a^2-b^2) \text{AppellF1} \left[\frac{1}{2}, -\frac{1}{2}-\frac{n p}{2}, 1, \frac{3}{2}, -\tan[e+f x]^2, \frac{b^2 \tan[e+f x]^2}{a^2-b^2} \right] + \right. \right. \\
& \quad \left. \left. 2 b^2 \text{AppellF1} \left[\frac{3}{2}, -\frac{1}{2}-\frac{n p}{2}, 2, \frac{5}{2}, -\tan[e+f x]^2, \frac{b^2 \tan[e+f x]^2}{a^2-b^2} \right] + \right)
\end{aligned}$$

$$\begin{aligned}
& 6 b^2 \operatorname{AppellF1}\left[\frac{5}{2}, \frac{1}{2} - \frac{n p}{2}, 2, \frac{7}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2}\right] \\
& \sec[e + f x]^2 \tan[e + f x] - \frac{6}{5} \left(\frac{1}{2} - \frac{n p}{2}\right) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{3}{2} - \frac{n p}{2}, 1, \frac{7}{2},\right. \\
& \left. -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2}\right] \sec[e + f x]^2 \tan[e + f x]\Bigg)\Bigg)\Bigg)\Bigg) \\
& \left(\left(3 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{1}{2} - \frac{n p}{2}, 1, \frac{3}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2}\right] + \right.\right. \\
& \left. \left(2 b^2 \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{1}{2} - \frac{n p}{2}, 2, \frac{5}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2}\right] + \right.\right. \\
& (a^2 - b^2) (1 + n p) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2} - \frac{n p}{2}, 1, \frac{5}{2}, -\tan[e + f x]^2,\right. \\
& \left. \left. \frac{b^2 \tan[e + f x]^2}{a^2 - b^2}\right)\right] \tan[e + f x]^2 \left(-a^2 + b^2 (1 + \tan[e + f x]^2)\right)\Bigg) - \\
& \left(3 a b^2 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{n p}{2}, 1, \frac{3}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2}\right]\right. \\
& \left. \left(1 + \tan[e + f x]^2 \right)^{\frac{n p}{2}}\right. \\
& \left. \left(2 \left(2 b^2 \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{n p}{2}, 2, \frac{5}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2}\right] + \right.\right. \right. \\
& (a^2 - b^2) n p \operatorname{AppellF1}\left[\frac{3}{2}, 1 - \frac{n p}{2}, 1, \frac{5}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2}\right]\Bigg) \\
& \sec[e + f x]^2 \tan[e + f x] + 3 (a^2 - b^2) \left(\frac{1}{3 (a^2 - b^2)} 2 b^2 \operatorname{AppellF1}\left[\frac{3}{2},\right. \right. \\
& \left. \left. -\frac{n p}{2}, 2, \frac{5}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2}\right] \sec[e + f x]^2 \tan[e + f x] + \right. \\
& \left. \frac{1}{3} n p \operatorname{AppellF1}\left[\frac{3}{2}, 1 - \frac{n p}{2}, 1, \frac{5}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2}\right]\right. \\
& \left. \sec[e + f x]^2 \tan[e + f x]\right) + \tan[e + f x]^2 \left(2 b^2 \left(\frac{1}{5 (a^2 - b^2)} 12 b^2 \operatorname{AppellF1}\left[\frac{5}{2},\right. \right. \right. \\
& \left. \left. \left. -\frac{n p}{2}, 3, \frac{7}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2}\right] \sec[e + f x]^2 \tan[e + f x] + \right.\right. \\
& \left. \left. \frac{3}{5} n p \operatorname{AppellF1}\left[\frac{5}{2}, 1 - \frac{n p}{2}, 2, \frac{7}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2}\right]\right.\right. \\
& \left. \left. \sec[e + f x]^2 \tan[e + f x]\right) + (a^2 - b^2) n p \left(\frac{1}{5 (a^2 - b^2)}\right.\right. \\
& \left. \left. 6 b^2 \operatorname{AppellF1}\left[\frac{5}{2}, 1 - \frac{n p}{2}, 2, \frac{7}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2}\right]\right.\right. \\
& \left. \left. \sec[e + f x]^2 \tan[e + f x] - \frac{6}{5} \left(1 - \frac{n p}{2}\right) \operatorname{AppellF1}\left[\frac{5}{2}, 2 - \frac{n p}{2}, 1, \frac{7}{2},\right.\right. \right. \\
& \left. \left. \left. -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2}\right] \sec[e + f x]^2 \tan[e + f x]\right)\right)\right)\Bigg)
\end{aligned}$$

$$\left(\left(3 (a^2 - b^2) \text{AppellF1} \left[\frac{1}{2}, -\frac{n p}{2}, 1, \frac{3}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2} \right] + \right. \right. \\ \left. \left. \left(2 b^2 \text{AppellF1} \left[\frac{3}{2}, -\frac{n p}{2}, 2, \frac{5}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2} \right] + \right. \right. \\ \left. \left. (a^2 - b^2) n p \text{AppellF1} \left[\frac{3}{2}, 1 - \frac{n p}{2}, 1, \frac{5}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2} \right] \right) \right) \\ \left. \left. \left. \tan[e + f x]^2 \right)^2 (a^2 - b^2 (1 + \tan[e + f x]^2)) \right) \right)$$

Problem 241: Result more than twice size of optimal antiderivative.

$$\int \frac{(c (d \sec[e + f x])^p)^n}{(a + b \sec[e + f x])^2} dx$$

Optimal (type 6, 322 leaves, 10 steps):

$$-\frac{1}{(a^2 - b^2)^2 f} 2 a b \text{AppellF1} \left[\frac{1}{2}, \frac{1}{2} (-2 + n p), 2, \frac{3}{2}, \sin[e + f x]^2, \frac{a^2 \sin[e + f x]^2}{a^2 - b^2} \right] \\ (\cos[e + f x]^2)^{\frac{n p}{2}} (c (d \sec[e + f x])^p)^n \sin[e + f x] + \frac{1}{(a^2 - b^2)^2 f} \\ a^2 \text{AppellF1} \left[\frac{1}{2}, \frac{1}{2} (-3 + n p), 2, \frac{3}{2}, \sin[e + f x]^2, \frac{a^2 \sin[e + f x]^2}{a^2 - b^2} \right] \cos[e + f x] \\ (\cos[e + f x]^2)^{\frac{1}{2} (-1+n p)} (c (d \sec[e + f x])^p)^n \sin[e + f x] + \frac{1}{(a^2 - b^2)^2 f} \\ b^2 \text{AppellF1} \left[\frac{1}{2}, \frac{1}{2} (-1 + n p), 2, \frac{3}{2}, \sin[e + f x]^2, \frac{a^2 \sin[e + f x]^2}{a^2 - b^2} \right] \\ \cos[e + f x] (\cos[e + f x]^2)^{\frac{1}{2} (-1+n p)} (c (d \sec[e + f x])^p)^n \sin[e + f x]$$

Result (type 6, 10678 leaves):

$$\left((c (d \sec[e + f x])^p)^n \left(-\frac{1}{a^3} 2 b \text{Hypergeometric2F1} \left[\frac{1}{2}, \frac{1}{2} - \frac{n p}{2}, \frac{3}{2}, -\tan[e + f x]^2 \right] \tan[e + f x] + \right. \right. \\ \left. \left. \frac{1}{a^2} \text{Hypergeometric2F1} \left[\frac{1}{2}, 1 - \frac{n p}{2}, \frac{3}{2}, -\tan[e + f x]^2 \right] \tan[e + f x] - \right. \right. \\ \left. \left. 6 b^3 (a^2 - b^2) \text{AppellF1} \left[\frac{1}{2}, -\frac{1}{2} - \frac{n p}{2}, 2, \frac{3}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2} \right] \right. \right. \\ \left. \left. \tan[e + f x] (1 + \tan[e + f x]^2)^{\frac{1}{2} (1+n p)} \right) \right/ \\ \left(a \left(3 (a^2 - b^2) \text{AppellF1} \left[\frac{1}{2}, -\frac{1}{2} - \frac{n p}{2}, 2, \frac{3}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2} \right] + \right. \right. \\ \left. \left. 4 b^2 \text{AppellF1} \left[\frac{3}{2}, -\frac{1}{2} - \frac{n p}{2}, 3, \frac{5}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2} \right] + \right. \right. \\ \left. \left. (a^2 - b^2) (1 + n p) \text{AppellF1} \left[\frac{3}{2}, \frac{1}{2} - \frac{n p}{2}, 2, \frac{5}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2} \right] \right) \right)$$

$$\begin{aligned}
& \left. \left(\frac{\tan(e + fx)^2}{a^2 - b^2} \left(1 + \tan(e + fx)^2 \right)^2 \right) + \right. \\
& \left. \left(6b^2 (a^2 - b^2) \text{AppellF1} \left[\frac{1}{2}, -\frac{np}{2}, 2, \frac{3}{2}, -\tan(e + fx)^2, \frac{b^2 \tan(e + fx)^2}{a^2 - b^2} \right] \right. \\
& \left. \left. \tan(e + fx) \left(1 + \tan(e + fx)^2 \right)^{\frac{np}{2}} \right) \right/ \\
& \left(\left(3 (a^2 - b^2) \text{AppellF1} \left[\frac{1}{2}, -\frac{np}{2}, 2, \frac{3}{2}, -\tan(e + fx)^2, \frac{b^2 \tan(e + fx)^2}{a^2 - b^2} \right] + \right. \right. \\
& \left. \left. \left(4b^2 \text{AppellF1} \left[\frac{3}{2}, -\frac{np}{2}, 3, \frac{5}{2}, -\tan(e + fx)^2, \frac{b^2 \tan(e + fx)^2}{a^2 - b^2} \right] + (a^2 - b^2) np \right. \right. \\
& \left. \left. \text{AppellF1} \left[\frac{3}{2}, 1 - \frac{np}{2}, 2, \frac{5}{2}, -\tan(e + fx)^2, \frac{b^2 \tan(e + fx)^2}{a^2 - b^2} \right] \right) \tan(e + fx)^2 \right) \\
& \left. \left(a^2 - b^2 \left(1 + \tan(e + fx)^2 \right)^2 \right) \right) + \left(6b^3 (a^2 - b^2) \text{AppellF1} \left[\frac{1}{2}, -\frac{1}{2} - \frac{np}{2}, 1, \frac{3}{2}, \right. \right. \\
& \left. \left. -\tan(e + fx)^2, \frac{b^2 \tan(e + fx)^2}{a^2 - b^2} \right] \tan(e + fx) \left(1 + \tan(e + fx)^2 \right)^{\frac{1}{2}(1+np)} \right) \right/ \\
& \left(a^3 \left(3 (a^2 - b^2) \text{AppellF1} \left[\frac{1}{2}, -\frac{1}{2} - \frac{np}{2}, 1, \frac{3}{2}, -\tan(e + fx)^2, \frac{b^2 \tan(e + fx)^2}{a^2 - b^2} \right] + \right. \right. \\
& \left. \left. \left(2b^2 \text{AppellF1} \left[\frac{3}{2}, -\frac{1}{2} - \frac{np}{2}, 2, \frac{5}{2}, -\tan(e + fx)^2, \frac{b^2 \tan(e + fx)^2}{a^2 - b^2} \right] + \right. \right. \\
& \left. \left. (a^2 - b^2) (1 + np) \text{AppellF1} \left[\frac{3}{2}, \frac{1}{2} - \frac{np}{2}, 1, \frac{5}{2}, -\tan(e + fx)^2, \frac{b^2 \tan(e + fx)^2}{a^2 - b^2} \right] \right) \right. \\
& \left. \tan(e + fx)^2 \right) \left(-a^2 + b^2 \left(1 + \tan(e + fx)^2 \right) \right) \right) - \\
& \left(3b^2 (a^2 - b^2) \text{AppellF1} \left[\frac{1}{2}, -\frac{np}{2}, 1, \frac{3}{2}, -\tan(e + fx)^2, \frac{b^2 \tan(e + fx)^2}{a^2 - b^2} \right] \right. \\
& \left. \tan(e + fx) \left(1 + \tan(e + fx)^2 \right)^{\frac{np}{2}} \right) \right/ \\
& \left(a^2 \left(3 (a^2 - b^2) \text{AppellF1} \left[\frac{1}{2}, -\frac{np}{2}, 1, \frac{3}{2}, -\tan(e + fx)^2, \frac{b^2 \tan(e + fx)^2}{a^2 - b^2} \right] + \right. \right. \\
& \left. \left. \left(2b^2 \text{AppellF1} \left[\frac{3}{2}, -\frac{np}{2}, 2, \frac{5}{2}, -\tan(e + fx)^2, \frac{b^2 \tan(e + fx)^2}{a^2 - b^2} \right] + (a^2 - b^2) np \right. \right. \\
& \left. \left. \text{AppellF1} \left[\frac{3}{2}, 1 - \frac{np}{2}, 1, \frac{5}{2}, -\tan(e + fx)^2, \frac{b^2 \tan(e + fx)^2}{a^2 - b^2} \right] \right) \tan(e + fx)^2 \right) \\
& \left. \left(-a^2 + b^2 \left(1 + \tan(e + fx)^2 \right) \right) \right) \right) \right/ \left(f (a + b \sec(e + fx))^2 \right. \\
& \left(-\frac{1}{a^3} 2b \text{Hypergeometric2F1} \left[\frac{1}{2}, \frac{1}{2} - \frac{np}{2}, \frac{3}{2}, -\tan(e + fx)^2 \right] \sec(e + fx)^2 + \right. \\
& \left. \frac{1}{a^2} \right. \\
& \left. \text{Hypergeometric2F1} \left[\frac{1}{2}, 1 - \frac{np}{2}, \frac{3}{2}, -\tan(e + fx)^2 \right] \sec(e + fx)^2 - \right)
\end{aligned}$$

$$\begin{aligned}
& \left(24 b^5 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{2}, -\frac{1}{2} - \frac{n p}{2}, 2, \frac{3}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2} \right] \right. \\
& \quad \left. \sec[e + f x]^2 \tan[e + f x]^2 (1 + \tan[e + f x]^2)^{\frac{1}{2}(1+n p)} \right) / \\
& \left(a \left(3 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{2}, -\frac{1}{2} - \frac{n p}{2}, 2, \frac{3}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2} \right] + \right. \right. \\
& \quad \left. \left(4 b^2 \operatorname{AppellF1} \left[\frac{3}{2}, -\frac{1}{2} - \frac{n p}{2}, 3, \frac{5}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2} \right] + \right. \\
& \quad \left. \left. (a^2 - b^2) (1 + n p) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{1}{2} - \frac{n p}{2}, 2, \frac{5}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2} \right] \right) \right. \\
& \quad \left. \tan[e + f x]^2 \right) (a^2 - b^2 (1 + \tan[e + f x]^2))^3 \right) + \\
& \left(24 b^4 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{2}, -\frac{n p}{2}, 2, \frac{3}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2} \right] \right. \\
& \quad \left. \sec[e + f x]^2 \tan[e + f x]^2 (1 + \tan[e + f x]^2)^{\frac{n p}{2}} \right) / \\
& \left(\left(3 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{2}, -\frac{n p}{2}, 2, \frac{3}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2} \right] + \right. \right. \\
& \quad \left. \left(4 b^2 \operatorname{AppellF1} \left[\frac{3}{2}, -\frac{n p}{2}, 3, \frac{5}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2} \right] + (a^2 - b^2) n p \right. \\
& \quad \left. \operatorname{AppellF1} \left[\frac{3}{2}, 1 - \frac{n p}{2}, 2, \frac{5}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2} \right] \right) \tan[e + f x]^2 \right) \\
& \quad (a^2 - b^2 (1 + \tan[e + f x]^2))^3 \Bigg) - \left(6 b^3 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{2}, -\frac{1}{2} - \frac{n p}{2}, 2, \frac{3}{2}, \right. \right. \\
& \quad \left. \left. -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2} \right] \sec[e + f x]^2 (1 + \tan[e + f x]^2)^{\frac{1}{2}(1+n p)} \right) / \\
& \left(a \left(3 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{2}, -\frac{1}{2} - \frac{n p}{2}, 2, \frac{3}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2} \right] + \right. \right. \\
& \quad \left. \left(4 b^2 \operatorname{AppellF1} \left[\frac{3}{2}, -\frac{1}{2} - \frac{n p}{2}, 3, \frac{5}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2} \right] + \right. \\
& \quad \left. \left. (a^2 - b^2) (1 + n p) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{1}{2} - \frac{n p}{2}, 2, \frac{5}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2} \right] \right) \right. \\
& \quad \left. \tan[e + f x]^2 \right) (a^2 - b^2 (1 + \tan[e + f x]^2))^2 \right) - \left(6 b^3 (a^2 - b^2) \tan[e + f x] \right. \\
& \quad \left(\frac{1}{3 (a^2 - b^2)} 4 b^2 \operatorname{AppellF1} \left[\frac{3}{2}, -\frac{1}{2} - \frac{n p}{2}, 3, \frac{5}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2} \right] \right. \\
& \quad \left. \sec[e + f x]^2 \tan[e + f x] - \frac{2}{3} \left(-\frac{1}{2} - \frac{n p}{2} \right) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{1}{2} - \frac{n p}{2}, 2, \frac{5}{2}, -\tan[e + f x]^2, \right. \right. \\
& \quad \left. \left. \frac{b^2 \tan[e + f x]^2}{a^2 - b^2} \right] \sec[e + f x]^2 \tan[e + f x] \right) (1 + \tan[e + f x]^2)^{\frac{1}{2}(1+n p)} \Bigg) / \\
& \left(a \left(3 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{2}, -\frac{1}{2} - \frac{n p}{2}, 2, \frac{3}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2} \right] + \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left(4 b^2 \operatorname{AppellF1} \left[\frac{3}{2}, -\frac{1}{2} - \frac{n p}{2}, 3, \frac{5}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2} \right] + \right. \\
& \quad \left(a^2 - b^2 \right) \left(1 + n p \right) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{1}{2} - \frac{n p}{2}, 2, \frac{5}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2} \right] \Big) \\
& \quad \left. \tan[e + f x]^2 \right) \left(a^2 - b^2 \left(1 + \tan[e + f x]^2 \right) \right)^2 \Big) - \\
& \left(6 b^3 \left(a^2 - b^2 \right) \left(1 + n p \right) \operatorname{AppellF1} \left[\frac{1}{2}, -\frac{1}{2} - \frac{n p}{2}, 2, \frac{3}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2} \right] \right. \\
& \quad \left. \sec[e + f x]^2 \tan[e + f x]^2 \left(1 + \tan[e + f x]^2 \right)^{-1 + \frac{1}{2}(1+n p)} \right) \Big) / \\
& \left(a \left(3 \left(a^2 - b^2 \right) \operatorname{AppellF1} \left[\frac{1}{2}, -\frac{1}{2} - \frac{n p}{2}, 2, \frac{3}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2} \right] + \right. \right. \\
& \quad \left(4 b^2 \operatorname{AppellF1} \left[\frac{3}{2}, -\frac{1}{2} - \frac{n p}{2}, 3, \frac{5}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2} \right] + \right. \\
& \quad \left. \left. \left(a^2 - b^2 \right) \left(1 + n p \right) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{1}{2} - \frac{n p}{2}, 2, \frac{5}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2} \right] \right) \right. \\
& \quad \left. \tan[e + f x]^2 \right) \left(a^2 - b^2 \left(1 + \tan[e + f x]^2 \right) \right)^2 \Big) + \\
& \left(6 b^2 \left(a^2 - b^2 \right) \operatorname{AppellF1} \left[\frac{1}{2}, -\frac{n p}{2}, 2, \frac{3}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2} \right] \right. \\
& \quad \left. \sec[e + f x]^2 \left(1 + \tan[e + f x]^2 \right)^{\frac{n p}{2}} \right) \Big) / \\
& \left(\left(3 \left(a^2 - b^2 \right) \operatorname{AppellF1} \left[\frac{1}{2}, -\frac{n p}{2}, 2, \frac{3}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2} \right] + \right. \right. \\
& \quad \left(4 b^2 \operatorname{AppellF1} \left[\frac{3}{2}, -\frac{n p}{2}, 3, \frac{5}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2} \right] + \left(a^2 - b^2 \right) n p \right. \\
& \quad \left. \operatorname{AppellF1} \left[\frac{3}{2}, 1 - \frac{n p}{2}, 2, \frac{5}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2} \right] \right) \tan[e + f x]^2 \Big) \\
& \quad \left(a^2 - b^2 \left(1 + \tan[e + f x]^2 \right) \right)^2 \Big) + \left(6 b^2 \left(a^2 - b^2 \right) \tan[e + f x] \right. \\
& \quad \left. \left(\frac{1}{3 \left(a^2 - b^2 \right)} 4 b^2 \operatorname{AppellF1} \left[\frac{3}{2}, -\frac{n p}{2}, 3, \frac{5}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2} \right] \sec[e + f x]^2 \right. \right. \\
& \quad \left. \tan[e + f x] + \frac{1}{3} n p \operatorname{AppellF1} \left[\frac{3}{2}, 1 - \frac{n p}{2}, 2, \frac{5}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2} \right] \right. \\
& \quad \left. \sec[e + f x]^2 \tan[e + f x] \right) \left(1 + \tan[e + f x]^2 \right)^{\frac{n p}{2}} \Big) \Big) / \\
& \left(\left(3 \left(a^2 - b^2 \right) \operatorname{AppellF1} \left[\frac{1}{2}, -\frac{n p}{2}, 2, \frac{3}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2} \right] + \right. \right. \\
& \quad \left(4 b^2 \operatorname{AppellF1} \left[\frac{3}{2}, -\frac{n p}{2}, 3, \frac{5}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2} \right] + \right. \\
& \quad \left. \left. \left(a^2 - b^2 \right) n p \operatorname{AppellF1} \left[\frac{3}{2}, 1 - \frac{n p}{2}, 2, \frac{5}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2} \right] \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left. \left(\frac{\tan(e + f x)^2}{a^2 - b^2} \left(1 + \tan(e + f x)^2 \right)^2 \right) + \right. \\
& \left(6 b^2 (a^2 - b^2) n p \text{AppellF1} \left[\frac{1}{2}, -\frac{n p}{2}, 2, \frac{3}{2}, -\tan(e + f x)^2, \frac{b^2 \tan(e + f x)^2}{a^2 - b^2} \right] \right. \\
& \left. \left. \sec(e + f x)^2 \tan(e + f x)^2 \left(1 + \tan(e + f x)^2 \right)^{-1+\frac{n p}{2}} \right) \right/ \\
& \left(\left(3 (a^2 - b^2) \text{AppellF1} \left[\frac{1}{2}, -\frac{n p}{2}, 2, \frac{3}{2}, -\tan(e + f x)^2, \frac{b^2 \tan(e + f x)^2}{a^2 - b^2} \right] + \right. \right. \\
& \left. \left. \left(4 b^2 \text{AppellF1} \left[\frac{3}{2}, -\frac{n p}{2}, 3, \frac{5}{2}, -\tan(e + f x)^2, \frac{b^2 \tan(e + f x)^2}{a^2 - b^2} \right] + \right. \right. \\
& \left. \left. \left(a^2 - b^2 \right) n p \text{AppellF1} \left[\frac{3}{2}, 1 - \frac{n p}{2}, 2, \frac{5}{2}, -\tan(e + f x)^2, \frac{b^2 \tan(e + f x)^2}{a^2 - b^2} \right] \right) \right. \\
& \left. \left. \tan(e + f x)^2 \right) \left(a^2 - b^2 \left(1 + \tan(e + f x)^2 \right)^2 \right) \right) - \\
& \left(12 b^5 (a^2 - b^2) \text{AppellF1} \left[\frac{1}{2}, -\frac{1}{2} - \frac{n p}{2}, 1, \frac{3}{2}, -\tan(e + f x)^2, \frac{b^2 \tan(e + f x)^2}{a^2 - b^2} \right] \right. \\
& \left. \left. \sec(e + f x)^2 \tan(e + f x)^2 \left(1 + \tan(e + f x)^2 \right)^{\frac{1}{2}(1+n p)} \right) \right/ \\
& \left(a^3 \left(3 (a^2 - b^2) \text{AppellF1} \left[\frac{1}{2}, -\frac{1}{2} - \frac{n p}{2}, 1, \frac{3}{2}, -\tan(e + f x)^2, \frac{b^2 \tan(e + f x)^2}{a^2 - b^2} \right] + \right. \right. \\
& \left. \left. \left(2 b^2 \text{AppellF1} \left[\frac{3}{2}, -\frac{1}{2} - \frac{n p}{2}, 2, \frac{5}{2}, -\tan(e + f x)^2, \frac{b^2 \tan(e + f x)^2}{a^2 - b^2} \right] + \right. \right. \\
& \left. \left. \left(a^2 - b^2 \right) (1 + n p) \text{AppellF1} \left[\frac{3}{2}, \frac{1}{2} - \frac{n p}{2}, 1, \frac{5}{2}, -\tan(e + f x)^2, \frac{b^2 \tan(e + f x)^2}{a^2 - b^2} \right] \right) \right. \\
& \left. \left. \tan(e + f x)^2 \right) \left(-a^2 + b^2 \left(1 + \tan(e + f x)^2 \right)^2 \right) \right) + \\
& \left(6 b^4 (a^2 - b^2) \text{AppellF1} \left[\frac{1}{2}, -\frac{n p}{2}, 1, \frac{3}{2}, -\tan(e + f x)^2, \frac{b^2 \tan(e + f x)^2}{a^2 - b^2} \right] \right. \\
& \left. \left. \sec(e + f x)^2 \tan(e + f x)^2 \left(1 + \tan(e + f x)^2 \right)^{\frac{n p}{2}} \right) \right/ \\
& \left(a^2 \left(3 (a^2 - b^2) \text{AppellF1} \left[\frac{1}{2}, -\frac{n p}{2}, 1, \frac{3}{2}, -\tan(e + f x)^2, \frac{b^2 \tan(e + f x)^2}{a^2 - b^2} \right] + \right. \right. \\
& \left. \left. \left(2 b^2 \text{AppellF1} \left[\frac{3}{2}, -\frac{n p}{2}, 2, \frac{5}{2}, -\tan(e + f x)^2, \frac{b^2 \tan(e + f x)^2}{a^2 - b^2} \right] + \right. \right. \\
& \left. \left. \left(a^2 - b^2 \right) n p \text{AppellF1} \left[\frac{3}{2}, 1 - \frac{n p}{2}, 1, \frac{5}{2}, -\tan(e + f x)^2, \frac{b^2 \tan(e + f x)^2}{a^2 - b^2} \right] \right) \right. \\
& \left. \left. \tan(e + f x)^2 \right) \left(-a^2 + b^2 \left(1 + \tan(e + f x)^2 \right)^2 \right) \right) + \\
& \left(6 b^3 (a^2 - b^2) \text{AppellF1} \left[\frac{1}{2}, -\frac{1}{2} - \frac{n p}{2}, 1, \frac{3}{2}, -\tan(e + f x)^2, \frac{b^2 \tan(e + f x)^2}{a^2 - b^2} \right] \right. \\
& \left. \left. \sec(e + f x)^2 \left(1 + \tan(e + f x)^2 \right)^{\frac{1}{2}(1+n p)} \right) \right/
\end{aligned}$$

$$\begin{aligned}
& \left(a^3 \left(3 (a^2 - b^2) \text{AppellF1} \left[\frac{1}{2}, -\frac{1}{2} - \frac{n p}{2}, 1, \frac{3}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2} \right] + \right. \right. \\
& \quad \left(2 b^2 \text{AppellF1} \left[\frac{3}{2}, -\frac{1}{2} - \frac{n p}{2}, 2, \frac{5}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2} \right] + \right. \\
& \quad \left. \left. (a^2 - b^2) (1 + n p) \text{AppellF1} \left[\frac{3}{2}, \frac{1}{2} - \frac{n p}{2}, 1, \frac{5}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2} \right] \right) \right. \\
& \quad \left. \tan[e + f x]^2 \right) (-a^2 + b^2 (1 + \tan[e + f x]^2)) \Bigg) + \\
& \left(6 b^3 (a^2 - b^2) \tan[e + f x] \left(\frac{1}{3 (a^2 - b^2)} 2 b^2 \text{AppellF1} \left[\frac{3}{2}, -\frac{1}{2} - \frac{n p}{2}, 2, \frac{5}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2} \right] \right. \right. \\
& \quad \left. \left. \sec[e + f x]^2 \tan[e + f x] - \frac{2}{3} \left(-\frac{1}{2} - \frac{n p}{2} \right) \text{AppellF1} \left[\frac{3}{2}, \frac{1}{2} - \frac{n p}{2}, 1, \frac{5}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2} \right] \right. \right. \\
& \quad \left. \left. \sec[e + f x]^2 \tan[e + f x] \right) (1 + \tan[e + f x]^2)^{\frac{1}{2} (1+n p)} \right) \Bigg) \Bigg/ \\
& \left(a^3 \left(3 (a^2 - b^2) \text{AppellF1} \left[\frac{1}{2}, -\frac{1}{2} - \frac{n p}{2}, 1, \frac{3}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2} \right] + \right. \right. \\
& \quad \left(2 b^2 \text{AppellF1} \left[\frac{3}{2}, -\frac{1}{2} - \frac{n p}{2}, 2, \frac{5}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2} \right] + \right. \\
& \quad \left. \left. (a^2 - b^2) (1 + n p) \text{AppellF1} \left[\frac{3}{2}, \frac{1}{2} - \frac{n p}{2}, 1, \frac{5}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2} \right] \right) \right. \\
& \quad \left. \tan[e + f x]^2 \right) (-a^2 + b^2 (1 + \tan[e + f x]^2)) \Bigg) + \\
& \left(6 b^3 (a^2 - b^2) (1 + n p) \text{AppellF1} \left[\frac{1}{2}, -\frac{1}{2} - \frac{n p}{2}, 1, \frac{3}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2} \right] \right. \\
& \quad \left. \sec[e + f x]^2 \tan[e + f x]^2 (1 + \tan[e + f x]^2)^{-1+\frac{1}{2} (1+n p)} \right) \Bigg) \Bigg/ \\
& \left(a^3 \left(3 (a^2 - b^2) \text{AppellF1} \left[\frac{1}{2}, -\frac{1}{2} - \frac{n p}{2}, 1, \frac{3}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2} \right] + \right. \right. \\
& \quad \left(2 b^2 \text{AppellF1} \left[\frac{3}{2}, -\frac{1}{2} - \frac{n p}{2}, 2, \frac{5}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2} \right] + \right. \\
& \quad \left. \left. (a^2 - b^2) (1 + n p) \text{AppellF1} \left[\frac{3}{2}, \frac{1}{2} - \frac{n p}{2}, 1, \frac{5}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2} \right] \right) \right. \\
& \quad \left. \tan[e + f x]^2 \right) (-a^2 + b^2 (1 + \tan[e + f x]^2)) \Bigg) - \\
& \left(3 b^2 (a^2 - b^2) \text{AppellF1} \left[\frac{1}{2}, -\frac{n p}{2}, 1, \frac{3}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2} \right] \right. \\
& \quad \left. \sec[e + f x]^2 (1 + \tan[e + f x]^2)^{\frac{n p}{2}} \right) \Bigg) \Bigg/ \\
& \left(a^2 \left(3 (a^2 - b^2) \text{AppellF1} \left[\frac{1}{2}, -\frac{n p}{2}, 1, \frac{3}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2} \right] + \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left(2 b^2 \text{AppellF1} \left[\frac{3}{2}, -\frac{n p}{2}, 2, \frac{5}{2}, -\tan[e+f x]^2, \frac{b^2 \tan[e+f x]^2}{a^2-b^2} \right] + (a^2-b^2) n p \right. \\
& \quad \left. \text{AppellF1} \left[\frac{3}{2}, 1-\frac{n p}{2}, 1, \frac{5}{2}, -\tan[e+f x]^2, \frac{b^2 \tan[e+f x]^2}{a^2-b^2} \right] \right) \tan[e+f x]^2 \\
& \left(-a^2 + b^2 (1 + \tan[e+f x]^2) \right) - \left(3 b^2 (a^2-b^2) \tan[e+f x] \right. \\
& \quad \left(\frac{1}{3 (a^2-b^2)} 2 b^2 \text{AppellF1} \left[\frac{3}{2}, -\frac{n p}{2}, 2, \frac{5}{2}, -\tan[e+f x]^2, \frac{b^2 \tan[e+f x]^2}{a^2-b^2} \right] \sec[e+f x]^2 \right. \\
& \quad \left. \tan[e+f x] + \frac{1}{3} n p \text{AppellF1} \left[\frac{3}{2}, 1-\frac{n p}{2}, 1, \frac{5}{2}, -\tan[e+f x]^2, \frac{b^2 \tan[e+f x]^2}{a^2-b^2} \right] \right. \\
& \quad \left. \sec[e+f x]^2 \tan[e+f x] \right) (1 + \tan[e+f x]^2)^{\frac{n p}{2}} \Bigg) / \\
& \left(a^2 \left(3 (a^2-b^2) \text{AppellF1} \left[\frac{1}{2}, -\frac{n p}{2}, 1, \frac{3}{2}, -\tan[e+f x]^2, \frac{b^2 \tan[e+f x]^2}{a^2-b^2} \right] + \right. \right. \\
& \quad \left. \left. \left(2 b^2 \text{AppellF1} \left[\frac{3}{2}, -\frac{n p}{2}, 2, \frac{5}{2}, -\tan[e+f x]^2, \frac{b^2 \tan[e+f x]^2}{a^2-b^2} \right] + \right. \right. \\
& \quad \left. \left. (a^2-b^2) n p \text{AppellF1} \left[\frac{3}{2}, 1-\frac{n p}{2}, 1, \frac{5}{2}, -\tan[e+f x]^2, \frac{b^2 \tan[e+f x]^2}{a^2-b^2} \right] \right) \right. \\
& \quad \left. \tan[e+f x]^2 \right) (-a^2 + b^2 (1 + \tan[e+f x]^2)) \Bigg) - \\
& \left(3 b^2 (a^2-b^2) n p \text{AppellF1} \left[\frac{1}{2}, -\frac{n p}{2}, 1, \frac{3}{2}, -\tan[e+f x]^2, \frac{b^2 \tan[e+f x]^2}{a^2-b^2} \right] \right. \\
& \quad \left. \sec[e+f x]^2 \tan[e+f x]^2 (1 + \tan[e+f x]^2)^{-1+\frac{n p}{2}} \right) / \\
& \left(a^2 \left(3 (a^2-b^2) \text{AppellF1} \left[\frac{1}{2}, -\frac{n p}{2}, 1, \frac{3}{2}, -\tan[e+f x]^2, \frac{b^2 \tan[e+f x]^2}{a^2-b^2} \right] + \right. \right. \\
& \quad \left. \left. \left(2 b^2 \text{AppellF1} \left[\frac{3}{2}, -\frac{n p}{2}, 2, \frac{5}{2}, -\tan[e+f x]^2, \frac{b^2 \tan[e+f x]^2}{a^2-b^2} \right] + \right. \right. \\
& \quad \left. \left. (a^2-b^2) n p \text{AppellF1} \left[\frac{3}{2}, 1-\frac{n p}{2}, 1, \frac{5}{2}, -\tan[e+f x]^2, \frac{b^2 \tan[e+f x]^2}{a^2-b^2} \right] \right) \right. \\
& \quad \left. \tan[e+f x]^2 \right) (-a^2 + b^2 (1 + \tan[e+f x]^2)) \Bigg) + \frac{1}{a^2} \sec[e+f x]^2 \\
& \left(-\text{Hypergeometric2F1} \left[\frac{1}{2}, 1-\frac{n p}{2}, \frac{3}{2}, -\tan[e+f x]^2 \right] + (1 + \tan[e+f x]^2)^{-1+\frac{n p}{2}} \right) - \frac{1}{a^3} 2 \\
& b \sec[e+f x]^2 \\
& \left(-\text{Hypergeometric2F1} \left[\frac{1}{2}, \frac{1}{2}-\frac{n p}{2}, \frac{3}{2}, -\tan[e+f x]^2 \right] + (1 + \tan[e+f x]^2)^{-\frac{1}{2}+\frac{n p}{2}} \right) - \\
& \left(6 b^3 (a^2-b^2) \text{AppellF1} \left[\frac{1}{2}, -\frac{1}{2}-\frac{n p}{2}, 1, \frac{3}{2}, -\tan[e+f x]^2, \frac{b^2 \tan[e+f x]^2}{a^2-b^2} \right] \right. \\
& \quad \left. \tan[e+f x] (1 + \tan[e+f x]^2)^{\frac{1}{2}(1+n p)} \right. \\
& \quad \left. \left(2 \left(2 b^2 \text{AppellF1} \left[\frac{3}{2}, -\frac{1}{2}-\frac{n p}{2}, 2, \frac{5}{2}, -\tan[e+f x]^2, \frac{b^2 \tan[e+f x]^2}{a^2-b^2} \right] + (a^2-b^2) (1+n p) \right. \right. \right. \\
& \quad \left. \left. \left. \tan[e+f x] (1 + \tan[e+f x]^2)^{\frac{1}{2}(1+n p)} \right) \right) \right)
\end{aligned}$$

$$\begin{aligned}
& p) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2} - \frac{n p}{2}, 1, \frac{5}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2}\right] \sec[e + f x]^2 \\
& \tan[e + f x] + 3(a^2 - b^2) \left(\frac{1}{3(a^2 - b^2)} 2b^2 \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{1}{2} - \frac{n p}{2}, 2, \frac{5}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2}\right] \sec[e + f x]^2 \tan[e + f x] - \frac{2}{3} \left(-\frac{1}{2} - \frac{n p}{2}\right) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2} - \right. \right. \\
& \left. \left. \frac{n p}{2}, 1, \frac{5}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2}\right] \sec[e + f x]^2 \tan[e + f x]\right) + \\
& \tan[e + f x]^2 \left(2b^2 \left(\frac{1}{5(a^2 - b^2)} 12b^2 \operatorname{AppellF1}\left[\frac{5}{2}, -\frac{1}{2} - \frac{n p}{2}, 3, \frac{7}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2}\right] \sec[e + f x]^2 \tan[e + f x] - \frac{6}{5} \left(-\frac{1}{2} - \frac{n p}{2}\right) \operatorname{AppellF1}\left[\frac{5}{2}, \right. \right. \right. \\
& \left. \left. \left. \frac{1}{2} - \frac{n p}{2}, 2, \frac{7}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2}\right] \sec[e + f x]^2 \tan[e + f x]\right) + \right. \\
& (a^2 - b^2)(1 + n p) \left(\frac{1}{5(a^2 - b^2)} 6b^2 \operatorname{AppellF1}\left[\frac{5}{2}, \frac{1}{2} - \frac{n p}{2}, 2, \frac{7}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2}\right] \sec[e + f x]^2 \tan[e + f x] - \frac{6}{5} \left(\frac{1}{2} - \frac{n p}{2}\right) \operatorname{AppellF1}\left[\frac{5}{2}, \frac{3}{2} - \right. \right. \\
& \left. \left. \frac{n p}{2}, 1, \frac{7}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2}\right] \sec[e + f x]^2 \tan[e + f x]\right) \Bigg) \Bigg) \Bigg) \\
& \left(a^3 \left(3(a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{1}{2} - \frac{n p}{2}, 1, \frac{3}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2}\right] + \right. \right. \\
& \left. \left. \left(2b^2 \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{1}{2} - \frac{n p}{2}, 2, \frac{5}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2}\right] + (a^2 - b^2) \right. \right. \right. \\
& \left. \left. \left. (1 + n p) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2} - \frac{n p}{2}, 1, \frac{5}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2}\right]\right) \right. \right. \\
& \left. \left. \tan[e + f x]^2\right)^2 (-a^2 + b^2 (1 + \tan[e + f x]^2))\right) + \\
& \left(6b^3 (a^2 - b^2) \operatorname{AppellF1}\left[\frac{1}{2}, -\frac{1}{2} - \frac{n p}{2}, 2, \frac{3}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2}\right] \right. \\
& \left. \tan[e + f x] (1 + \tan[e + f x]^2)^{\frac{1}{2}(1+n p)} \right. \\
& \left. \left(2 \left(4b^2 \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{1}{2} - \frac{n p}{2}, 3, \frac{5}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2}\right] + (a^2 - b^2) (1 + n p) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2} - \frac{n p}{2}, 2, \frac{5}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2}\right]\right) \sec[e + f x]^2 \right. \right. \\
& \left. \left. \tan[e + f x] + 3(a^2 - b^2) \left(\frac{1}{3(a^2 - b^2)} 4b^2 \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{1}{2} - \frac{n p}{2}, 3, \frac{5}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2}\right] \sec[e + f x]^2 \tan[e + f x] - \frac{2}{3} \left(-\frac{1}{2} - \frac{n p}{2}\right) \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2} - \right. \right. \right. \right. \\
& \left. \left. \left. \left. \frac{n p}{2}, 2, \frac{5}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2}\right] \sec[e + f x]^2 \tan[e + f x]\right) + \right. \right. \right. \right.
\end{aligned}$$

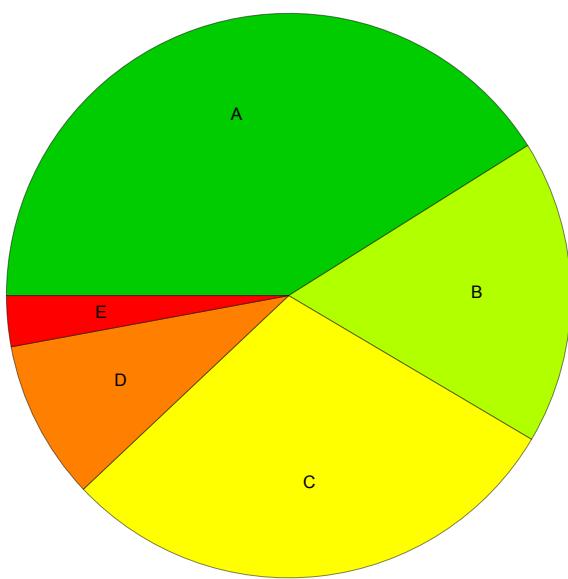
$$\begin{aligned}
& \left(\frac{1}{5(a^2 - b^2)} 12 b^2 \operatorname{AppellF1} \left[\frac{5}{2}, -\frac{1}{2} - \frac{n p}{2}, 4, \frac{7}{2}, -\tan[e + f x]^2, \right. \right. \\
& \left. \left. \frac{b^2 \tan[e + f x]^2}{a^2 - b^2} \right] \sec[e + f x]^2 \tan[e + f x] - \frac{6}{5} \left(-\frac{1}{2} - \frac{n p}{2} \right) \operatorname{AppellF1} \left[\frac{5}{2}, \right. \right. \\
& \left. \left. \frac{1}{2} - \frac{n p}{2}, 3, \frac{7}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2} \right] \sec[e + f x]^2 \tan[e + f x] \right) + \\
& (a^2 - b^2) (1 + n p) \left(\frac{1}{5(a^2 - b^2)} 12 b^2 \operatorname{AppellF1} \left[\frac{5}{2}, \frac{1}{2} - \frac{n p}{2}, 3, \frac{7}{2}, -\tan[e + f x]^2, \right. \right. \\
& \left. \left. \frac{b^2 \tan[e + f x]^2}{a^2 - b^2} \right] \sec[e + f x]^2 \tan[e + f x] - \frac{6}{5} \left(\frac{1}{2} - \frac{n p}{2} \right) \operatorname{AppellF1} \left[\frac{5}{2}, \frac{3}{2} - \right. \right. \\
& \left. \left. \frac{n p}{2}, 2, \frac{7}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2} \right] \sec[e + f x]^2 \tan[e + f x] \right) \Bigg) \Bigg) \Bigg) \\
& \left(a \left(3 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{2}, -\frac{1}{2} - \frac{n p}{2}, 2, \frac{3}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2} \right] + \right. \right. \\
& \left. \left. \left(4 b^2 \operatorname{AppellF1} \left[\frac{3}{2}, -\frac{1}{2} - \frac{n p}{2}, 3, \frac{5}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2} \right] + (a^2 - b^2) \right. \right. \\
& \left. \left. (1 + n p) \operatorname{AppellF1} \left[\frac{3}{2}, \frac{1}{2} - \frac{n p}{2}, 2, \frac{5}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2} \right] \right) \right. \\
& \left. \left. \tan[e + f x]^2 \right)^2 (a^2 - b^2 (1 + \tan[e + f x]^2))^2 \right) + \\
& \left(3 b^2 (a^2 - b^2) \operatorname{AppellF1} \left[\frac{1}{2}, -\frac{n p}{2}, 1, \frac{3}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2} \right] \right. \\
& \left. \left. \tan[e + f x] (1 + \tan[e + f x]^2)^{\frac{n p}{2}} \right. \right. \\
& \left. \left. \left(2 \left(2 b^2 \operatorname{AppellF1} \left[\frac{3}{2}, -\frac{n p}{2}, 2, \frac{5}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2} \right] + \right. \right. \right. \right. \\
& \left. \left. \left. \left. (a^2 - b^2) n p \operatorname{AppellF1} \left[\frac{3}{2}, 1 - \frac{n p}{2}, 1, \frac{5}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2} \right] \right) \right. \right. \\
& \left. \left. \left. \sec[e + f x]^2 \tan[e + f x] + 3 (a^2 - b^2) \left(\frac{1}{3(a^2 - b^2)} 2 b^2 \operatorname{AppellF1} \left[\frac{3}{2}, -\frac{n p}{2}, 2, \right. \right. \right. \right. \\
& \left. \left. \left. \left. \frac{5}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2} \right] \sec[e + f x]^2 \tan[e + f x] + \frac{1}{3} n p \operatorname{AppellF1} \left[\right. \right. \right. \right. \\
& \left. \left. \left. \left. \frac{3}{2}, 1 - \frac{n p}{2}, 1, \frac{5}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2} \right] \sec[e + f x]^2 \tan[e + f x] \right) + \right. \right. \\
& \left. \left. \left. \tan[e + f x]^2 \left(2 b^2 \left(\frac{1}{5(a^2 - b^2)} 12 b^2 \operatorname{AppellF1} \left[\frac{5}{2}, -\frac{n p}{2}, 3, \frac{7}{2}, -\tan[e + f x]^2, \right. \right. \right. \right. \right. \right. \\
& \left. \left. \left. \left. \left. \left. \frac{b^2 \tan[e + f x]^2}{a^2 - b^2} \right] \sec[e + f x]^2 \tan[e + f x] + \frac{3}{5} n p \operatorname{AppellF1} \left[\frac{5}{2}, 1 - \frac{n p}{2}, \right. \right. \right. \right. \right. \right. \\
& \left. \left. \left. \left. \left. \left. 2, \frac{7}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2} \right] \sec[e + f x]^2 \tan[e + f x] \right) + \right. \right. \right. \\
& \left. \left. \left. \left. (a^2 - b^2) n p \left(\frac{1}{5(a^2 - b^2)} 6 b^2 \operatorname{AppellF1} \left[\frac{5}{2}, 1 - \frac{n p}{2}, 2, \frac{7}{2}, -\tan[e + f x]^2, \right. \right. \right. \right. \right. \right. \\
& \left. \left. \left. \left. \left. \left. \frac{b^2 \tan[e + f x]^2}{a^2 - b^2} \right] \sec[e + f x]^2 \tan[e + f x] \right) \right) \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \frac{b^2 \tan[e+f x]^2}{a^2 - b^2} \] \sec[e+f x]^2 \tan[e+f x] - \frac{6}{5} \left(1 - \frac{n p}{2}\right) \text{AppellF1}\left[\frac{5}{2}, 2 - \frac{n p}{2}, \right. \\
& \left. 1, \frac{7}{2}, -\tan[e+f x]^2, \frac{b^2 \tan[e+f x]^2}{a^2 - b^2} \] \sec[e+f x]^2 \tan[e+f x]\right)\right)\Bigg) \\
& \left(a^2 \left(3 (a^2 - b^2) \text{AppellF1}\left[\frac{1}{2}, -\frac{n p}{2}, 1, \frac{3}{2}, -\tan[e+f x]^2, \frac{b^2 \tan[e+f x]^2}{a^2 - b^2}\right] + \right. \right. \\
& \left. \left(2 b^2 \text{AppellF1}\left[\frac{3}{2}, -\frac{n p}{2}, 2, \frac{5}{2}, -\tan[e+f x]^2, \frac{b^2 \tan[e+f x]^2}{a^2 - b^2}\right] + (a^2 - b^2) n p \right. \right. \\
& \left. \left.\text{AppellF1}\left[\frac{3}{2}, 1 - \frac{n p}{2}, 1, \frac{5}{2}, -\tan[e+f x]^2, \frac{b^2 \tan[e+f x]^2}{a^2 - b^2}\right]\right) \tan[e+f x]^2\right)^2 \\
& \left(-a^2 + b^2 (1 + \tan[e+f x]^2)\right) - \left(6 b^2 (a^2 - b^2) \text{AppellF1}\left[\frac{1}{2}, -\frac{n p}{2}, 2, \frac{3}{2}, \right. \right. \\
& \left. \left.-\tan[e+f x]^2, \frac{b^2 \tan[e+f x]^2}{a^2 - b^2}\right] \tan[e+f x] (1 + \tan[e+f x]^2)^{\frac{n p}{2}} \right. \\
& \left(2 \left(4 b^2 \text{AppellF1}\left[\frac{3}{2}, -\frac{n p}{2}, 3, \frac{5}{2}, -\tan[e+f x]^2, \frac{b^2 \tan[e+f x]^2}{a^2 - b^2}\right] + \right. \right. \\
& \left. \left.(a^2 - b^2) n p \text{AppellF1}\left[\frac{3}{2}, 1 - \frac{n p}{2}, 2, \frac{5}{2}, -\tan[e+f x]^2, \frac{b^2 \tan[e+f x]^2}{a^2 - b^2}\right]\right) \\
& \sec[e+f x]^2 \tan[e+f x] + 3 (a^2 - b^2) \left(\frac{1}{3 (a^2 - b^2)} 4 b^2 \text{AppellF1}\left[\frac{3}{2}, -\frac{n p}{2}, 3, \right. \right. \\
& \left. \left.\frac{5}{2}, -\tan[e+f x]^2, \frac{b^2 \tan[e+f x]^2}{a^2 - b^2}\right] \sec[e+f x]^2 \tan[e+f x] + \frac{1}{3} n p \text{AppellF1}\left[\right. \right. \\
& \left. \left.\frac{3}{2}, 1 - \frac{n p}{2}, 2, \frac{5}{2}, -\tan[e+f x]^2, \frac{b^2 \tan[e+f x]^2}{a^2 - b^2}\right] \sec[e+f x]^2 \tan[e+f x]\right) + \\
& \tan[e+f x]^2 \left(4 b^2 \left(\frac{1}{5 (a^2 - b^2)} 18 b^2 \text{AppellF1}\left[\frac{5}{2}, -\frac{n p}{2}, 4, \frac{7}{2}, -\tan[e+f x]^2, \right. \right. \right. \\
& \left. \left.\left.\frac{b^2 \tan[e+f x]^2}{a^2 - b^2}\right] \sec[e+f x]^2 \tan[e+f x] + \frac{3}{5} n p \text{AppellF1}\left[\frac{5}{2}, 1 - \frac{n p}{2}, \right. \right. \\
& \left. \left.3, \frac{7}{2}, -\tan[e+f x]^2, \frac{b^2 \tan[e+f x]^2}{a^2 - b^2}\right] \sec[e+f x]^2 \tan[e+f x]\right) + \\
& (a^2 - b^2) n p \left(\frac{1}{5 (a^2 - b^2)} 12 b^2 \text{AppellF1}\left[\frac{5}{2}, 1 - \frac{n p}{2}, 3, \frac{7}{2}, -\tan[e+f x]^2, \right. \right. \\
& \left. \left.\frac{b^2 \tan[e+f x]^2}{a^2 - b^2}\right] \sec[e+f x]^2 \tan[e+f x] - \frac{6}{5} \left(1 - \frac{n p}{2}\right) \text{AppellF1}\left[\frac{5}{2}, 2 - \frac{n p}{2}, \right. \right. \\
& \left. \left.2, \frac{7}{2}, -\tan[e+f x]^2, \frac{b^2 \tan[e+f x]^2}{a^2 - b^2}\right] \sec[e+f x]^2 \tan[e+f x]\right)\right)\Bigg) \\
& \left(\left(3 (a^2 - b^2) \text{AppellF1}\left[\frac{1}{2}, -\frac{n p}{2}, 2, \frac{3}{2}, -\tan[e+f x]^2, \frac{b^2 \tan[e+f x]^2}{a^2 - b^2}\right] + \right. \right. \\
& \left. \left(4 b^2 \text{AppellF1}\left[\frac{3}{2}, -\frac{n p}{2}, 3, \frac{5}{2}, -\tan[e+f x]^2, \frac{b^2 \tan[e+f x]^2}{a^2 - b^2}\right] + (a^2 - b^2)\right)
\end{aligned}$$

$$\begin{aligned} & n p \operatorname{AppellF1}\left[\frac{3}{2}, 1 - \frac{n p}{2}, 2, \frac{5}{2}, -\tan[e + f x]^2, \frac{b^2 \tan[e + f x]^2}{a^2 - b^2}\right] \\ & \tan[e + f x]^2\right)^2 \left(a^2 - b^2 \left(1 + \tan[e + f x]^2\right)\right)^2\right]\right) \end{aligned}$$

Summary of Integration Test Results

241 integration problems



A - 99 optimal antiderivatives

B - 42 more than twice size of optimal antiderivatives

C - 71 unnecessarily complex antiderivatives

D - 22 unable to integrate problems

E - 7 integration timeouts